APPLICATION POSSIBILITIES FOR THE HIPPARCOS INTERMEDIATE ASTROMETRIC DATA

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ABSTRACT

Examples of how and when to use the Hipparcos intermediate astrometric data are presented. It is shown how, through the use of the intermediate astrometric data, correlation problems can be properly accounted for. Applications are in general limited to three kinds: calibrations of a common parallax and proper motion for stars in clusters, calibrations of common luminosity properties for stars distributed over the entire sky, and the determination of orbital motion caused by an unseen companion. In the first case, the main advantage is the possibility to eliminate correlations between astrometric parameters and the possibility to incorporate ground-based observations. In the second case, the advantage is in the reduction of degrees of freedom and the homogeneity in which all original observations are incorporated in one solution. In both cases a much improved distinction of faulty measurements is possible. The analysis of possible orbital motions and the detection of unseen companions is not treated here.

Key words: space astrometry; star clusters; luminosity calibrations; intermediate astrometric data.

1. INTRODUCTION

The Hipparcos astrometric data may seem at first glance to be 'just' a catalogue of positions, proper motions and parallaxes, ready to use by any astronomer as required. There is, however, much more to it than that. First of all, a proper understanding of the astrometric data and the way is has been obtained is needed whenever using parallax values close to the detection limits. In particular for disturbed objects (multiple stars, stars embedded in bright and irregular nebulae etc) considerable care has to be taken in how to interpret and use a Hipparcos determination of astrometric parameters. Often significant correlations exist between the determinations of parallax, proper motion and position, and these correlations will have to be accounted for when using combined proper motion and parallax information for groups of stars.

The Hipparcos astrometric data for single stars have been derived from one-dimensional abscissa measurements or intermediate astrometric data, which have been preserved and were made available on the ASCII CD-ROM (disc 5, files `hip_i.dat` and `hip_rgc.dat`). The intermediate astrometric data data can be used to reconstruct any single star solution given in the Hipparcos catalogue. They can, equally well, be used in alternative solutions, such as solving for one common proper motion for all stars in the Magellanic Clouds.

The great circle reduction process obtained abscissae along a reference great circle, using the phase estimates from the analysis of the main detector photon counts, and calibrating at the same time the attitude along the scanning circle and the instrument parameters. This process introduced an inevitable correlation between the errors on abscissa measurements obtained on the same reference great circle. These correlations are most pronounced at angular distances near to $n \times 58°$, $n = 0, 1, 2, 3, \ldots$ The character of these correlations is further explained in van Leeuwen & Evans (1997). They are the strongest for stars at small separations (less than 3° to 4°) on a great circle, where they can amount to 20 to 40 per cent. An example of these correlations is shown in Figure 1. When examining a small area of the sky, one has to take into account an accumulation of these correlations in the derived astrometric parameters, which themselves become correlated. Therefore, when examining the parallaxes or proper motions of stars in e.g. an open cluster, such correlations have to be accounted for.

A secure way to account for the correlations is through the use of the intermediate astrometric data. They can either be used to derive a set of uncorrelated astrometric parameters for individual stars, or to eliminate correlations in a combined solution for a group of stars. In the same type of solution, other information can be incorporated, such as very precise ground-based differential proper motions.

When luminosity calibrations are done using Hipparcos parallax data, there is usually a large quantity of very marginal determinations including a number of negative parallaxes. In the common-parameter solution, this problem of negative parallaxes does not occur. It is possible to write a direct relation between the actual observation (the abscissa measurement) and the parameters of the luminosity calibration function. It may be needed to iterate this solution, but in the end it provides not only the requested distance scale value, but also its accuracy and a unit

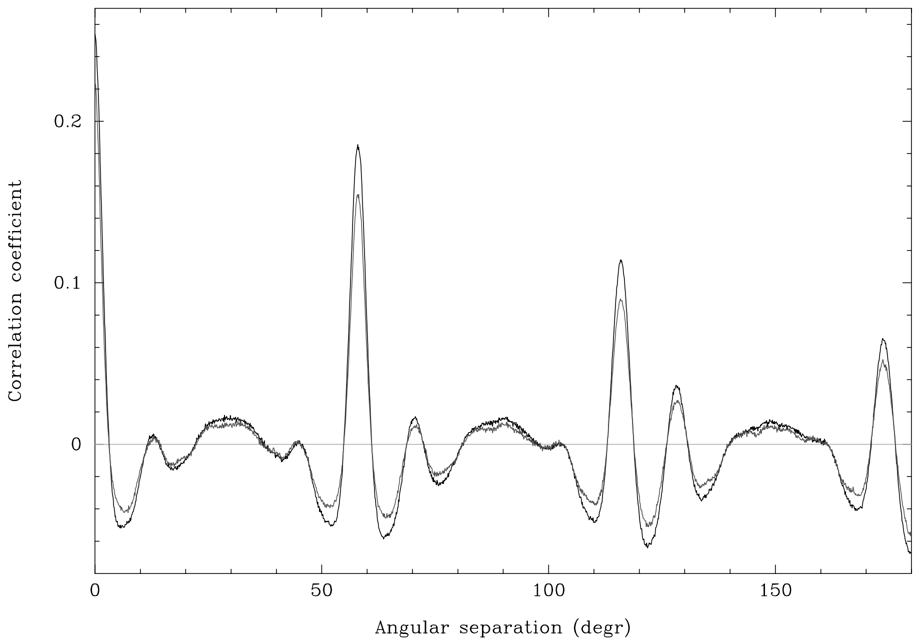

Figure 1. The correlations between abscissae residuals for different separations on a great circle. The curves are the average over all great circles. The NDAC curve can be distinguished from the FAST curve by its lower minima and higher maxima. The peaks at intervals of 58° reflect the overlapping fields of view, separated by the basic angle.

weight error which indicates the applicability of the model that was used.

2. THE BASIC EQUATIONS

All single star solutions as obtained from the intermediate astrometric data describe the changes in the abscissa positions v_i (collected in the vector $\mathbf{v}$ and with ϵ as the associated vector of estimated standard errors) as a function of corrections to model parameters Δa_j:

$$\Delta\mathbf{v} = \sum_{j=1}^{n} \frac{\partial \mathbf{v}}{\partial a_j} \Delta a_j + \epsilon \tag{1}$$

In the most common case there are 5 model parameters: two for positional corrections, one for parallax correction, and two for proper motion corrections. Equation 1 is solved through minimizing the sum of the squares of the residuals, χ^2:

$$\chi^2 = \left(\Delta\mathbf{v} - \sum_{j=1}^{n} \frac{\partial \mathbf{v}}{\partial a_j} \Delta a_j\right)' \mathbf{V}^{-1} \left(\Delta\mathbf{v} - \sum_{j=1}^{n} \frac{\partial \mathbf{v}}{\partial a_j} \Delta a_j\right) \tag{2}$$

as a function of the parameter corrections Δa_j. The covariance matrix of the residuals is given by $\mathbf{V}$, and can be constructed from data provided in the abscissa data file and van Leeuwen & Evans (1997). Defining the inverse of the covariance matrix as $\mathbf{U} = \mathbf{V}^{-1}$, the minimization of χ^2 produces $k = 1, n$ equations:

$$\sum_{j=1}^{n} \left(\frac{\partial \mathbf{v}}{\partial a_k}\right)' \mathbf{U} \left(\frac{\partial \mathbf{v}}{\partial a_j}\right) \Delta a_j = \left(\frac{\partial \mathbf{v}}{\partial a_k}\right)' \mathbf{U} \Delta\mathbf{v} \tag{3}$$

which constitute the normal equations. The symmetric matrix $\mathbf{U}$ can be written as the product of a lower triangular matrix $\mathbf{T}$ and its inverse (see van Leeuwen & Evans 1997):

$$\mathbf{U} = \mathbf{T}'\mathbf{T} \tag{4}$$

After replacing $\mathbf{U}$ by $\mathbf{T}'\mathbf{T}$ and some reorganization the following equations are obtained ($k = 1, n$):

$$\sum_{j=1}^{n} \left(\mathbf{T}\frac{\partial \mathbf{v}}{\partial a_k}\right)' \left(\mathbf{T}\frac{\partial \mathbf{v}}{\partial a_j}\right) \Delta a_j = \left(\mathbf{T}\frac{\partial \mathbf{v}}{\partial a_k}\right)' (\mathbf{T}\Delta\mathbf{v}) \tag{5}$$

Thus, by multiplying the observation equations with the lower triangular matrix $\mathbf{T}$ a set of uncorrelated observation equations is obtained, which can be implemented in any sort of least-squares solution.

The solution for a single star is obtained from the intermediate astrometric data from both consortia. As these data were obtained from the same observations, but were independently reduced, they are correlated. The levels of correlation were determined in the merging of the data, as a function of magnitude, estimated errors and time. The adopted correlation coefficient q_{ij} as well as the estimated error is given in each abscissa record, together with the abscissa residual and the values of $\frac{\partial v_i}{\partial a_j}$ for the five astrometric parameters a_j. A flag indicates the consortium from which the measurement originates and whether the observation has been accepted. Each observation is identified with a reference great circle, and provides one observation equation. When two observations originate from the same reference great circle, they have to be de-correlated through multiplication with the matrix $\mathbf{T}$:

$$\mathbf{T} = \begin{pmatrix} \frac{1}{\sigma_i} & 0 \\ \frac{-q_{ij}}{\sigma_i\sqrt{1-q_{ij}^2}} & \frac{1}{\sigma_j\sqrt{1-q_{ij}^2}} \end{pmatrix} \tag{6}$$

where the indices i and j represent the two observations. The two equations thus obtained are uncorrelated and provide the combined information for this observation by the satellite. When applying this mechanism correctly to the data for a star with a simple five-parameter astrometric solution, the values obtained will be corrections to the estimated astrometric parameters and should all be very close to zero. The estimated errors and the covariance matrix as found from such solution should be the same as those produced in the main astrometric catalogue. This is a useful test for the correct implementation of solutions.

3. APPLICATION TO OPEN CLUSTERS AND THE MAGELLANIC CLOUDS

In order to determine the parallax and/or proper motion for a group of stars using the abscissa data, Equation 1 is used. Instead of implementing 5 parameters per star, only two parameters are solved for, namely the position corrections. The remaining parameters are solved for as common parameters. Each abscissa measurement i for star k contributes an observation equation:

$$\Delta v_{ik} = \sum_{j=1}^{2} \left(\frac{\partial v_{ik}}{\partial a_{jk}}\right) \Delta a_{jk} + \sum_{j=1}^{3} \left(\frac{\partial v_{ik}}{\partial b_j}\right) \Delta b_j \qquad (7)$$

In the case of the Magellanic Clouds only two parameters b_j are solved for, as the parallax can be considered equal to zero for the Hipparcos measurement accuracy. Equation 7 is accumulated for all data obtained within the same reference great circle, and cleaned from the correlations using the methods described above. The cleaned equations for all relevant reference great circles are accumulated in one least-squares solution, which provides the estimates for the parameters b_j. Computer algorithms for the various steps have been provided by van Leeuwen & Evans 1997.

Before Equation 7 can be implemented for common parameters, the abscissae data have to be corrected to one and the same reference solution. The problem is as follows. In the intermediate astrometric data file the abscissa data are given as residuals relative to the solution given in the header record for each star. This is a solution for a position, linear proper motion and parallax. The solution of Equation 7 provides a correction to these astrometric parameters. When proper motion and parallax are solved for as common parameters, then the corrections for each individual star need to be the same. Corrections for the abscissa residuals to a common reference proper motion and reference parallax can be obtained by reversing Equation 7, using the differences Δb_{jk} between the applied ($\hat{b}_{jk}$) and the reference values ($\tilde{b}_j$) of the common parameters (with $\Delta b_{jk} = \tilde{b}_j - \hat{b}_{jk}$) as coefficients to obtain corrections to $\Delta v'_{ik}$, the published abscissa residuals:

$$\Delta v_{ik} = \Delta v'_{ik} - \sum_{j=1}^{3} \frac{\partial v_{ik}}{\partial b_{jk}} \Delta b_{jk} \qquad (8)$$

In actual fact, not the proper motion but the space velocity should be considered the constant quantity. The proper motions are affected by projection, both on the sky and along the line of sight. The projection effects on the sky can be partly accounted for by referring the local proper motion to the proper motion of the cluster centre (indicated by the subscript c):

$$\begin{aligned} \mu_\alpha \cos\delta &\approx (\mu_\alpha \cos\delta)_c + (\mu_\delta)_c \sin\delta \sin(\alpha - \alpha_c) \\ \mu_\delta &\approx (\mu_\delta)_c - (\mu_\alpha \cos\delta)_c \sin\delta \sin(\alpha - \alpha_c) \end{aligned} \qquad (9)$$

In the case of the Hyades the radial velocities and parallaxes have to be taken into account too.

In general only stars that were solved using the basic five astrometric parameters should be used, although in a number of cases it will also be worthwhile to take along at least in trials stars that were solved using a non-linear proper motion. In not all cases was such non-linearity significant. Stars solved for in double stars solutions must be avoided, as for these the abscissae have lost their meaning as a direct measure of position. An example of the application to an open cluster can be found in van Leeuwen & Hansen-Ruiz (1997).

4. APPLICATION TO LUMINOSITY CALIBRATIONS

In a luminosity calibration one or more parameters describing the relation between parallax and other independent observables are determined. A luminosity calibration should always be carried out in 'parallax space', that is to say: the description of any relations is expressed with respect to parallax, not distance or distance modulus. The reason for this is that the errors on the parallaxes are well defined and approximately Gaussian, while on derived quantities this is not the case, in particular when the parallax values become small relative to their accuracies.

Making a solution for a luminosity calibration using abscissae data takes this consideration automatically into account, and in fact takes it one step further.

The principle of a luminosity calibration is based on a prediction of the parallaxes based on a set of assumed model parameters, and describing changes to those predicted parallaxes as changes in the model parameters. The solution using the abscissa data then solves for those changes to the model parameters. In general, relations between parallax changes and model parameters will be non linear, and the process will have to be iterated.

The simplest example is the calibration of the absolute magnitudes of RR Lyrae stars or the calibration of the zero point in a period-luminosity relation. The parallax (in mas) is derived from the observed magnitude, corrected for reddening (m_V) and the predicted absolute magnitude $M_V(RR)$:

$$\begin{aligned} \hat{\pi} &= 100 \times 10^{-0.2(m_V - \hat{M}_V(RR))} \\ &= 100 \times e^{-0.46(m_V - \hat{M}_V(RR))} \end{aligned} \qquad (10)$$

The abscissae observations for all selected stars are corrected for the differences between the published parallax ϖ and the predicted parallax $\hat{\pi}$ using Equation 8. A correction $\Delta M_V(RR)$ to the assumed value of $\hat{M}_V(RR)$ to obtain the best estimate that can be obtained from the available data ($\tilde{M}_V(RR)$) translates in a correction to the predicted parallaxes:

$$\tilde{\pi} = \hat{\pi} \times (1 - 0.46\Delta M_V(RR)) \equiv \hat{\pi}(1 - \nu) \qquad (11)$$

giving corrections to the assumed parallaxes of $-\hat{\pi}\nu$. Thus, the luminosity calibration has been transformed into a calibration of the distance scale factor ν, which has to be the same for all stars used in the solution.

In a more general form, with the absolute magnitudes given as a function of the parameters a_i, the parallax correction can be expressed as:

$$\Delta\pi = -0.46\hat{\pi} \sum_{i=1}^{n} \frac{\partial M}{\partial a_i} \Delta a_i \qquad (12)$$

The criterion for the best solution should be the lowest χ^2 value for the complete solution, and never the accuracy on the distance scale parameters, as these accuracies themselves are a function of the these same parameters.

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