AN ALGORITHM FOR CALCULATING FRAME ROTATIONS AND ITS APPLICATION TO HIPPARCOS

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ABSTRACT

Given the extragalactic radio reference system, it is possible to extend this system to optical wavelengths by linking the provisional Hipparcos frame to the radio reference frame. Among the varying methods of frame connection we discuss here the usage of radio stars observed by the Very Large Array (VLA) relative to extragalactic radio sources. A general adjustment algorithm has been developed for the simultaneous determination of orientation angles and angular rates of rotation. From case studies we have derived the orientation angles of the provisional Hipparcos frame with respect to the VLA reference frame for radio star observations. Typical orientation angles about the three axes of the adopted equatorial coordinate system are −12.8, −19.1, +5.5 milliarcsec (mas), respectively. Evidence is shown that an offset exists between the VLA frame and the International Celestial Reference Frame (ICRF). Especially in the z-axis the offset is significant amounting to about 16 mas. From simulated data it is inferred that some 50 VLA observed radio stars are instrumental in linking optical and radio reference frames, with a precision of 0.5 mas/yr for the angular rates.

Key words: reference frames; radio stars; Hipparcos.

1. INTRODUCTION

On a global scale the internal precision of the provisional Hipparcos system is 0.1 mas in positions and annual proper motions, as may be inferred from Lindegren & Kovalevsky (1995). This system has been chosen so that it agrees globally with the position and proper motion system of the FK5. To take full advantage of the properties of the International Celestial Reference System (ICRS) it is necessary to link the Hipparcos frame to the ICRF, the International Celestial Reference Frame that is based on extragalactic radio sources. Apart from the quasar 3C273B the two frames are represented by objects whose intersection is empty. Therefore, linking of the frames requires observations of external objects that may be related to them. Various indirect methods for linking have been discussed (Lindegren & Kovalevsky (1995)), and it occured that the Hipparcos Catalogue (ESA 1997) is based on the synthesis of these methods in the sense of a weighted mean.

In principle, the link parameters are determinable from the positions and proper motions of a couple of radio-emitting Hipparcos programme stars whose positions and proper motions are also determined from radio observations with respect to the ICRF. So far some 10 Hipparcos stars observed by Very Long Baseline Interferometry (VLBI) proved to be useful link objects due to the high precisions in position and annual proper motion in the range of about 0.3 to 1.2 mas. Accordingly, the orientation of the final Hipparcos Catalogue may be dominated by this dozen of stars. Here, we investigate the employment of radio stars observed by the Very Large Array (VLA) for establishing the link. With reference to a former methodical paper (Walter et al. 1993) we present in the following sections an improved algorithm for the linking of frames and apply it to Hipparcos and VLA data of radio stars aiming at estimating the orientation parameters as well as the relationship between ICRF and the extragalactic frame used for the VLA observations. These results and simulations based on a realistic number of 50 stars strengthen the confidence in the suitability of the VLA stars to contribute to the link and its maintenance.

2. DESCRIPTION OF THE ALGORITHM

Assuming a linear model the observations l_i represented by the vector $\mathbf{l}$ interact with the vector $\mathbf{x}$ of unknowns by means of the matrix of coefficients $\mathbf{M}$, thus forming the observation equations $\mathbf{l} = \mathbf{Mx}$. Usually, to each observation a specific weight is allocated depending essentially on the *a priori* estimation of the observation error. All weights are arranged in a weight matrix $\mathbf{P}$ whose off-diagonal elements are zero if the observations are uncorrelated. Otherwise off-diagonal elements have also to be considered. The unknowns are estimated by standard least-squares adjustment, i.e. $\mathbf{M^T PMx} = \mathbf{M^T Pl}$. In the present case the unknowns $\mathbf{x}$ are the angles of orientation A_1, A_2, A_3 and the angular rates of rotation $\dot{A}_1$, $\dot{A}_2$, $\dot{A}_3$ denoting rotations from the Hipparcos frame to the VLA radio frame. Since both coordinate frames are nearly coincident the 'small-angle approximation' is applicable. It works without loss of accuracy if the relative angles of orientation are so small that each product of two of them can be neglected within the required accuracy. In our case, however, both coordinate frames already coincide to better than 0.1 arcseconds in all axes, and therefore the small-angle approximation is adequate. If, for example, any two angles of rotation are less than 2 arcseconds, equivalent to 1×10^{-5}, the small angle approximation is accurate to 10^{-10} or 0.02 mas.

For an individual star the observation equations are:

$$\begin{pmatrix} \Delta\alpha\cos\delta \\ \Delta\delta \\ \Delta\mu_\alpha\cos\delta \\ \Delta\mu_\delta \end{pmatrix} =$$

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} & 0 & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 & 0 & 0 \\ m_{31} & m_{32} & 0 & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & 0 & m_{44} & m_{45} & 0 \end{pmatrix} \cdot \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ \dot{A}_1 \\ \dot{A}_2 \\ \dot{A}_3 \end{pmatrix}$$

The 'observations' Δ denote differences between positions or proper motions referring to the VLA and Hipparcos frames. The weight matrix of the above observation equations is derived from the covariance matrices of the Hipparcos and VLA-observed parameters by adding them in accordance with the error propagation law. Due to correlations between Hipparcos positions and proper motions the resulting weight matrix shows off-diagonal elements different from zero. Currently no correlations are quoted for the VLA quantities.

The non-zero elements of the matrix of coefficients are:

$$\begin{aligned}
m_{11} &= +\sin\delta\,\cos\alpha \\
m_{12} &= +\sin\delta\,\sin\alpha \\
m_{13} &= -\cos\delta \\
m_{21} &= -\sin\alpha \\
m_{22} &= +\cos\alpha \\
m_{31} &= -\sin\delta\,\sin\alpha\;\mu_\alpha + \cos\alpha/\cos\delta\;\mu_\delta \\
m_{32} &= +\sin\delta\,\cos\alpha\;\mu_\alpha + \sin\alpha/\cos\delta\;\mu_\delta \\
m_{34} &= +\sin\delta\,\cos\alpha \\
m_{35} &= +\sin\delta\,\sin\alpha \\
m_{36} &= -\cos\delta \\
m_{41} &= -\cos\alpha\;\mu_\alpha \\
m_{42} &= -\sin\alpha\;\mu_\alpha \\
m_{44} &= -\sin\alpha \\
m_{45} &= +\cos\alpha
\end{aligned}$$

Each star provides four equations if angles of orientation as well as angular rates of rotation are to be determined.

3. THE INPUT DATA

A set of 46 VLA-observed radio stars available in the Hipparcos frame was chosen for the present investigation. For a part of it radio positions had been determined from single epoch observations during 1982 to 1989 by the United States Naval Observatory (USNO) (see Florkowski et al. 1985). Published data are compiled in the radio star catalogue (Walter et al. 1997) which serves as our data base for radio positions and their mean errors. Each position component has a mean error of 20–30 mas.

Epoch transformations to the Hipparcos Catalogue epoch J1991.25 are performed by using Hipparcos proper motions since VLA proper motions are presently not available. In view of the mean errors given in the Hipparcos Catalogue (ESA 1997) it follows that the covariance matrix and consequently the weight matrix are largely dominated by radio position errors. Mean errors of Hipparcos positions contribute infinitely small, and those of proper motions add less than 10 mas to the radio position error at epoch J1991.25, due to the epoch transformation covering 3 to 9 years.

Not all 46 stars are suitable candidates for determining link parameters. A critical examination is required, above all with respect to multiple stars. Further discrepancies may be caused by variable stars or observational errors. Obvious troublesome candidates can be found by checking post-fit residuals. Additional information is provided by computing spherical arcs between pairs of stars. This is done separately in the radio and optical domain. A star is rejected on statistical grounds using the differences of the optical and radio arclengths between the star under consideration and the remaining stars of the set. If the differences exceed the expected error by 50 per cent in the majority of combinations, the star is classified unsuitable for the link process. Eventually, some 20 stars appear to be appropriate for the link. More stars may become eligible after modelling of the radio-optical offsets.

4. RESULTS

In applying the algorithm to the optical positions of the Hipparcos Catalogue (ESA 1997) and to VLA positions taken from Walter et al. (1997) we find for a set of 23 stars the following angles of rotation from the provisional Hipparcos reference frame (CERGA 1996) to the USNO extragalactic frame at J1991.25:

$$\begin{aligned}
A_1 &= -12.8 \pm 5.4 \text{ mas} \\
A_2 &= -19.1 \pm 5.9 \text{ mas} \\
A_3 &= +\ 5.5 \pm 6.2 \text{ mas}
\end{aligned}$$

Using the same set of data one gets for the rotation angles from the final Hipparcos frame (ICRF) to the USNO frame at J1991.25:

$$\begin{aligned}
A_1 &= +\ 3.1 \pm 4.5 \text{ mas} \\
A_2 &= -\ 9.1 \pm 5.2 \text{ mas} \\
A_3 &= -15.7 \pm 5.4 \text{ mas}
\end{aligned}$$

Note that the USNO frame referred to here reflects the state of the art of the early eighties when it was adopted for the reduction of VLA observations of radio stars relative to calibrators represented by quasars (Florkowski et al. 1985).

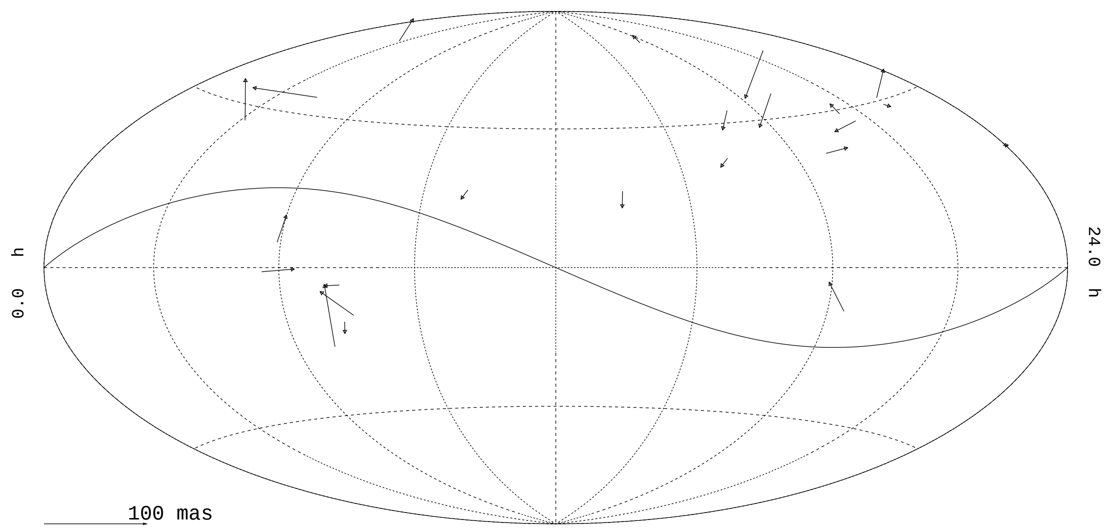

Figure 1. Vectorial residuals pointing from the Hipparcos frame to the USNO radio frame. The Hammer projection is subdivided in areas of 45°×45°; the ecliptic plane is represented by the sinusoidal line.

The post-fit residuals VLA minus Hipparcos are plotted in Figure 1 showing that the link procedure is dominated by the northern hemisphere. Systematic effects in lengths and directions of the difference vectors cannot be inferred.

The combination of the two aforementioned rotations yields the angles $A_1 = -15.9$ mas, $A_2 = -10.0$ mas and $A_3 = 21.2$ mas defining the rotation of the provisional Hipparcos frame into the ICRF provided that the angles of the second rotation can be confirmed without reference to optical positions of radio stars in the ICRF. In other words, the determination of the link using VLA star positions, which refer to the ICRF, would provide the above angles with precisions of about 5 mas in each direction. This demonstrates the suitability of VLA positions and proper motions for the establishment of the link to the ICRF.

On comparing the above triples of rotation angles, differences of the errors are noticeable. Evidently a change of one or other Hipparcos position has taken place at the transition from the provisional to the final frame. In the last case one would expect no significant deviation from zero on the assumption that the Hipparcos frame represented by ICRF coincides with the USNO frame. This is demonstrated for the x- and y-axes showing insignificance within a 1- and 2-σ-level. However, the angle of orientation around the z-axis indicates that the two reference frames do not really coincide. It will be useful to see how different sets of stars affect the angles of orientation. For example, a group consisting of only 10 stars out of the 23 gives the results listed below:

$$
\begin{aligned}
A_1 &= +0.8 \pm 9.1 \text{ mas} \\
A_2 &= +0.5 \pm 8.5 \text{ mas} \\
A_3 &= -1.9 \pm 10.0 \text{ mas}
\end{aligned}
$$

All angles are now far from being significant, but due to the smaller number of objects the mean errors have grown. Other sets have been analysed. They underline the general tendency that the angle of orientation around the z-axis becomes more significant with increasing star numbers.

On the other hand, the suspected relation identified above between the USNO radio frame, to which the VLA radio positions refer, and the ICRF is not of prime importance for us; it is the order of magnitude of the mean errors of the angles of orientation which assess the accuracy of the linking procedure. The values obtained so far may have a bias because of the usage of Hipparcos proper motions for epoch transformations. It remains also to be proven that the radio centres of emission coincide with their optical counterparts within negligible limits.

5. PROSPECTS

At the present stage VLA-observed radio stars do not offer an alternative to the determination of the link. This situation probably improves within the next years as soon as the number of VLA-observed radio stars has increased and positions are supplemented by consistent proper motions. A numerical simulation using 50 radio stars shows a decrease of the statistical error estimates to better than 4 mas for the angles of orientation, and to 0.5 mas/yr for the angular rates of rotation.

6. IN CONCLUSION

The detected but not yet fully confirmed difference between the ICRF and the USNO extragalactic frame used for calibration of the VLA observations suggests a re-reduction of the VLA measurements. A sample of 50 to 100 VLA-observed radio stars seems to be instrumental in linking firmly optical and radio reference frames.

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