UNBIASED LUMINOSITY CALIBRATIONS FOR HIPPARCOS DATA*

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ABSTRACT

One of the main uses of the Hipparcos astrometric parameters is the calibration of stellar luminosities against colour indices. The high quality of Hipparcos parameters would however be useless if selection biases were not taken into account.

Using parametrical probability density functions, a maximum likelihood algorithm has been developed. It takes into account limiting magnitude or limiting parallax in order to compute unbiased estimates of various parameters: absolute magnitude (as a function of colour indices, metallicity, $v \sin i$), galactic scale height, and first- and second-order moment of the velocity ellipsoid. As a by-product, improved estimates of distances can be obtained.

A multi-platform, tcl/tk based user interface has been built which allows to determine the relevant parameters using Hipparcos data. The algorithm is described, together with the adopted parametrical models, and several applications are given.

Key words: Luminosity calibration.

1. INTRODUCTION

The Hipparcos data will give a considerable insight into the physical stellar properties as well as into the spatial or kinematical properties of stellar populations.

These properties may be described by a model through the use of parametrical probability density functions (pdf) where the parameters Θ to be estimated are:

- the absolute magnitude as a linear function of colour indices, and its dispersion,
- the galactic scale height h_Z, scale length h_R,
- the moments of the velocity ellipsoid $(\overline{U}, \overline{V}, \overline{W})$, $(\sigma_{UU}, \sigma_{VV}, \sigma_{WW}, \sigma_{UV}, \sigma_{UW}, \sigma_{VW})$,
- the zero-point and unit-weight error of observed parallaxes.

These parameters are estimated using all the available observations O: astrometry (position, parallax, proper motion), photometry, colour indices, rotational velocity,...

However, the estimated parameters can be biased, due to truncations of the distributions of some of the observables O: selection in apparent magnitude, observed parallax $\pi_{\rm H}$ (or alternatively of relative parallax error), proper motion, etc. In order to derive unbiased estimates of the parameters, selection effects must then be taken into account.

The method which has been adopted is similar to those of Ratnatunga & Casertano (1991) or Luri et al. (1996).

2. THE METHOD

The conditional probability $g(O|\Theta)$ to observe the quantities O measured for a given star, given the parameters Θ, may be written conditionally to the distance or parallax:

$$g(O|\Theta) = \int_0^{+\infty} h(O|\pi, \Theta) p(\pi) d\pi \qquad (1)$$

The pdf h can be expressed as a product of independent functions:

$$h(O|\pi,\Theta) = p_1(\pi_{\rm H}|\pi,\Theta).p_2(m|\pi,\Theta). \\ p_3(\mu_{\alpha*}, \mu_\delta, V_R|\pi,\Theta).p_4(l,b|\pi,\Theta) \qquad (2)$$

so that all the available information can be taken into account in one of the pdf.

In a first step, the correlations between astrometric parameter errors, as given in the Hipparcos Catalogue, have been neglected. A more appropriate method would use a conditional multidimensional Gaussian:

$$G(\pi_{\rm H}, \alpha_{\rm H}, \delta_{\rm H}, \mu_{\alpha*{\rm H}}, \mu_{\delta{\rm H}}|\pi, l, b, \mu_{\alpha*}, \mu_\delta) \qquad (3)$$

2.1. Adopted Models

Although all observables and parameters are mentioned in Equation 2, at least one of the p_i is needed,

*Based on observations made with ESA Hipparcos satellite.

if no other information is available. Up to now, the assumed models are the following:

- a Gaussian for p_1, the parameters being the zero-point of the parallaxes and their unit-weight error;
- a Gaussian in absolute magnitude for p_2, the unknowns being the mean absolute magnitude and the coefficients of the various photometric indices, together with the dispersion in absolute magnitude; the interstellar extinction is accounted for either using photometric calibrations or with a tridimensional calibration (Arenou et al. 1992);
- a Gaussian velocity ellipsoid is assumed for p_3, where the mean velocity components, together with the 6 second-order moments may be determined;
- an exponential or cosecant law, with an unknown scale height, or simply $\frac{1}{\pi^4}$, for the pdf $p_4(b|\pi)p(\pi)$. This is based on the assumption of an uniform distribution of the stars in the galactic plane, so that $f(X,Y,Z) \propto f(Z) \propto p_4(b|\pi)p(\pi)$.

Where appropriate, these pdf take into account a possible selection bias on an observable o (e.g. $\pi_{\rm H}$, $m_{\rm H}$, $\mu_{\alpha*{\rm H}}$ or $\mu_{\delta{\rm H}}$), through the use of an appropriate normalisation:

$$p_j(o|\pi,\Theta) = \left| \begin{array}{l} \frac{q_j(o|\pi,\Theta)}{\int_{o_-}^{o_+} q_j(o|\pi,\Theta)do} \text{ if } o \in [o_-, o_+] \\ 0 \text{ otherwise.} \end{array} \right. \quad (4)$$

Whereas the selection in apparent magnitude is unavoidable, there is no reason to select stars on the basis of the observed parallax or of their relative parallax error. Even small and negative parallaxes may be used: although their weight is smaller than the weight of nearby stars, their contribution is not negligible.

2.2. Fit of Parameters

Instead of using $g(O|\Theta)$, the method has been applied to the conditional probability of observing $\pi_{\rm H}$ given the other observables and parameters, so that the normalisation factor which appears in p_4 vanishes:

$$f(\pi_{\rm H}|O,\Theta) = \frac{g(O|\Theta)}{\int_{-\infty}^{+\infty} g(O|\Theta)d\pi_{\rm H}} \quad (5)$$

The unknown parameters are then found numerically by maximum likelihood. Their formal errors and correlations are obtained with the inverse of the Fisher information matrix. The residual between the observed and the 'predicted' parallax

$$\delta_i = \pi_{{\rm H}i} - \int_{-\infty}^{+\infty} \pi_{\rm H} f(\pi_{\rm H}|O,\Theta)d\pi_{\rm H} \quad (6)$$

is computed, together with its formal error. Using simulations, a confidence interval for δ/σ_δ is estimated, so that outliers (i.e. normalised residuals outside of the confidence interval) in the studied sample are found and rejected.

2.3. Estimation of the True Parallax

Once the calibration has been done, an estimate of the parallax, or alternatively the distance, of each star may be obtained. This may be of interest for distant stars where the observed parallax is too uncertain. One estimate of distance is its *a posteriori* expectation, which however depends on the properties of the used sample:

$$\hat{r} = E[\frac{1}{\pi}|O,\Theta] = \frac{\int_0^{+\infty} \frac{1}{\pi} h(O|\pi,\Theta)p(\pi)d\pi}{\int_0^{+\infty} h(O|\pi,\Theta)p(\pi)d\pi} \quad (7)$$

with associated variance

$$V(\frac{1}{\pi}|O,\Theta) = \frac{\int_0^{+\infty} (\frac{1}{\pi} - \hat{r})^2 h(O|\pi,\Theta)p(\pi)d\pi}{\int_0^{+\infty} h(O|\pi,\Theta)p(\pi)d\pi} \quad (8)$$

2.4. User Interface

Since many observables, parameters and density laws may be chosen, the configuration of this program could be rather complicated. In order to ease the configuration, a graphical user interface (GUI) has been built (Figure 1). Written in Tcl/Tk, the GUI may be used with few (if any) changes on the major operating systems: Windows, Macintosh, Unix.

After having selected the input data file, the user can define the name of the observable in each field, together with its physical units and the upper and lower limit of a possible truncation. The presence of one or several given observables allow to choose a calibration model, which in turn depends on one or several parameters. The user can either consider the parameter as a constant, giving its value, or can to determine its value using the maximum likelihood program.

3. APPLICATIONS

Two different examples of application are presented below, the first concerning a check of Hipparcos parallaxes, and the second is related to the calibration of the absolute magnitude.

3.1. The Zero-point of the Hipparcos Parallaxes

The described method has been applied to the Hipparcos data after 30 months of mission (Arenou 1995), then to the final Hipparcos Catalogue (Arenou et al. 1997). This application was intended to verify the accuracy and precision of the Hipparcos parallaxes.

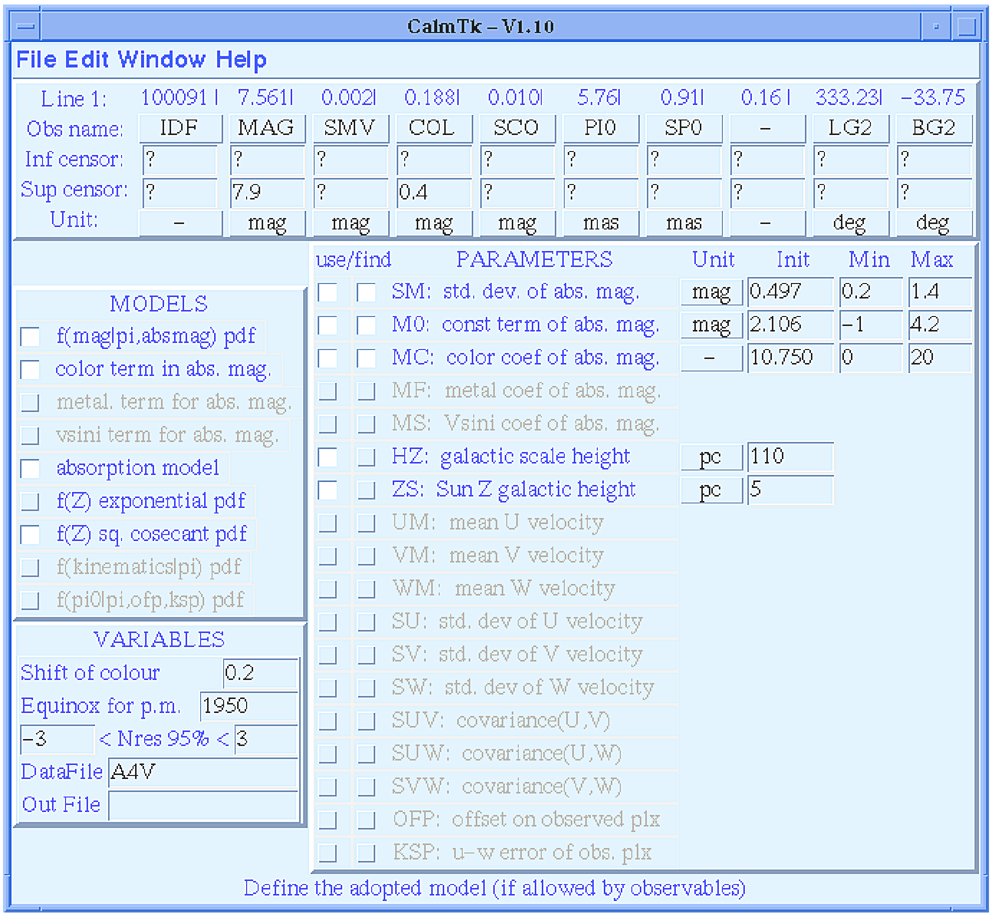

Figure 1. The user interface to the calibration program.

For this purpose, a sample of distant stars was defined using a selection in distance modulus, where the absolute magnitude and interstellar extinction were obtained through the use of $uvby\beta$ photometric calibrations. Distant stars were needed since in this case the trigonometric parallax reflects merely its measurement error, whereas a possible zero-point error of the ground-based absolute magnitudes has a very small effect on the photometric parallaxes.

Two unknowns were searched for: the zero-point of Hipparcos parallaxes and their unit-weight error. In both cases they were found to be not significantly different from the expected values, 0 and 1 respectively (Arenou et al. 1997).

3.2. Corrections to the $uvby\beta$ Absolute Magnitudes

The program has been applied to a sample of A3-A9 luminosity class V stars. All suspected binaries have been excluded. The sample is considered complete up to $m = 7.9$ mag, from which all stars having $uvby\beta$ photometry were kept. The photometry was found in an updated version of Hauck and Mermilliod (1990) Catalogue.

Our purpose was to check whether the absolute magnitude calibration for late A type stars by Crawford (1979) needed or not to be revised. His calibration is $M_V = M_{\rm ZAMS}(\beta) - 9\delta c_0$.

In order to find whether the zero-point was biased or not, the photometric distance moduli was used and a correction δM was calibrated against β. No extinction model has been applied since individual reddening was computed using Crawford calibration. A correction $\delta M = .08 - 1.24(\beta - 2.8) \pm .04$ has been found.

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