ABSOLUTE MAGNITUDE CALIBRATION FOR THE W UMA-TYPE CONTACT BINARY STARS*

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ABSTRACT

Hipparcos parallax data for 19 contact binary stars of the W UMa-type (with $\epsilon M_V < 0.25$) are used to check the previous absolute-magnitude calibration of the form $M_V = M_V(\log P, B-V)$, and to derive a new one. The new calibration covers the ranges $0.15 < (B-V)_0 < 0.93$, $0.27 < P < 0.65$ day, $1.7 < M_V < 5.5$, and gives practically identical predictions for M_V as the previous one, although it is based on entirely independent M_V data, with only 2 systems common to both input datasets. The main limitations of the calibration are the inadequate quality of the ground-based photometric data and its sensitivity to the inclusion of rare systems which lie far from the general period–colour relation.

Key words: contact binary stars; absolute magnitudes.

1. INTRODUCTION

Contact binary stars of the W UMa-type hide the complexity of their structure inside a common envelope which has the external properties of an equipotential surface of the restricted three-body problem, the 'Roche lobe'. Such systems are effectively single objects with two mass centres. The simplicity of such a configuration and the common observational property of the W UMa-type systems of existing close to the main sequence were the underlying reasons for establishing an absolute-magnitude calibration of these systems (Rucinski 1994 (*CAL1*)). The simplest such calibration of absolute magnitudes at maximum-light utilizes two observational quantities, the orbital period P and an intrinsic colour, e.g. $(B-V)_0$ or $(V-I)_0$. Both quantities are correlated through the combined effects of similar geometry, Kepler's third law and main-sequence relationships, although the correlation is not perfect because of the evolutionary spread in sizes within the main sequence.

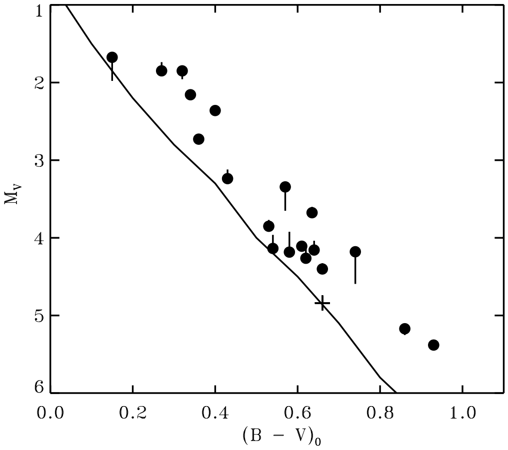

Figure 1. The colour–luminosity relation for the Hipparcos systems with errors smaller than 0.25 magnitude which form the basis of this study. The line represents the main sequence for single solar-type stars, with the Sun marked by a cross. The short vertical lines point at the predicted absolute magnitudes according to CAL4a (see Section 2).

A calibration of the form:

$$M_V = a_P \log P + a_C\, C + \text{const}$$

(where C is any colour index) can also be considered as an attempt to shift the stress from the colour-derived temperature dependence of the normal main sequence (which presents operational difficulties, as it is strongly affected by interstellar reddening) into a period dependence which is controlled by Kepler's law. For the absolute magnitude calibration and determination, the period can be considered error-free because it can be precisely determined by studying accumulated deviations of minimum times from a given ephemeris. It should be noted, that no mass–luminosity relation is involved in the calibration, and that it results from simple geometrical relations.

The main applications of the period–colour–luminosity relation are:

- it serves as a consistency check for the membership of individual, newly discovered contact

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*Based on data from the ESA Hipparcos astrometry satellite.

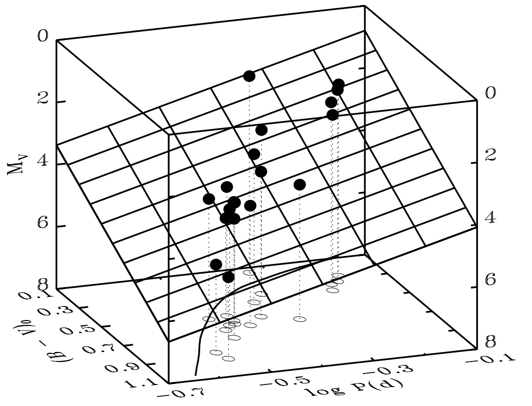

Figure 2. The period–colour–luminosity relation, based on the calibration CAL4a, is shown as an inclined plane. The data of the 19 Hipparcos systems used for the calibration are shown as dots. The rms deviation of the observed values of M_V from the plane is 0.17 magnitude. The data are also projected into the horizontal period–colour plane, where they exhibit the well-known period-colour relation.

systems in star clusters, when the metallicity dependence is also taken into account (Rucinski 1995 (*CAL2*)) – this application being so far the most frequent one;

- it is used for distance determinations (e.g. in the ‘pencil-beam’ search volume of the micro-lensing project OGLE (Rucinski 1997a (*CAL3*), Rucinski 1997b). Contact binaries have a high potential for galactic-structure applications since they are some 24 000 times more common in the solar neighborhood than RR Lyr stars (*CAL3*);
- it helps to re-estimate the spatial density of contact systems on the basis of a volume-limited sample in the solar neighbourhood (Duerbeck 1997), and allows comparison with the OGLE statistics (*CAL3*, Rucinski 1997b).

2. THE NEW ABSOLUTE-MAGNITUDE CALIBRATION

When the previous calibration (*CAL1*) is applied to the Hipparcos data, listed in Table 1, the standard error per point ($\sigma = 0.22$) is smaller than that of the data used for the first calibration ($\sigma = 0.24$). Clearly, a new determination of the calibration coefficients is warranted to make use of this gain in accuracy.

Three new calibrations, designated *CAL4a–c*, were obtained by least-squares, 3-parameter fits for 19 contact systems with Hipparcos parallaxes (ESA 1997) accurate enough to give $\epsilon M_V < 0.25$; these systems have orbital periods in the range $0.27 < P < 0.65$ day. The absolute magnitudes were determined from $M = m + 5\log\pi - 3.1\,E(B-V) - 10$, with the parallaxes π in milliarcsec. Observed maximum-light magnitudes (m) were of three types leading to 3 separate calibrations: (a) $V_{\rm max}$ gleaned from the literature, (b) V derived from $Hp_{\rm max}$, as tabulated in the Hipparcos output data for the upper 5 per cent of the brightness, and corrected with the Hp to V transformation, and (c) $Hp_{\rm max}$, as observed. The maximum-light entries for the cases (b) and (c) could be derived entirely on the basis of the published Hipparcos data, using the Hp to V transformation as in Tables 1.3.5 and 1.3.6 in Volume 1 (draft version, release 1, December 1996) of ESA 1997; these transformations involve a small $(V-I)$-term which was estimated from the adopted $(B-V)$ via the colour–colour calibrations of Bessell (1979). The orbital periods were taken from the General Catalogue of Variable Stars, or more recent publications. The same dereddened colours were used for all three calibrations, $(B-V)_0 = (B-V) - E(B-V)$. The novel approach in this study is the estimation of $E(B-V)$ from the hydrogen column density, $N_{\rm H\,I}$, which resulted in systematically lower values of reddening than assumed before. It should be noted that the observed maximum magnitudes and the colour indices, as collected from the literature, are of varying quality, since in most cases the main effort of the observers was to collect complete light curves relative to some comparison star.

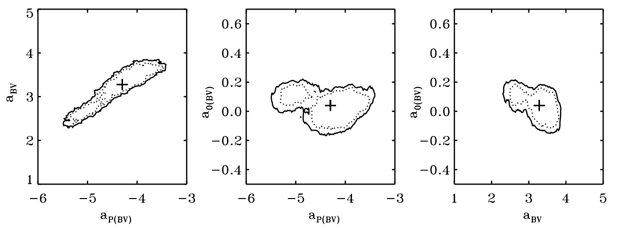

Figure 3. The results from the bootstrap re-sampling experiment for the first of the new calibrations (CAL4a). The naming of the coefficients is the same as in CAL1, in that BV in parentheses indicates that this is a B–V based calibration. The solid contours encompass 68.3 per cent of all bootstrap solutions of the coefficients, a level which is normally associated with the one-sigma standard-error uncertainty. The dotted lines give similar levels for 50 per cent, which is normally associated with the probable errors. Note the two ‘islands’ of solutions which were also present in the previous calibration CAL1.

The new calibrations, called *CAL4a–CAL4c* (with the respective *rms* errors σ = 0.17, 0.18 and 0.18) are:

$$\begin{aligned} M_V &= -4.30\log P + 3.28\,(B-V)_0 + 0.04 \\ M_V^{Hp} &= -4.18\log P + 3.51\,(B-V)_0 - 0.07 \\ M_{Hp} &= -4.18\log P + 3.51\,(B-V)_0 + 0.07 \end{aligned}$$

The colour–luminosity relation is shown in Figure 1. The full 3-dimensional period–colour–luminosity relation, with the resulting planar fit for the first of the new calibrations (*CAL4a*), are shown in Figure 2. For all of these calibrations, the formal errors of the coefficients are very large and very strongly non-Gaussian. This is illustrated in Figure 3 for *CAL4a*, taken here as a typical case. This figure shows the spread in the coefficients obtained from 10 000 bootstrap re-sampling solutions (random selection of points with repetition). As we can see, the one-sigma ranges for the coefficients are very large. This, however, does not mean that the predicted absolute magnitudes are equally uncertain. Interparametric correlations result in cancellation of the contributing uncertainties. Results of Monte Carlo experiments (Figure 4) show surprisingly small uncertainties, as small as 0.1 magnitude, which is unexpected since the stars may have spots and the mass-ratio dependence was not explicitly taken into account in the calibration. The actual deviations on the same plot are larger, indicating that systematic rather than random errors dominate in determining the uncertainty levels.

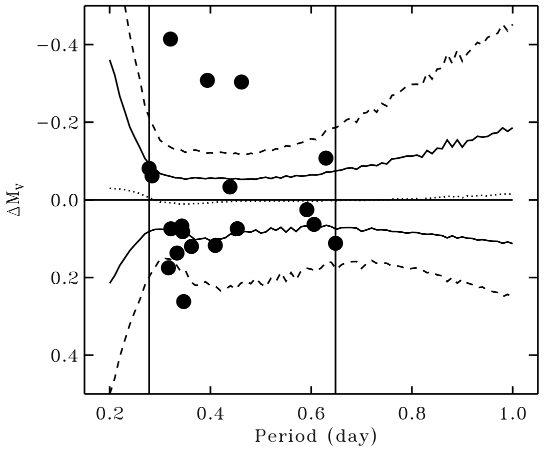

Figure 4. The uncertainty levels in M_V for CAL4a predicted by Monte Carlo simulations are shown by lines whereas the filled circles give the actual deviations. The solid lines give the one-sigma range (68.3 per cent of all cases), the broken lines give the two-sigma range (95.4 per cent of all cases) and the median for the random samplings is shown by the dotted line. The two vertical lines delineate the region defined by systems with the extreme values of the orbital period in the Hipparcos sample. This plot clearly shows that choice of calibrating systems and systematic deviations dominate in setting the uncertainty levels of the calibration.

The high quality of the calibration, judged by the statistical errors, does not imply that it is free from systematic errors, in particular those related to the choice of systems used for the calibration. It is noted that all systems in the sample closely follow the period–colour relation. The sample does not contain evolved contact systems, such as the system V371 Cep (V5 in NGC 188) (see Figure 5). It is not obvious that systems like V5 should be used in the derivation of the calibration, as they may simply be not in contact. A comparison of *CAL4a* with the first and most commonly used calibration *CAL1* shows agreement in the predicted values of M_V to ±0.25 magnitude, without strong systematic trends (Figure 6, left panels), which can be considered highly encouraging. To see how sensitive the results would be to inclusion of V371 Cep, a tour-de-force experiment was made by combining the 19 systems with the Hipparcos data with the *CAL1* data for this system. The resulting calibration (*CAL4d*, $\sigma = 0.20$) is:

$$M_V = -3.17 \log P + 3.94\,(B-V)_0 + 0.16$$

The coefficients of such a calibration are quite different from those of *CAL4a*, but the systematic and random differences *CAL4d*–*CAL1* (Figure 6, right panels) are smaller.

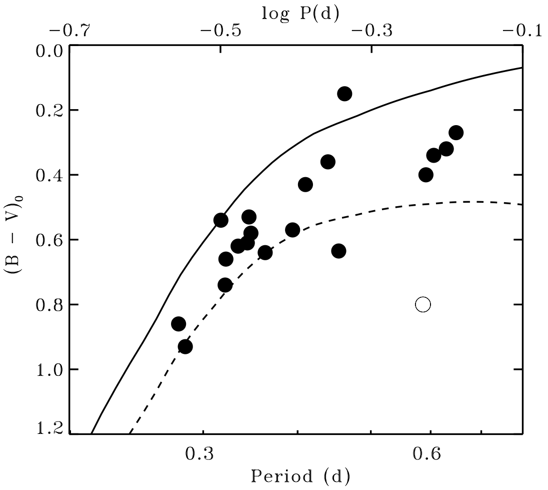

Figure 5. The period–colour diagram for the W UMa-type systems observed by Hipparcos (filled circles). The solid line is the blue–short-period (BSP) envelope from CAL3, transformed to the $(B-V)$ colour. The broken line gives the red edge of the band used for the Monte Carlo simulations for estimating the spread in M_V, as in Figure 4. The open circle marks the system V371 Cep (NGC 188 V5) whose relation to the genuine contact systems is somewhat ambiguous as it is either a contact system with poor energy exchange between components, or a semi-detached system just mimicking contact. If the calibration applies to this system, then its inclusion very strongly constrains the solution and prevents pivoting of the calibration plane around the period–colour relation, as explained in CAL1. The calibration CAL4d was calculated by combining the data for the 19 systems observed by Hipparcos with the data for V371 Cep, as in CAL1.

3. CONCLUSIONS

The old (*CAL1*) and the Hipparcos-based new calibration (*CAL4a*) are basically equally suitable for predicting the absolute magnitudes of W UMa-type systems. This is a very encouraging result which gives credence to the previous calibration, already used in many studies. However, the disparity between the observed deviations and uncertainty levels predicted by Monte Carlo simulations and the strong dependence of the calibration coefficients on the inclusion or exclusion of evolved systems such as V371 Cep (which provide a wider baseline for separating the effects of periods and colours) indicate that the calibrations may be improved. In particular, it

Table 1. W UMa-type systems with ϵ M_V <0.25.

	π $0''.001$	$\epsilon\pi$ $0''.001$	$B-V$	$E(B-V)$	Hp max	V max	P day
S Ant	13.3	0.7	0.32	0.05	6.37	6.38	0.6484
V535 Ara	8.9	0.9	0.34	0.02	7.24	7.17	0.6293
44i Boo B*	78.4	1.0		0	5.98	5.9	0.2678
RR Cen	9.8	0.8	0.36	0.02	7.40	7.27	0.6057
V757 Cen	14.2	1.1	0.64	0.03	8.50	8.44	0.3432
V759 Cen	15.9	0.9	0.59	0.02	7.56	7.40	0.3940
VW Cep	36.2	1.0	0.86	0	7.54	7.38	0.2783
ϵ CrA	33.4	0.9	0.40	0	4.82	4.74	0.5914
SX Crv	10.9	1.2	0.56	0.02	9.05	9.01	0.3166
YY Eri	18.0	1.2	0.66	0	8.30	8.13	0.3215
SW Lac	12.3	1.3	0.75	0.01	8.80	8.76	0.3207
XY Leo	15.9	1.8	0.95	0.02	9.64	9.44	0.2841
TY Men	5.9	0.6	0.25	0.10	8.17	8.11	0.4617
V502 Oph	11.8	1.2	0.645	0.01	8.51	8.34	0.4534
V566 Oph	14.0	1.1	0.44	0.01	7.55	7.54	0.4096
AE Phe	20.5	0.8	0.64	0	7.69	7.60	0.3624
V781 Tau	12.3	1.4	0.58	0.05	8.68	8.55	0.3449
W UMa	20.2	1.0	0.62	0	7.85	7.74	0.3336
AW UMa	15.1	0.9	0.36	0	6.91	6.83	0.4387
GR Vir	18.8	1.2	0.58	0	7.93	7.81	0.3470

* 44i Boo B was not used because no reliable value of $(B-V)$ was available.

may be advantageous to carry out a calibration based on poorer Hipparcos parallax data, perhaps with errors $\epsilon M_V \simeq 0.5$. Such a sample includes systems with a wider range of periods and colours than the present one. The close similarity of the coefficients for calibrations giving absolute magnitudes in V, V-from-Hp and Hp systems (*CAL4b*, *CAL4c*) confirms that the selection of calibrating objects remains the most important factor in determining the quality of the calibrations.

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REFERENCES

Bessell, M.S., 1979, PASP, 91, 589

Duerbeck, H.W., 1997, in preparation

ESA, 1997, The Hipparcos and Tycho Catalogues, ESA SP–1200

Rucinski, S.M., 1994, PASP, 106, 462 (*CAL1*)

Rucinski, S.M., 1995, PASP, 107, 648 (*CAL2*)

Rucinski, S.M., 1997a, AJ, 113, 407 (*CAL3*)

Rucinski, S.M., 1997b, AJ, 113, 1112

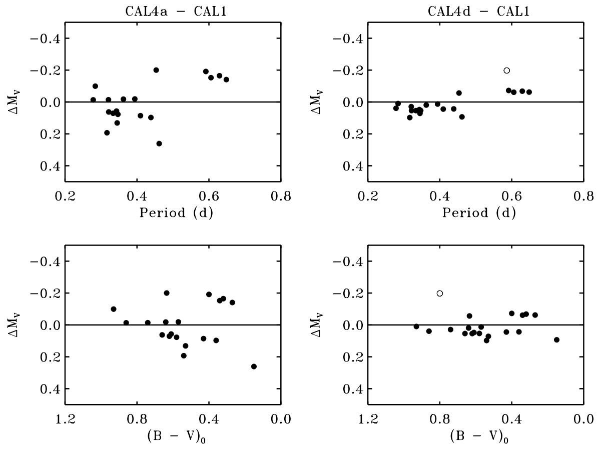

Figure 6. The differences between predicted values of M_V from the new (CAL4a and CAL4d) and the old (CAL1) calibrations, plotted versus the orbital period and the B–V colour index. CAL4d differs from CAL4a by inclusion of the system V371 Cep (V5 in NGC 188), as explained in the text.