DETERMINATION OF THE PPN PARAMETER γ WITH THE HIPPARCOS DATA*

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ABSTRACT

In the processing of the Hipparcos data, relativistic effects in the propagation of light, like the aberration and the bending of light rays, were introduced at an early level in the modelling. Thanks to the accumulation of very accurate measurements of star positions at various elongations from the Sun, it was possible to assess by how much the PPN parameter γ deviates from its General Relativity value. A series of tests has been implemented to determine this parameter and evaluate the magnitude of the possible sources of errors. From the results we can conclude that the coefficient γ is within 0.3 per cent of unity, with $\gamma = 0.997 \pm 0.003$. This value is not significantly different from unity, as predicted by General Relativity.

Keywords: Astrometry, General Relativity.

1. INTRODUCTION

The accuracy of the Hipparcos measurements implied that the relativistic effect in the light propagation had to be included in the modelling of the data reduction. This concerned the second order light aberration which is of the order of 1 mas (1 mas=0.001 arcsec) and the light deflection whose magnitude is $\sim$ 4.07 mas at right angles to the solar direction for an observer at one astronomical unit from the Sun. After a careful calibration of the instrument parameters, each abscissa on a reference great-circle (RGC) had a typical precision of 3 mas for a star of 8–9 mag. Each consortium has generated for the full mission about 3.5 million abscissae. Thus, ideally, if the reduction is without systematic errors and by neglecting all kind of correlations, an unknown angular parameter in the abscissa modelling common to all stars could be determined with a precision as good as:

$$\frac{3}{\sqrt{3.5 \times 10^6}} = 0.0016 \text{ mas}$$

Assuming that the deflection for all stars is of the form:

$$4 \times \frac{(1+\gamma)}{2} \text{ mas}$$

γ could be known to better than 0.001. Unfortunately, the correlation between the global parallax zero point and γ is very large (-0.92) and increases the expected formal error of γ by an estimated factor of about 2.5, so that it becomes 0.003. However, such a result would rank among the best determinations obtained by various means.

A fairly large number of determinations of γ have appeared in the past twenty years (Will 1981, 1984, 1987) involving both the bending of light, the Shapiro time delay in the solar system or the motion of the Moon.

Concerning the measurement of the bending in the visible from the Hipparcos data, it is simply a repetition (at large angles) of the celebrated 1919 expeditions to Brazil and to Principe Island (Dyson et al. 1920), which took advantage of a solar eclipse to observe stars with respect to each other in the solar neighbourhood. This was followed by a subsequent comparison to the same stellar field without the Sun. Since then, a total of nine solar eclipses have been used for light deflection measurements. In this kind of experiment, the error is very large and generally exceeds 10 per cent. In addition to the intrinsic difficulty in observing at small distances from the Sun, in the adverse conditions of an eclipse, the main source of uncertainty is the astrometry over two independent exposures of the same field with two different techniques, involving a delicate calibration of scaling factors.

The advent of very–long–base interferometry at radio wavelengths has produced greatly improved determinations of γ, thanks to the capability of measuring the distance between radio–sources as they pass very close to the Sun. Somewhat akin to Hipparcos in its principle, a recent global processing of the VLBI data used to monitor the Earth's rotation includes the determination of γ from observations carried out over almost the entire celestial sphere. This yields (Robertson & Carter 1984, Robertson et al. 1991):

$$\frac{(1+\gamma)}{2} = 1.000 \pm 0.001$$

*Based on observations made with the ESA Hipparcos satellite.

In all these observations, the refraction of the radio waves in the solar corona, which bends the ray more strongly than in the visible, limits the accuracy as one must model this additional bending to correct the data. The Hipparcos data retains the best of the two above methods: the optical wavelength where no corona effect is to be feared and the accuracy and full sky coverage of the VLBI technique.

In addition to its bending, a light signal takes a longer time to travel a given distance in a gravitational field than in the vacuum and the magnitude of this 'time delay' (Shapiro 1964) depends also on γ. The analysis by Reasenberg et al. (1979) of the radar time-delay using Mariner 9 and the Viking landers and orbiters has resulted in:

$$\frac{(1+\gamma)}{2} = 1.000 \pm 0.001$$

Despite the use of a dual-frequency (at 2.3 and 8.4 GHz), the propagation in the solar corona remains the major factor limiting the accuracy.

A global evaluation of the PPN parameters by Hellings (1984) has allowed the use of a large data sample of solar system observations: meridian transits of planets and the Moon, lunar laser ranging measurements, radar range measurements on Mercury, Venus and Mars. The dynamics of all these objects and the observations were modelled in the PPN formalism and the best fit yielded:

$$\frac{(1+\gamma)}{2} = 0.9994 \pm 0.0008$$

However the great heterogeneity of the observational material makes the assessment of the true uncertainty difficult.

From the measurement of the rate of change of the fringe delays of radiosources during a solar occultation, Lebach et al. (1995) have obtained:

$$\frac{(1+\gamma)}{2} = 0.9998 \pm 0.0008$$

where the quoted error includes the standard error of the fit and accounts for systematic errors brought about by the source structure and the propagation effects.

Very recently Williams et al. (1996) have made several determinations of PPN parameters from lunar laser ranging. The test of geodetic precession based on the rotation of the lunar orbit can be taken as a 1 per cent test of γ. In addition, a careful discussion of the different terms in the lunar motion yields eventually $\gamma = 1.000 \pm 0.005$ as the present LLR result.

With the Hipparcos data, the information related to the γ is unique in several respects:

- The measurements are carried out in the visible and are then free from the large bending of radio waves by the solar corona, which limits the accuracy of the astrometry by VLBI;
- All the observations are obtained at very large angular distance from the Sun, between 47 and 133 degrees. At first glance, this is a drawback, but it allows a better randomisation of the systematic errors thanks to the large coverage in elongation;
- This is the first optical measurement of γ performed without need for solar eclipses;
- All observations treated have been made with the same instrument, well calibrated all over the sky and over a period of 37 months.

2. THE LIGHT DEFLECTION MODEL

The general derivation of the equation of light-rays in the PPN formalism is given in Soffel (1989) and yields the measurable angle between two sources in a gravitational field as a function of the coordinates. Let us define the coordinate deflection $\delta\theta_1$ as the angle between the unperturbed and perturbed paths, by:

$$\delta\theta_1 = \frac{2GM_\odot}{rc^2} \times \frac{(1+\gamma)}{2} \times \frac{\sin\chi}{(1-\cos\chi)} \qquad (1)$$

where G is the gravitational constant, $M_\odot$ the solar mass, r the distance between the satellite and the center of the Sun and c the speed of light. The angle χ is the angular distance between the star and the Sun.

The effect of the light deflection on the RGC abscissa is given by:

$$\delta\varphi_1 = \delta\theta_1 \,\cos\psi$$

and

$$\delta\varphi_1 = \frac{2GM_\odot}{rc^2} \times \frac{(1+\gamma)}{2} \times \frac{\cos\varepsilon\,\sin\phi}{(1-\cos\varepsilon\,\cos\phi)} \qquad (2)$$

where ε is the latitude of the Sun with respect to the RGC and ϕ the abscissa difference on the RGC between the star and the Sun as shown in Figure 1. In the Hipparcos data processing a value of $\gamma = 1$ has been used consistently, so that the true unknown to be used in the analysis of the residuals is $\gamma - 1$. The geometry of the light deflection and that of the parallactic effect are very similar, differing in direction and in their dependence in χ, with $\delta\theta_2 = \pi \sin\chi$ for the parallactic effect and the expression of Equation 1 for the deflection. This difference proves essential in separating the two effects in the observation equations. However the separation is not perfect, despite the good coverage in elongation, and the two effects remain largely correlated.

3. OBSERVATION EQUATIONS

We have used all the star abscissa residuals resulting from the FAST and NDAC solutions, expressed in the Hipparcos catalogue system. For each measurement the calibrated standard errors and the correlation between the two nearly simultaneous measurements are available in the Hipparcos and Tycho Catalogues (ESA 1997, Volume 1, Section 2.8). In this data set, we have selected only the single star's

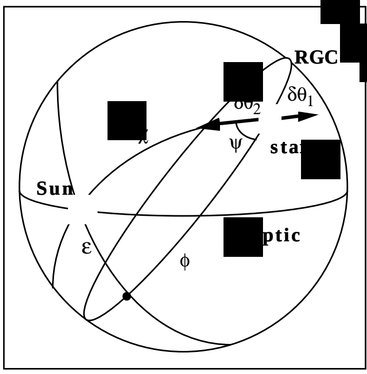

Figure 1. Geometry of the light deflection and stellar parallax. $\delta\theta_1$ stands for the light deflection and $\delta\theta_2$ for the parallactic effect. They differ in direction and in their analytical dependence in χ.

measurements, which are on the average of better quality than those of the double and multiple stars. The total number of abscissae used in this analysis is 5 429 065. The observation equations express the difference between the observed and computed abscissae in terms of the different unmodelled factors. Presently we have considered three kinds of terms:

- An error in the reference value of γ;
- A possible zero point of the parallaxes, common to all stars and unmodelled in the astrometric reduction;
- An error in the chromaticity value.

The corrected abscissa for each star i observed on the jth RGC is:

$$\Delta\varphi_{ij} = \sum_{k=1}^{5} \frac{\partial\varphi_{ij}}{\partial ,_k} d ,_k$$

with the partial derivatives for the unknown $,_1 = \gamma - 1$:

$$\frac{\partial\varphi_{ij}}{\partial ,_1} = 2.036 \frac{\cos\varepsilon_j \sin\phi_{ij}}{(1-\cos\varepsilon_j \cos\phi_{ij})} \quad \text{(mas)} \qquad (3)$$

$$\frac{\partial\varphi_{ij}}{\partial ,_2} = -\cos\varepsilon_j \sin\phi_{ij} \qquad (4)$$

for the global zero point error of the parallaxes:

$$\frac{\partial\varphi_{ij}}{\partial ,_3} = (V-I)_i - 0.5 \qquad (5)$$

$$\frac{\partial\varphi_{ij}}{\partial ,_4} = [(V-I)_i - 0.5]^2 \qquad (6)$$

and

$$\frac{\partial\varphi_{ij}}{\partial ,_5} = [(V-I)_i - 0.5]\,(T_j - T_0)$$

where $T_0 = 1991.25$ in the last of the three chromatic terms.

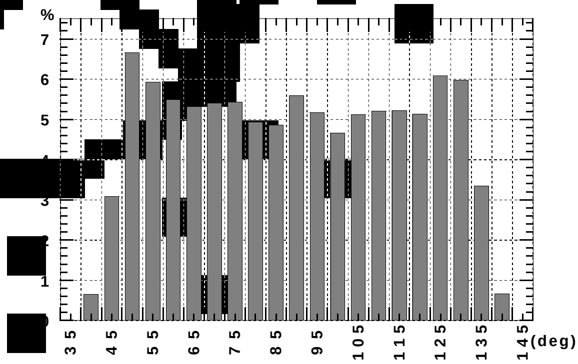

Figure 2. Distribution of the angular distance between the stars and the Sun.

4. DETERMINATION OF THE γ PARAMETER

We have designed several experiments with the data to assess the reliability of the solution through its stability according to the model fitted to the observations.

4.1. The Experiments

• **Experiment 1:** Half of the single stars were used and the parameters fitted were restricted to $\gamma - 1$ and the zero point parallax. The distribution of the stars is fairly uniform on the sky and that of the elongation with respect to the Sun is regular between the two extreme possible values of 47 and 133 degrees as shown in Figure 2. After the least squares we recover the expected correlation between γ and the zero point of the parallaxes.

• **Experiment 2:** In this experiment we have used the same data set, but three general parameters were considered and solved together with $\gamma - 1$. The solution gives in particular the correlations between the chromatic parameters and γ. These coefficients are indeed very small (0.003, 0.012, 0.000).

• **Experiment 3:** All the stars known to be non-problem stars in the data reduction were selected, which amounts to a sample of 87 382 out of the 117 955 in the catalogue. Only observations on even RGCs were retained in the least squares solution.

• **Experiment 4:** Same data set of stars as in experiment 3, but only observations on odd RGCs were used.

• **Experiment 5:** Same data set of stars as in experiment 3 with all the observations. The unknowns are $\gamma - 1$ and the zero point of the parallaxes

• **Experiment 6:** Same data set of stars and observations as in experiment 5. General parameters, zero point parallax and $\gamma - 1$ were fitted.

Table 1. Summary of the results. The eight columns give respectively: the number of the experiment, the number of stars in each experiment (in 10^3), the number of abscissae used (in 10^6), the number of independent parameters fitted to the data, the estimates of $\gamma - 1$ and σ and the values of the zero point of the parallaxes with their σ (in μas).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	44	2.7	2	-0.0049	0.0043	-11	11
2	44	2.7	5	-0.0052	0.0043	-11	11
3	87	2.7	5	-0.0017	0.0044	-12	11
4	87	2.7	5	-0.0053	0.0043	- 4	11
5	87	5.4	2	-0.0033	0.0031	- 7	8
6	87	5.4	5	-0.0035	0.0031	- 8	8

4.2. Results

The main parameters of each experiment together with $\gamma - 1$ and the formal errors are listed in table 1. It appears that in no case γ differs significantly from its General Relativity value. The formal errors depend primarily on the number of observations and on the correlations between the parallaxes and γ. As long as one can consider the formal error as representative of a true uncertainty, the Hipparcos based determination of γ is the best ever derived from the light deflection in the visible and the first to use observations made at wide angles from the Sun. The comparison of experiments differing only by the number of adjusted parameters (Experiments 1–2 and 5–6) shows that the results are rather insensitive to the introduction or neglect of general parameters which are weakly correlated with the parallaxes and γ, thanks to a wide coverage of the configurations. The formal error is significantly better than the results obtained separately by FAST and NDAC (ESA 1997, Volume 3) with their own sphere solution before the astrometric catalogue merging.

The Hipparcos solution is plotted in Figure 3 with the other determinations cited in the Introduction. Although the Hipparcos determination is not as accurate as the best determinations of γ based on the Shapiro effect, it constitutes a first full proof of the possibility of determining the space curvature from global astrometric measurements that could be performed in the future by a mission like GAIA. Moreover, the fact that we reach with Hipparcos a result which does not depart significantly from the GR value, may be taken as an additional indication of the absence of systematic bias in the Hipparcos data.

5. CONCLUSION

A determination of the parameter γ of the PPN formulation of the gravitation theory has been made by using all the astrometric observations carried out by Hipparcos. Various numerical experiments were designed in order to control the effect of hidden correlations on the results and the stability of the solution with varying data sets. The final result gives

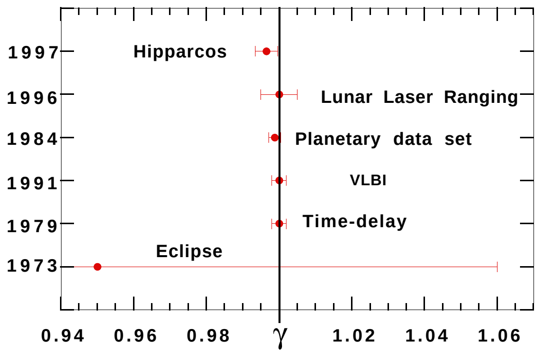

Figure 3. The best determinations of γ in each technique.

$\gamma = 0.997 \pm 0.003$. This is the best determination based on the light deflection in the visible and is comparable in accuracy to the measurement reported by Robertson & Carter (1984) from the VLBI observations of the deflection in radio wavelengths. It is still short by a factor three of the best estimates of γ, but should help spur future space astrometric missions like GAIA with an expected improvement by at least two orders of magnitude.

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