DETERMINATION OF LIMB-DARKENING FROM HIPPARCOS OBSERVATIONS

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ABSTRACT

We show in this paper how observations of an extended stellar or planetary object carried out with a periodic grid may also yield valuable information on its size and on the light distribution over its surface. We discuss the overall principles of this method and derive an analytical formulation for the 'modulation function' defined in the text. We present the results obtained from Hipparcos main grid observations of minor planet (1) Ceres and planetary satellite S6-Titan, i.e. two of the largest bodies observed through the 1.2074 arcsec periodic main grid. The limb-darkening or Minnaert parameter of (1) Ceres is found to be $k = 0.61$, and for Titan $k = 0.92$.

Key words: Photometry; asteroids; scattering.

1. INTRODUCTION

Asteroids are objects of particular interest for the study of the origin of our solar system. Parameters such as their mass, spin axis orientation and shape are still barely known. A more comprehensive determination of their diameters was made thanks to the IRAS survey. Radiometric diameters however remains restricted to the IR domain, and comparable results in the visible (e.g. occultations, speckle) remain limited in number.

The image provided by the astrometric Hipparcos satellite was modulated by a periodic grid yielding in the end the astrometric and photometric information. The method proved very efficient despite the very modest aperture of the on-board telescope, less than 30 cm in diameter. In the case of Hipparcos it has been shown conclusively that the use of a particular combination of the amplitudes of the modulated signal, used primarily as an estimator of the magnitude of point sources, was also a powerful means for detecting binary stars and determining their astrometric parameters (Mignard et al. 1995).

The modulated signal of an extended object seen through a periodic grid departs from the signal of a point source. This difference is essentially influenced by the apparent size of the object and its surface brightness distribution (Morando 1987, Lindegren 1987, Morando & Lindegren 1989, Hestroffer et al. 1995). We give in the first section the analytical formulation of the 'modulation function' for a spherical object with surface scattering properties following the empirical law of Minnaert. Applications to the Hipparcos observations of minor planet (1) Ceres and Saturnian satellite S6-Titan are given by relating the modulation function to the two magnitudes $H_{\rm dc}$ and $H_{\rm ac}$. This enables us to investigate the limb-darkening of these objects.

2. MODULATION FUNCTION

During the crossing of the slits, the light of a point source is modulated in a regular way. Hence the photometer records a periodic signal of frequency ω, which can be expanded in a Fourier series as:

$$S(t) = I + \sum_{m>0} I\, M_m^o\, \cos\left[m\,(\omega\, t + \varphi_m)\right] \qquad (1)$$

where I is the total intensity and M_m^o and φ_m respectively the modulation coefficients and the phase offset of the m-th harmonic. The modulation coefficients M_m^o, for a point-like source, are linked to the instrument transfer function and thus can be calibrated with natural or artificial point sources. Moreover the phases φ_m of a point source do not depend on the harmonic rank and are all equal to $\varphi_1 = \varphi$.

Similarly, an extended source can be viewed as a continuum of closely packed sources and the resulting signal has amplitudes and phases depending on the source apparent size and its surface scattering properties or albedo distribution. Writing Equation 1 for each surface element $d\sigma$ of intensity I_σ and phase φ_σ regardless of the harmonic, we have (Hestroffer & Mignard 1997):

$$\begin{aligned} S(t) &= \iint_{\mathcal{D}} \Big(I_\sigma + \sum_{m>0} I_\sigma\, M_m^o\, \cos\left[m\,(\omega\, t + \varphi_\sigma)\right] \Big)\, d\sigma \\ &= I + \sum_{m>0} I\, M_m\, \cos\left[m\,(\omega\, t + \phi_m)\right] \qquad (2) \end{aligned}$$

where the integration holds over the illuminated part of the body visible from the telescope. The modulation function, for any harmonic m, is the actual modulation coefficient scaled by the reference value, that is to say M_m/M_m^o.

The intensity I_σ is directly related to the light distribution over the surface. The phase offset φ_σ depends on the size of the object and the geometry of its projection in the scanning direction. It can be written as a function of the spatial frequency $x = \pi\rho/s$, where ρ is the apparent diameter (for a spherical object) and s the grid period.

We now consider the simple case of geometric scattering given by the empirical law of Minnaert. Then the brightness distribution, normalised to the centre, is:

$$I_\sigma = \mu_o^k \mu^{k-1} \tag{3}$$

where μ_o and μ are the cosines of the angles between the surface normal and respectively the incident and reflected ray. Assuming the body to be spherical and the brightness distribution with azimuthal symmetry, (i.e. $\mu_o = \mu$), the modulation function is:

$$M_m/M_m^o = |\,{}_0F_1(k + 3/2;\, -(m\,x)^2/4)\,| \tag{4}$$

where ${}_0F_1$ is the hypergeometric function.

3. APPLICATION TO HIPPARCOS

The Hipparcos observations carried out over more than 3 years provide good opportunities to apply this method. With a grid period of $s = 1.2074$ arcsec only few solar system objects are resolved by the instrument. The photometric reduction process provides two magnitude estimators $H_{\rm dc}$ and $H_{\rm ac}$, where the difference or magnitude bias is:

$$\Delta H = H_{\rm ac} - H_{\rm dc} = -2.5 \log \frac{M_1\, M_1^o + M_2\, M_2^o}{(M_1^o)^2 + (M_2^o)^2} \tag{5}$$

For moderately large values of ρ ($\lesssim$ 0.7 arcsec) we have the first order approximation $\Delta H \sim a\,\rho^2$ where a depends on the limb darkening parameter k.

Since the Hipparcos observations do not occur at the opposition but at large solar phase angles, the above formulation has to be corrected for the phase effect (no radial symmetry). This is preferably done by numerical integration for each normal point, i.e. an average point over successive transits. The corrective terms are small and depend on the limb darkening, the phase angle, and the scanning direction. The resulting observational data is listed in Table 1.

Next we fit to the data a model where the two unknowns are the limb darkening parameter k and the diameter D. Unfortunately, as shown in Figure 1 for Ceres, the data points are not well distributed over the spatial frequency domain to allow a separate determination of both parameters. We have then fixed the diameter and searched the best value of the Minnaert parameter by minimising the absolute value of the residuals (L1 norm).

In the case of the Saturnian satellite, the magnitude $H_{\rm dc}$ is not measured but calculated from:

$$H_{\rm dc} = V(1,0) + 5 \log r\Delta + 0.004\,\alpha + 0.161 \tag{6}$$

Table 1. Observational data for Ceres and Titan. For each normal point, the epoch, the magnitude bias ΔH and the solar phase angle α are given.

Epoch JD−2.44 10^6	ΔH	α [deg]
(1) Ceres		
7920.72119	0.509	16.09
7928.77734	0.500	18.17
7963.72266	0.327	22.41
7994.31299	0.256	21.47
8013.15430	0.282	19.58
8275.92188	0.273	22.38
8298.88477	0.401	21.08
8426.65918	0.411	20.36
8476.77539	0.199	21.27
8516.54297	0.202	16.58
8522.87402	0.163	15.58
8709.31543	0.195	18.65
8732.37598	0.239	20.10
8933.23438	0.184	19.15
8970.77637	0.151	15.23
8977.54492	0.142	14.24
S6-Titan		
8144.53076	0.698	4.91
8163.70117	0.638	5.61
8191.24561	0.612	5.56
8393.44629	0.655	5.49
8410.64844	0.708	4.74
8539.43652	0.661	5.60
8566.58008	0.605	5.61
8712.86816	0.617	4.71
8760.00781	0.616	5.77
8935.95020	0.660	5.76

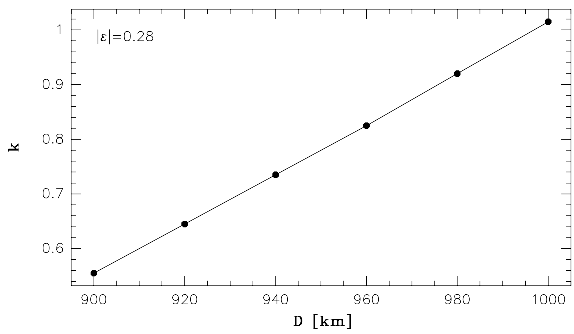

Figure 1. Value of Minnaert parameter k versus diameter D for (1) Ceres. Every pair of solutions yields the same L1 norm of the residuals $|\varepsilon|$. Thus only a single parameter can be determined.

where α is in degrees. The absolute magnitude $V(1,0) = -1.28$ taken from the Astronomical Almanac is in agreement with the observed value of Karkoschka (1994). The last term in the right-hand side accounts for the transformation between the Hipparcos Hp photometric system and the Johnson system based on a colour index $B - V = 1.28$.

3.1. Ceres

The largest minor planet (1) Ceres was frequently observed during the mission with an apparent diameter ranging from ~ 0.35 to ~ 0.7 arcsec. One must keep in mind that all the observations occurred at rather large phase angles, and consequently the Minnaert parameter is obtained for a mean value $\langle\alpha\rangle = 18°$, and should correspond to a larger limb-darkening than at full phase (French & Veverka 1983). Taking the diameter $D = 913$ km from Tedesco (1989), we find the best fit to the magnitude bias for a Minnaert parameter $k = 0.61$ (see Figures 1 and 2). Ceres is an almost uniformly bright object, which is consistent with the results of Lumme & Bowell (1981) from the analysis of the magnitude–phase relation, and also expected for a low albedo asteroid. On the other hand there is no contradiction with the stellar occultation results of Millis et al. (1989), who found a larger effective diameter, or with the results obtained from adaptive optics observations (Drummond & Christou 1994, Saint-Pé et al. 1993). Assuming that Ceres is not spherical and of larger size, we can only find a tentative fit yielding a slightly more pronounced limb-darkening. A more refined analysis will take into account the actual shape and scanning direction during each transit.

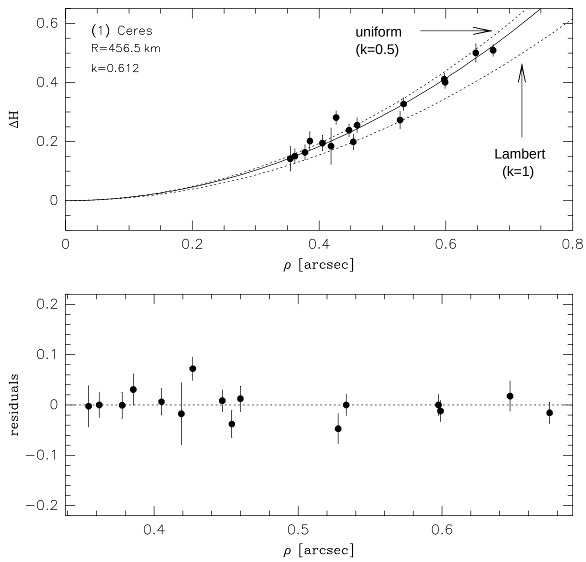

Figure 2. Magnitude bias ΔH for Hipparcos observations of (1) Ceres with a radius of 456.5 km. Each point is a mean of successive transits. The dotted lines correspond to the theoretical curves for a uniformly bright object and a scattering following Lambert's law. The solid line is the best Minnaert law fit; the corresponding residuals are given on the lower panel.

3.2. Titan

In contrast to Ceres, Titan has formed and retained a major atmosphere. It is thus expected to show a pronounced centre-to-limb darkening. The atmosphere is very opaque over a wide range of wavelengths and there is no significant brightness variation with orbital phase in the visible (Noland et al. 1974). Taking the diameter $D = 5720$ km from Smith et al. (1981) and Smith (1980), i.e. including the non-resolved higher haze layer, we find the Minnaert parameter $k = 0.92$ (see Figure 3), in good agreement with the (filters averaged) Pioneer results of Smith (1980) obtained at similar phase angle. It is stressed that the lower values $k = 0.83$ found by Sromovsky et al. (1981), and $k = 0.81$ found by Tomasko & Smith (1982) are not contradictory since they have been obtained at much larger phase angles ($\alpha > 28°$). Our result is also consistent with those obtained from the Lunar occultation technique (Elliot et al. 1975) and speckle interferometry (Nisenson et al. 1986).

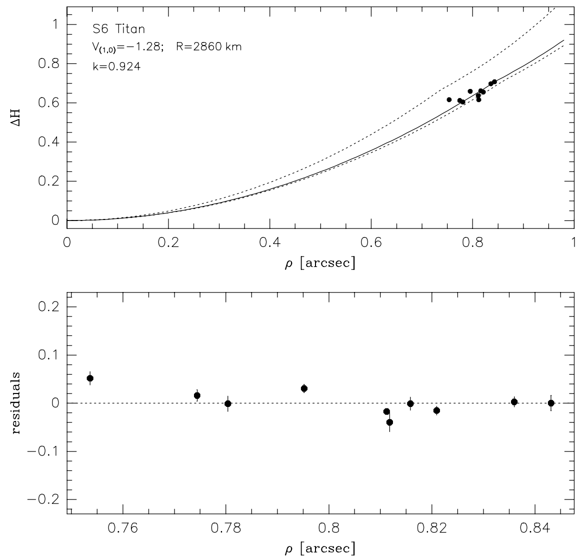

Figure 3. Same as Figure 2 for S6-Titan with a radius of 2860 km.

4. CONCLUSION

Photometric measurements, carried out with a periodic grid at the focal plane of a telescope, allows determination of the brightness properties of a celestial body. Application to Hipparcos observations of the minor planet (1) Ceres and the saturnian satellite Titan shows the validity of this method. Adopting the radius of these two objects, we have derived the Minnaert parameters $k(\langle\alpha\rangle{\sim}18°) = 0.61$ for Ceres, and $k(\langle\alpha\rangle{\sim}5.5°) = 0.92$ for Titan. However the Hipparcos scanning law was not optimal to perform this kind of observations. Also observations achieved at various grid steps should be preferred. Application of this method to ground-based observations, e.g. Multichannel Astrometric Photometer (Gatewood 1987), would constitute an important test of the above principles.

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