SOME CONSIDERATIONS IN MAKING FULL USE OF THE HIPPARCOS CATALOGUE

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ABSTRACT

This contribution is intended as a 'rough guide' to the Hipparcos Catalogue for the non-expert user. Some general aspects of the use of astrometric data are discussed as well as Hipparcos-specific applications. We discuss when and at what level one may expect systematic errors to occur in the Hipparcos Catalogue. Next we discuss the question of the interpretation of the measured parallaxes in terms of distances and luminosities of stars. What are the biases one should be aware of and how can these be corrected? When using the astrometric data to study the statistics of stars one should take the full covariance matrix of the errors on the astrometric parameters into account. We explain how to do this and discuss the specific case of a moving cluster. Finally, we address the question of the correlation of astrometric parameters over a given region of the sky. At present the Hipparcos Catalogue contains no identified systematic errors.

Key words: Space astrometry; Hipparcos; parallaxes; Luminosity calibration; statistics.

1. SYSTEMATIC ERRORS

As opposed to 'random error', the term 'systematic error' is generally understood to mean a statistical bias, i.e. that the error follows a distribution with mean value (or some other measure of location) different from zero. The application of this statistical concept to the Hipparcos Catalogue is far from trivial. To begin with, the Hipparcos Catalogue is unique and cannot be repeated. Is it then meaningful to speak of the bias of an individual data item in the catalogue? It probably is, as much as it is meaningful to speak of the standard error of a single datum: both depend on the notion that the observed value is 'drawn' from a population with a definite statistical distribution. In practice, however, the separation of random and systematic errors requires averaging, and the only averaging possible in our case is with respect to a sample of *different* stars. It is then necessary to assume that the stars in this sample share similar statistical properties.

Apart from these formal difficulties, the analysis of the Hipparcos Catalogue with respect to systematics faces a very severe practical problem. Systematic errors can generally only be revealed through comparison with independent data of at least similar quality. Very few such data exist and the tests that have been performed on the Hipparcos data are therefore limited in scope and precision. The results of several comparisons are summarized below; for a full description see Chapters 18 to 22 in Volume 3 of the Hipparcos Catalogue (ESA 1997).

The published catalogue is essentially the mean of the two separate reductions performed by the FAST and NDAC consortia. While a comparison of the two reductions does not prove anything about the systematic errors of the final catalogue, it gives considerable insight into the properties of the errors. Thus we may perhaps take the systematic FAST/NDAC differences (see Volume 3, Chapter 16) as an indication of what can be expected for the systematic errors in the Hipparcos Catalogue.

1.1. Position and Proper Motion

The positions and proper motions in the Hipparcos Catalogue formally refer to ICRS, the International Celestial Reference System replacing (although closely coinciding with) the 'equinox 2000' system. ICRS is defined by means of extra-galactic radio sources and great care was taken to link the Hipparcos Catalogue to this extra-galactic system (Kovalevsky 1997). The final uncertainty of the link corresponds to an orientation error of ± 0.6 milliarcsec (mas) for the system of positions at the epoch J1991.25, and to an error of ± 0.25 mas yr^{-1} for the global rotation of the proper motion system. For the epoch J2000 the uncertainty in the orientation of the Hipparcos positions with respect to ICRS will increase to $[0.6^2 + (8.75 \times 0.25)^2]^{1/2} \simeq \pm 2.3$ mas. The difference between the Hipparcos positions and proper motions (known as the Hipparcos reference frame) and the ICRS may be regarded as a systematic error of the catalogue. The uncertainties of the extra-galactic link quoted above are not included in the standard errors of the positions and proper motions of Hipparcos objects as given in the catalogue.

Other systematic errors in the positions and proper motions correspond to a distortion of the Hipparcos reference frame, and consequently affect e.g. the calculated angle between objects. Practically the only significant external check was achieved by means of the 12 radio stars observed by VLBI, yielding rms residuals of 1.7 mas in position (epoch J1991.25) and 0.8 mas yr^{-1} in proper motion. These are consistent with the formal standard errors (taking into account the known structure of two of the objects), indicating that the distortions of the Hipparcos reference frame are less than 1 mas and 0.5 mas yr^{-1}, respectively. Differences between the NDAC and FAST reductions suggest errors of a similar size on a very local scale (few degrees). Large-scale systematic differences are considerably smaller, e.g. < 0.1 mas or mas yr^{-1} on a scale of 90°.

1.2. Parallax

A global zero-point error in the Hipparcos parallaxes could in principle be produced by a specific harmonic of a systematic variation of the instrument with respect to the solar aspect angle. Such possible variations were guarded against in the satellite thermal design, and were carefully investigated during data reduction, leading to the conclusion that any global effect of this nature is probably less than 0.1 mas. *A priori* we thus expect the Hipparcos parallaxes to be absolute.

A comparison of Hipparcos parallaxes with the best ground-based optical parallaxes (88 stars; from the USNO 61-inch reflector) gives a median difference of $+0.2 \pm 0.35$ mas, suggesting the absence of systematic differences between the two techniques. Parallaxes of radio stars obtained by VLBI are also in very good agreement with Hipparcos. Comparison with other ground-based parallax programmes (see van Altena et al. 1995) shows systematic differences of up to several milliarcsec, especially for the southern sky; part of this may be related to the transformation from relative to absolute parallax in the ground-based programmes.

Using the photometric distances of open clusters more than 200 pc away, a parallax zero-point error of $+0.04 \pm 0.06$ mas was derived. For a sample of 467 field stars with $uvby\beta$ photometry, the statistical method of Arenou et al. (1995) gave a zero-point error of -0.05 ± 0.05 mas. From these comparisons the global zero-point error of the Hipparcos parallaxes is considered to be smaller than 0.1 mas. However, note that in general very red stars may exhibit various problems, including a possible zero point error. For details we refer to Chapters 20 and 21 of Volume 3 of the Hipparcos Catalogue.

1.3. Photometry

Although Hipparcos was not specifically designed for accurate photometry, the all-sky photometric survey in the Hp, B_T and V_T bands provides a data base of unprecedented homogeneity. No significant systematic errors are expected as a function of position. However, small non-linearities of the magnitude scales, partly due to a saturation effect in the Hipparcos measurements, are found through comparison with ground-based Johnson and Geneva photometry: for the Hp scale, a mean slope of -0.0017 mag mag^{-1} in the range $V = 3$ to 9 mag and departures up to 0.04 mag around $V = 0$; for B_T and V_T systematic deviations occur instead at the faint end as a result of statistical biases. For details refer to Chapter 21 of Volume 3 of the Hipparcos Catalogue.

The temporal stability of the magnitude scales is generally superb, permitting the detection of variability at the level of a few hundredths of a magnitude. However, radiation darkening of the optics caused a significant variation of the instrument passbands which had to be taken out in the photometric reductions. If the reduction was made with an erroneous $V - I$ colour index, this may have produced a spurious trend in the Hp magnitudes. The value of $V - I$ used for the reductions and a procedure for correcting any such trend if an improved $V - I$ becomes available, are given in the Hipparcos Catalogue (Volume 3, Chapter 14).

1.4. Outliers and External Accuracy

Related to the statistical distribution of the errors in the catalogue is the question of outliers (i.e. errors exceeding what can reasonably be expected of a Gaussian distribution) and external accuracy (i.e. the actual standard deviation of errors compared with the stated formal standard errors). A very small number of gross errors in position may exist, especially among the double-star components, as caused by grid-step errors (> 0.5 arcsec). The proper motions and parallaxes are generally less susceptible to this kind of error. For the proper motions one should however be aware that unresolved duplicity (astrometric binaries) may produce significant differences with respect to ground-based values (Lindegren 1997, Wielen 1997). For the parallaxes a similar effect can occur in the very rare case of an unrecognized binary with a period of about one year. In the epoch photometry, outliers occasionally occur, caused by satellite attitude errors (giving reduced flux) or parasitic stars from the complementary field of view (giving increased flux).

A meaningful check of the external accuracy has only been possible in the case of the parallaxes, through comparisons with photometric distances. These indicate that the external standard errors are about 1.05 ± 0.05 times larger than the standard errors given in the catalogue, at least for the brighter stars ($V < 9$ mag). From the general method by which the parallaxes were computed, it is reasonable to assume that the same factor applies to the standard errors in position and proper motion of single stars. The situation is much more complex for resolved double and multiple stars, but as a general rule it is believed that the errors are not underestimated by more than a factor 1.2.

2. CORRECT USE OF TRIGONOMETRIC PARALLAXES

Notwithstanding the unprecedented quality of the Hipparcos data, the correctness of the *astrophysical* results is not assured, as the estimation of stellar distances, absolute magnitudes and other physical quantities from trigonometric parallaxes is not a trivial process. The statistical properties of the relationships involved and the effects of sample selection hide several pitfalls that, if not avoided, lead to biased estimates.

We assume for this discussion that the Hipparcos parallaxes are unbiased, in the sense that their systematic errors are small compared to their random errors (see Section 1). Nevertheless, biases in the derived results may occur if an improper analysis of the data is done. In this section we present a brief review of the statistical properties of trigonometric parallaxes and derived quantities, as well as the effects of sample truncation(s). References given at the end of this paper may be consulted for the work done up to now on avoiding the various biases and making full use of the trigonometric parallaxes.

2.1. Selection Biases

A well-known selection bias is the Malmquist (1936) bias. In this case, a set of non-biased apparent magnitudes leads to a biased mean absolute magnitude due to the combination of the apparent magnitude limit of the sample and the intrinsic dispersion of absolute magnitudes (e.g., Luri et al. 1993). In statistical terms: the selection criteria make the mean absolute magnitude of the sample non-representative of that of the underlying parent population, thus introducing a bias, as faint stars are underrepresented. The use of the parallaxes of a truncated sample without caution may lead to similar biases in the derived results (see Luri, 1997, for some common examples).

Let us assume, for instance, that we want to check the systematic difference between Hipparcos ($\pi_{\rm H}$) and Tycho parallaxes ($\pi_{\rm T}$) and that for this purpose we select a sample containing only stars with $\pi_{\rm H} < 1$ mas. Computing the median difference $\pi_{\rm T} - \pi_{\rm H}$ on this sample results in 0.28 ± 0.01 mas, which suggests a significant systematic error in either the Tycho or Hipparcos parallaxes. However, this is only a selection bias due to the combination of the criterion $\pi_{\rm H} < 1$ mas, the non-uniformity of the parallax distribution and the random errors in $\pi_{\rm H}$ and $\pi_{\rm T}$. Indeed, the median difference $\pi_{\rm T} - \pi_{\rm H}$ without truncating the parallax distribution is not significantly different from zero. This example clearly illustrates how a truncation in the observed parallax distribution can introduce a bias in the sample so that, even if the individual parallaxes are not biased, the computed mean is biased. This example is based on truncation of the observed parallaxes, but the same argument applies to a selection based on the relative parallax errors $\sigma_{\pi_{\rm H}}/\pi_{\rm H}$.

Another type of bias is caused by an indirect truncation of the parallaxes. For instance, suppose that the spatial velocities of a given sample are computed and stars with high spatial velocity are selected. This will select stars with a truly high velocity but also stars with an overestimated distance ($\pi_{\rm H} \ll \pi$): the estimated distances of objects in this subsample will be biased in the mean. Consequently its estimated mean absolute magnitude will be too bright.

2.2. Biased Estimates

Several quantities, such as the stellar distance or the absolute magnitude, have a non-linear dependence, $h(\pi)$, on the parallax. In this case, the expectation value of the function, $E[h(\pi_{\rm H})]$, is in general different from $h(\pi)$, even if the individual Hipparcos parallaxes are unbiased, i.e. if $E[\pi_{\rm H}] \approx \pi$. In other words, $1/\pi_{\rm H}$ is a biased estimate of the star's true distance, and $m + 5\log\pi_{\rm H} + 5$ is a biased estimate of its absolute magnitude: $E[1/\pi_{\rm H}] \neq 1/\pi$ and $E[m + 5\log\pi_{\rm H} + 5] \neq m + 5\log\pi + 5$.

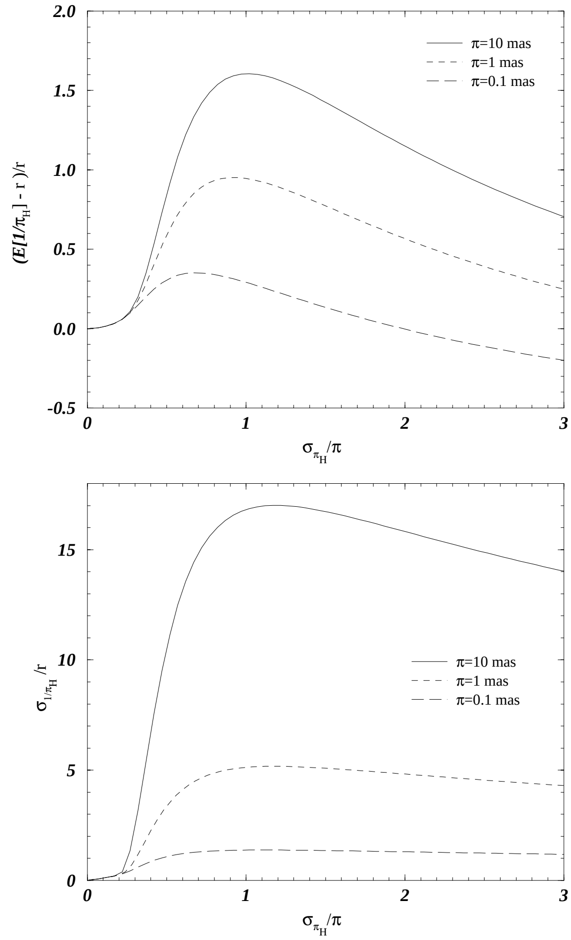

Figure 1. Relative bias (top) and relative precision (bottom) of computed distance as a function of the ratio of the parallax observational error to the true parallax.

To study this problem we will exclude negative and zero parallaxes, as will many users of the Hipparcos Catalogue. The lower bound used in the following calculations is 0.01 mas, which is the smallest nonzero value of parallax that can be found in the catalogue. In Figure 1 the value of the relative bias $(E[1/\pi_{\rm H}] - r)/r$ is shown as a function of the ratio

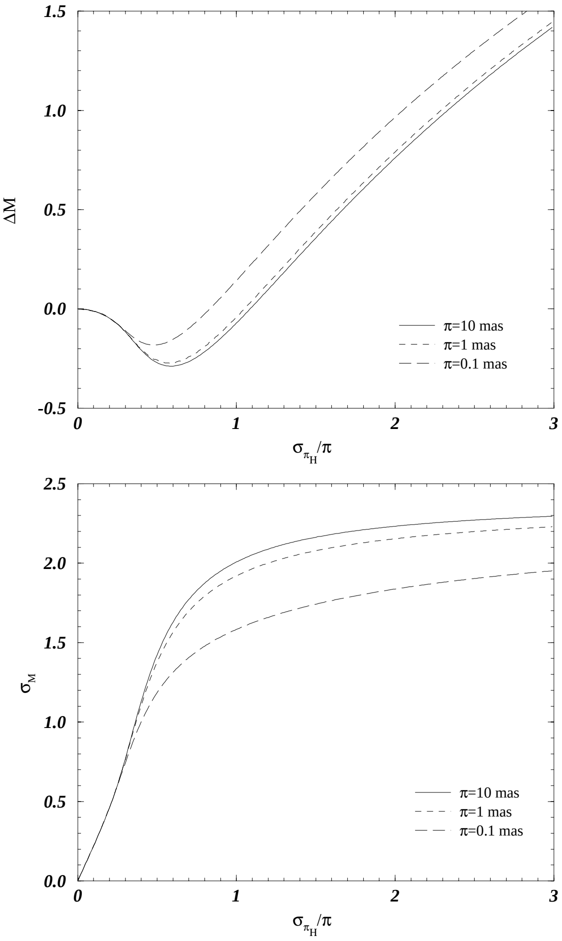

Figure 2. Bias (top) and precision (bottom) of computed absolute magnitude as a function of $\sigma_{\pi_{\rm H}}/\pi$.

of the observational error to the *true* parallax, for several values of this parallax. The computed distance may be overestimated by more than 100 per cent when the relative error in π is 100 per cent. Note that if negative parallaxes are not rejected, the bias, although reduced, is still present (Smith & Eichhorn 1996).

Since zero parallaxes have been rejected, the variance of $1/\pi_{\rm H}$ is not infinite and may be computed. The calculation is depicted in Figure 1. This is to be compared to the usual first order approximation $\sigma_r/r \approx \sigma_\pi/\pi$, which is valid to within ~ 25 per cent approximately up to a 20 per cent relative error.

Figure 1 shows that both bias and variance are negligible for relative errors better than about 10 per cent. For parallaxes with a higher relative error, one could naively hope to correct the computed distance from the bias shown above, but this is not possible because the bias is a function of $\sigma_{\pi_{\rm H}}/\pi$, where the *real* parallax π unknown. What is available is *not* the real relative parallax error but the observed one $\sigma_{\pi_{\rm H}}/\pi_{\rm H}$. Note that Figure 1 *also* indicates the relative bias of $\sigma_{\pi_{\rm H}}/\pi_{\rm H}$ as an estimate of $\sigma_{\pi_{\rm H}}/\pi$ and its relative precision. Given the uncertainty on the true relative error, a bias correction is simply not feasible. On the other hand, such a correction would only have a statistical meaning when applied to a sample, but would be questionable on an individual basis.

The situation is the same when considering absolute magnitudes (Figure 2). The absolute magnitudes computed from observed parallaxes are almost unbiased for small relative errors ($\sigma_{\pi_{\rm H}}/\pi \lesssim 0.1$), but are on the average 0.2 mag too bright when the relative parallax error is about 50 per cent, and 0.6 mag too faint for a 200 per cent relative error. Again, the correction for these biases would in principle require knowledge of the *true* parallaxes.

2.3. How to use Hipparcos Trigonometric Parallaxes

Various methods to use astrometric data with minimal biases have been proposed in the past and are summarized below, together with their positive and negative aspects:

- only stars with the best relative errors are kept. Keeping only stars with $\sigma_{\pi_{\rm H}}/\pi_{\rm H} < 10$ per cent means that more than 20 000 stars are still available. However, due to the implicit truncation of the parallax, a bias should still be expected;
- Smith & Eichhorn (1996) propose another estimator of distance, and absolute magnitude, based on a transformation of the observed parallax. Although the bias and variance of the new estimates are reduced, their physical meaning is questionable;
- models using all available information (photometry, position, proper motion) can be built in order to derive unbiased and precise estimates of physical data of interest: absolute magnitude, distance, kinematics (Ratnatunga & Casertano 1991, Luri et al. 1996, Arenou et al. 1995). The drawback is, of course, that the estimates found are model-dependent;
- finally, a recent approach using Hipparcos intermediate data has been proposed by van Leeuwen & Evans (1997), for the calibration of absolute magnitudes. Using no parametric model and all the available data, there remains however a correction to be done for magnitude-limited samples.

Summarizing, one can easily calculate the expected biases for a *given* true parallax. However, one only has the observed values, so the correction will depend on what kind of assumption one makes concerning the true values. In other words, the *distribution* of the true parallaxes has to be known, and this is an astrophysical question, not a statistical one! Hence, one cannot solve this problem just by statistics, but needs also some kind of modeling of the objects or sample under study. The reader is strongly encouraged to perform a detailed analysis of this sort *for each specific case* in order to obtain a correct estimation of any parameter of a star or a sample of stars using trigonometric parallaxes. This means in particular that one should neither ignore the possible biases nor apply blindly 'Malmquist' or 'Lutz-Kelker' corrections (Lutz & Kelker 1973).

3. USE OF THE COVARIANCE MATRIX

One of the unique features of the Hipparcos Catalogue is that not only the standard errors of the five astrometric parameters are provided but also their correlation coefficients. This allows the user to make full use of the information contained in the astrometric parameters. In the following we demonstrate briefly the use of the covariance matrix and we show the importance of using the matrix with a worked example. Here we concentrate on using the covariance matrix when interpreting the statistics of a particular data set. The covariance matrix is also necessary if one is interested in propagating the positions, proper motions and the corresponding standard errors within ICRS to an epoch different from the epoch (J1991.25) of the Hipparcos Catalogue. Propagation routines in C and Fortran are provided in the catalogue. For more information on the covariance matrix in relation to the astrometric parameters please refer to Sections 1.2 and 1.5 in Volume 1: Part 1 of the Hipparcos and Tycho Catalogues (ESA 1997).

If $\mathbf{x}$ is an observed vector with covariance matrix $\mathbf{C_x}$ then the confidence region around $\mathbf{x}$ is given by: $c = \mathbf{x'C_x^{-1}x}$, where the prime denotes matrix transposition. The distribution of c is described by a χ^2_ν probability distribution, where ν, the number of degrees of freedom, is equal to the dimension of $\mathbf{x}$. In the one-dimensional case this reduces to the well-known Gaussian distribution, where $c = 9$ corresponds to '3σ', the 99.73 per cent confidence level. For other values of ν the value of c will be higher for the same confidence level. It is 11.8 for $\nu = 2$ and 14.2 for $\nu = 3$. Note that the distribution of the errors around $\mathbf{x}$ is described by a multi-dimensional Gaussian and the equation above describes a confidence 'ellipsoid' around $\mathbf{x}$.

If the vector $\mathbf{y}$ is derived from $\mathbf{x}$ via some transformation $\mathbf{f}(\mathbf{x})$, the covariance matrix of $\mathbf{y}$ is: $\mathbf{C_y} = \mathbf{JC_xJ'}$. Here $\mathbf{J}$ is the Jacobian matrix of the transformation from $\mathbf{x}$ to $\mathbf{y}$: $[\mathbf{J}]_{ij} = \partial f_i/\partial x_j$. Thus one can calculate the covariance matrix of any set of variables derived from the observed astrometric parameters.

We now turn to the example of space velocities for cluster stars, specifically the Hyades. For the full details we refer the reader to Perryman et al. (1997). When deriving space velocities for cluster stars we make use of the observed vector $(\pi, \mu_{\alpha*}, \mu_\delta, V_\mathrm{R})$, where V_R is the radial velocity. This vector is transformed to a space velocity, implicitly invoking a transformation to $(V_{\alpha*}, V_\delta, V_\mathrm{R})$ ($V_{\alpha*} = \mu_{\alpha*}A_v/\pi, V_\delta = \mu_\delta A_v/\pi$, $A_v = 4.74047...$ km yr s^{-1}). To emphasize that using the covariance matrix is important even if the observed parameters are uncorrelated we shall proceed on the assumption that the astrometric errors are uncorrelated. Then the transformation of the observables to the vector $(\pi, V_{\alpha*}, V_\delta, V_\mathrm{R})$ yields the covariance matrix:

$$\begin{pmatrix} \mathbf{S} & \emptyset \\ \emptyset & \sigma^2_{V_\mathrm{R}} \end{pmatrix} \tag{1}$$

With $a = A_v/\pi^2$, $\mathbf{S}$ is given by:

$$\begin{pmatrix} \sigma^2_\pi & -\mu_{\alpha*}a\sigma^2_\pi & -\mu_\delta a\sigma^2_\pi \\ -\mu_{\alpha*}a\sigma^2_\pi & a^2\mu^2_{\alpha*}\sigma^2_\pi + A_v a\sigma^2_{\mu_{\alpha*}} & a^2\mu_{\alpha*}\mu_\delta\sigma^2_\pi \\ -\mu_\delta a\sigma^2_\pi & a^2\mu_{\alpha*}\mu_\delta\sigma^2_\pi & a^2\mu^2_\delta\sigma^2_\pi + A_v a\sigma^2_{\mu_\delta} \end{pmatrix}$$

Hence, even in the absence of correlations between astrometric errors, the parallaxes and velocity components $V_{\alpha*}$ and V_δ will in general be correlated. Moreover, because of the position of the convergent point of the Hyades with respect to the cluster centre, $\mu_{\alpha*}$ is positive and μ_δ is negative for most cluster members, and hence the product $\mu_{\alpha*}\mu_\delta$ is negative. Thus for the Hyades the correlated errors will lead to systematic behaviour of the uncertainties in the sample as a whole. These systematics will be transferred to the space velocities.

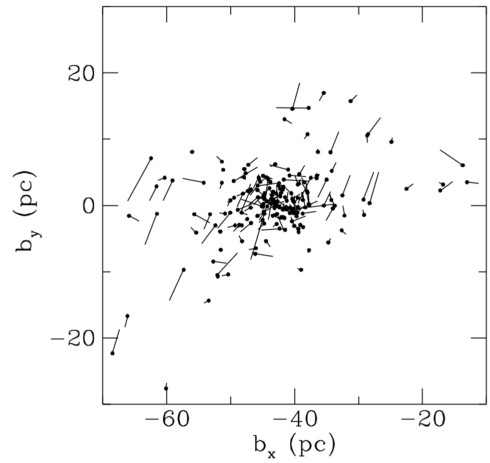

Figure 3. Projected velocities as a function of position for Hyades members. The residuals are given as vectors with respect to the mean velocity of the cluster in Galactic coordinates.

Figure 3 shows the velocity of the Hyades members with respect to the mean cluster motion plotted as vectors on the Galactic x-y plane. One immediately picks out a systematic motion, suggestive of rotation or shear. However, what one sees is a correlation between the velocity residuals (magnitude and direction) and the distances (parallaxes) of the stars. This can be understood as follows. The difference between the *observed* and *true* stellar parallaxes ($\Delta\pi = \pi_\mathrm{obs} - \pi_\mathrm{true}$) is not correlated with the true parallaxes. However, because all Hyades members have similar parallaxes, adding $\Delta\pi$ to π_true implies that, on average, the stars with positive $\Delta\pi$ will have the largest *observed* parallaxes (and vice versa for the stars with negative $\Delta\pi$). So the sign of the parallax error is correlated with the *observed* parallax. The correlation between $\Delta\pi$ and $V_{\alpha*}$ and V_δ, discussed above, will then lead to a correlation between the observed distances of the stars and the velocity residuals.

Figure 1 in Brown et al. (1997) shows how one can explain both the total spread and the correlations between velocity components by considering the covariance matrix of the observations. Hence, in the case of the Hyades both the overall distribution of the velocity residuals, as well as the correlation of the direction of the residuals with spatial position (the features in Figure 3), can be fully attributed to observational errors.

We stress here that ignoring the covariance matrix can easily lead to false interpretation of, for example, kinematic data. For cases other than the Hyades the way in which the features due to correlated errors enter may differ. It is important to carry out this kind of analysis and consider the implications for each case individually.

4. CORRELATION OF ASTROMETRIC PARAMETERS ON THE SKY

The Hipparcos data for stars concentrated in a small area of the sky have been derived from partly correlated observations (see Volume 3, Chapter 17 of ESA 1997). This means that proper motions and parallaxes of stars in open clusters or in the Magellanic Clouds, for example, cannot be interpreted as fully independent observations. For instance, the parallax errors of stars within a small ($< 2^\circ$) area of the sky in general have a positive statistical correlation ($\rho > 0$) because the stars were observed in more or less the same scans and part of their parallax errors derive from abscissa errors which were constant within each scan. Averaging the parallax errors of n stars in such an area will not quite produce the expected improvement by $n^{-1/2}$; in fact the error approaches (in principle) a certain limiting value as n is increased indefinitely, exactly as in the presence of a systematic error. Estimates suggest that the average of n stars improves as $n^{-0.35}$ for stars separated by less than about 2°.

The data from which the astrometric parameters have been derived have been preserved in the 'Hipparcos Intermediate Astrometric Data' file on Disc 5 of the ASCII CD-ROM set. Using those data, the correlations can be taken into account. Full details on this procedure, as well as more background information on the correlations in the Hipparcos observations are given by van Leeuwen & Evans (1997). The intermediate data also allow solutions in which information on astrometric parameters is linked within a selection of stars, such as stars in a cluster or stars sharing the same luminosity characteristics. In these cases, the individual parallax and/or proper motion solutions for the individual stars are replaced by the solution of a few common parameters for all stars involved, describing for example the parallax as a constant value (for a cluster) or as a function of photometric and spectroscopic parameters. For a specific example, where this is applied to the Pleiades, see van Leeuwen & Hansen-Ruiz (1997) and Mermilliod et al. (1997).

5. SELECTION AND COMPLETENESS

Finally, we want to end by emphasizing that in *any* quantitative study of a sample of stars it is *essential* to take *selection effects and completeness* into account. Unfortunately in the specific case of Hipparcos it is not at all trivial do so, even though a specific effort has been made to carry out part of the Hipparcos mission as a survey which is roughly complete to $V \sim 7$–8 mag. We will not discuss this issue here but refer the reader to Turon et al. (1992) and Volume II of ESA (1989), specifically Chapters 7 and 8. These references describe the details of the construction of the Hipparcos Input Catalogue and also go into the details of the catalogue completeness.

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