# DETERMINATION OF THE LUMINOSITY FUNCTION UP TO $M_V = 9$

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## ABSTRACT

The high quality of Hipparcos data and specially of parallaxes allows the accurate determination of the luminosity function in the solar neighbourhood. Complete samples must however be defined in order to derive unbiased distributions. For stars intrinsically brighter than $M_V = 5$ mag, the completeness is assured by the Hipparcos Survey. For stars intrinsically fainter, a distance-limited complete sample is established using the $V/V_{\max}$ method (Schmidt 1968).

The different biases due to sample selections and observational errors on the parallaxes are examined and a non-parametric statistical method is presented which takes them into account and corrects the observed distributions. A model for interstellar absorption can be included in the computation.

First results confirm that the luminosity function adopted in previous studies has been overestimated.

Key words: luminosity function; observational errors; bias.

## 1. INTRODUCTION

The luminosity function $\Phi(M)$, i.e. the number of stars of absolute magnitude $M$ per pc$^3$ is of fundamental importance. It allows to estimate the stellar mass function and the star formation rate.

First results obtained by Hipparcos have shown that previous distances have generally been underestimated, the most striking example being the Gliese catalogue, with more than 30 per cent of the stars lying outside the 25 pc sphere (Grenon 1983, Perryman et al. 1995). The direct consequence is that previous luminosity functions have been systematically overestimated.

The luminosity function can be computed empirically, by counting stars in each absolute magnitude interval. Complete samples are defined using the Hipparcos Survey for stars brighter than $M_V = 5$ mag or with the $V/V_{\max}$ method for fainter stars. However to take the bias due to selections and observational errors on parallaxes into account, a statistical method must be used. This method must be non-parametrical to avoid circular reasoning.

## 2. EMPIRICAL APPROACH

### 2.1. The Data

The Hipparcos Survey has been defined as a complete sample up to a limiting apparent magnitude $V_{\rm lim}$ depending on galactic latitude $b$ and spectral type ST:

$$V_{\rm lim} = 7.9 + 1.1|\sin b| \text{ for ST} \leq \text{G5} \quad (1)$$

$$V_{\rm lim} = 7.3 + 1.1|\sin b| \text{ for ST} > \text{G5} \quad (2)$$

The Survey contains 52 045 entries. With new apparent magnitudes V given in the catalogue, 3925 stars are rejected and 1544 added according to criteria (1) and (2).

The median standard error on Hipparcos parallaxes is less than 1 mas, the mean precision on apparent magnitude is 0.0015 mag, and for more than 40 000 stars, the relative error on Hipparcos parallaxes is smaller than 20 per cent. Within 150 pc, about 99 per cent of the stars have $\sigma_\pi/\pi < 0.3$, more than 97 per cent $\sigma_\pi/\pi < 0.2$, and about 65 per cent $\sigma_\pi/\pi < 0.1$. The corresponding errors $\sigma_M$ on absolute magnitudes $M = m + 5 + 5\log\pi$ are less than 0.65, 0.43 and 0.22 mag.

### 2.2. Sampling

The data are truncated in apparent magnitude so that the volume of completeness varies with the absolute magnitude and is greater for brighter stars.

- For $M_V \leq 5$ mag, the luminosity function is computed using Hipparcos Survey stars. Given an absolute magnitude interval $[M_1, M_2]$, the maximum volume of completeness $V_{\max}$ is defined according to the limiting magnitude $V_{\rm lim}$.

  The maximum distance at which a star of magnitude $M_2$ will still be included in the sample is (see Table 1):

$$d_{max} = 10^{-(M_2 - V_{\rm lim} - 5)/5} \quad (3)$$

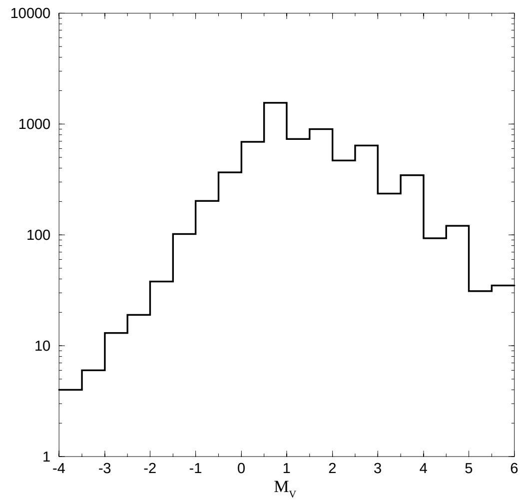


*Figure 1. Number of stars in the different bins for $M_V \leq$ 6 mag within the corresponding volume of completeness considered.*

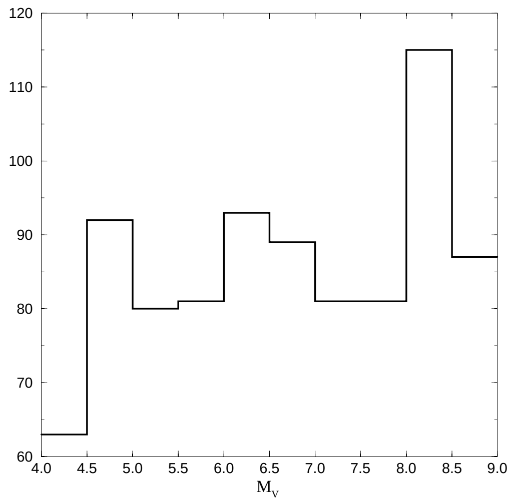


*Figure 2. Number of stars within 25 pc in the different bins for $M_V$ >4 mag.*

We consider only stars within 150 pc, even for $M_V \leq 1$ mag, so that the standard errors on absolute magnitudes are better than 0.2 mag for 65 per cent of stars. Volumes are chosen every 1 mag intervals.

- For $M_V > 5$ mag, we consider the stars within a sphere of 25 pc (1549 stars), which includes stars from the Gliese catalogue and other high proper motion objects. Within this limit, we test for completeness using the $V/V_{\max}$ method (Schmidt 1968). The test shows that the catalogue is roughly complete up to $M_V = 9$ mag.

Figures 1 and 2 give the number of stars in each absolute magnitude bin within the corresponding volume of completeness considered.

The luminosity function thus computed in bins of 0.5 mag is shown in Figure 3.

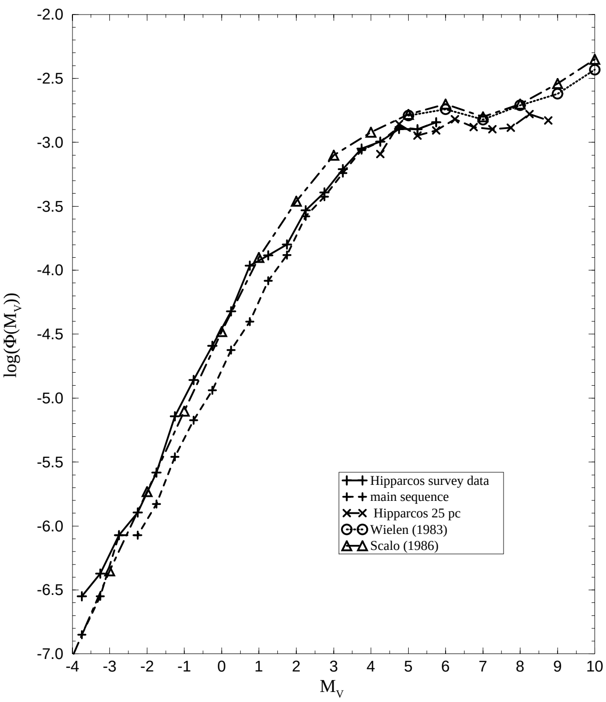


*Figure 3. The luminosity function up to $M_V = 9$ mag.*

### 2.3. Results

The luminosity function computed with Hipparcos stars differs from those of Scalo (1986) and Wielen et al. (1983): if giant stars are considered, the density is approximately the same up to $M_V = 1$ mag, but for stars brighter than $M_V = 1$ mag, the Scalo's luminosity function seems to be overestimated by a factor of about 1.5.

The giants clump appears clearly in the range $0.5 < M_V < 2$ mag. The link between the functions computed with the Survey and with the stars within 25 pc is good, and the flatness around $M_V = 7$ mag is well visible.

## 3. ERRORS AND BIASES

Due to observational errors on the parallaxes, absolute magnitude estimations made with a distance-

*Table 1.* $d_{max}$ *for* $V_{lim}$= *7.3 mag.*

| $M_2$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $d_{max}$ (pc) | 280 (150) | 180 (150) | 110 | 72 | 45 | 28 | 18 |
| total number of stars in $V_{max}$ | 21 156 | 21 156 | 13 925 | 6854 | 2520 | 819 | 275 |

limited sample are biased (Lutz & Kelker 1973). Some stars will be erroneously included in the sample and others omitted, as shown in Figure 4.

The observed parallaxes $\pi_o$ can be corrected using a Bayesian estimation of the true parallax $\pi_t$ if the observational errors are assumed to be Gaussian (Arenou 1993):

$$E[\pi_t|\pi_o] = \pi_c = \pi_o + \sigma_\pi^2 f'(\pi_o)/f(\pi_o) \qquad (4)$$

where $f(\pi_o)$ is the density distribution of the observed parallaxes. $f$ is estimated by the convolution kernel method.

This gives an estimation of the bias $\pi_c - \pi_o$. In Figure 5 the bias is represented according to the observed parallaxes, and the observed and corrected distributions $\phi_o(\pi)$ and $\phi_c(\pi)$ are shown in Figure 4.

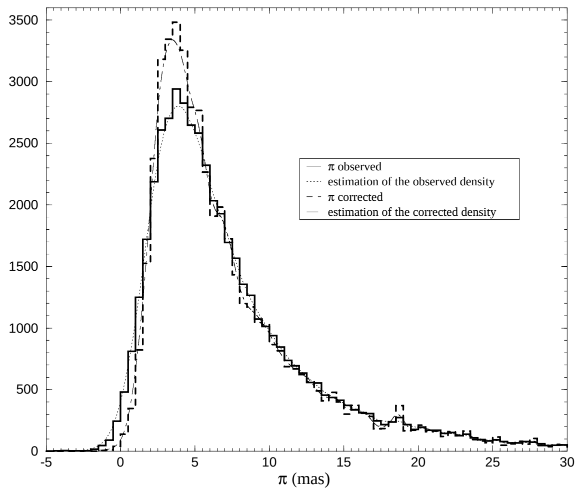


*Figure 4. Observed and corrected distributions of the Survey stars parallaxes.*

Figure 5 shows that for $\pi_o$ greater than $\approx 35$ mas, the bias is negligible. Its median value is smaller than 0.2 mas for $\pi_o > 2$ mas. Observed parallaxes are systematically overestimated for $\pi_o > 3$ mas, and underestimated for $\pi_o < 3$ mas. As implied by Equation 4, this value corresponds to the mode of the distribution. Below this limit, the bias increases rapidly.

Beyond approximately 150 pc the number of stars omitted becomes greater than the number of stars included. If we consider the cumulative distributions, there are no more than 4 per cent of stars erroneously

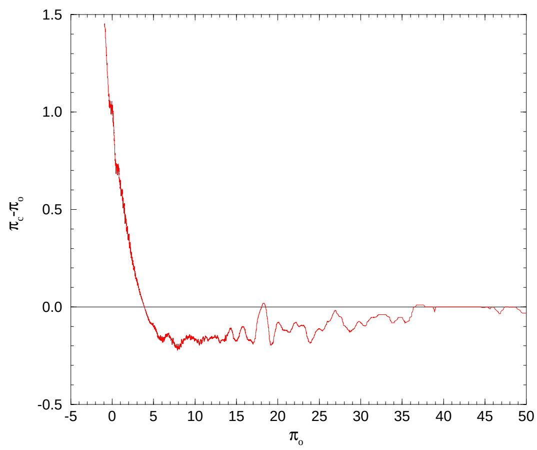


*Figure 5. The bias on parallaxes versus the observed parallax.*

included, but between 150 pc and 250 pc, they reach 7 per cent.

When we count stars in each absolute magnitude interval with different distance limits, the bias on parallaxes is different on each interval. Even within 150 pc, where the number of stars omitted roughly balances the number added, the bias is still present and its propagation on the absolute magnitude distribution must be studied.

## 4. STATISTICAL METHOD

Statistical corrections for distance-limited or magnitude-limited samples were estimated by many authors using parametric methods. But these works assume an *a priori* space distribution of stars and an *a priori* known conditional distribution of the absolute magnitudes given the spectral type of the star.

A non parametric statistical method is proposed here to compute the luminosity function taking bias into account. A model for interstellar absorption can also be included in the equations.

### 4.1. Correction of the Observed Absolute Magnitudes Distribution From the Bias due to Errors on Parallaxes

Let $F$ and $\phi$ denote respectively frequency functions and probability density functions ($F = N\phi$ where

$N$ is the the total number of stars in the population). The bivariate distribution function of absolute magnitudes and parallaxes $F(M,\pi)$ can be expressed through the bivariate distribution of apparent magnitudes and parallaxes in terms of conditional probability:

$$\begin{aligned} F(M,\pi) &= F(m(M,\pi),\pi) \\ &= F(m(M,\pi))\phi(\pi|m(M,\pi)) \end{aligned} \quad (5)$$

Let $F_o$ and $F_c$ denote respectively observed and corrected distributions. Errors on apparent magnitudes can be neglected and Equation 1 gives:

$$\begin{aligned} F_c(M,\pi) - F_o(M,\pi) = \\ F(m(M,\pi))[\phi_c(\pi|m(M,\pi)) - \phi_o(\pi|m(M,\pi))] \end{aligned} \quad (6)$$

The effect of the bias on parallaxes $F_c(M,\pi) - F_o(M,\pi)$ is estimated by Equation 6. As $\sigma_\pi$ depends mainly on the apparent magnitude, this allows to correct the parallaxes distribution with homogeneous samples in $\sigma_\pi$ and Equation 4 is a good approximation.

### 4.2. The Apparent Magnitude Truncation

The frequency function thus obtained with Hipparcos data corresponds to a truncated and incomplete sample in apparent magnitude and is not representative of the parent population if $(M,\pi)$ doesn't verify $M - 5 - 5\log\pi \leq m_{\mathrm{lim}}$ where $m_{\mathrm{lim}}$ is the limiting magnitude of the Survey.

Let $V(M, m_{\mathrm{lim}})$ be the volume for which $M - 5 - 5\log\pi \leq m_{\mathrm{lim}}$, i.e. $\pi > \pi_{\mathrm{lim}} = 10^{(M-m_{\mathrm{lim}}-5)/5}$. The number of stars of absolute magnitude $M$ per pc$^3$ is given by:

$$\Phi(M) = \frac{\int_{\pi>\pi_{\mathrm{lim}}} F_c(M,\pi)d\pi}{V(M, m_{\mathrm{lim}})} \quad (7)$$

### 4.3. The Interstellar Absorption

In the luminosity function presented in this paper, absorption was not taken into account. The luminosities are then systematically underestimated.

A tridimensional model of absorption $A_V(\pi,l,b)$ can be directly included in the previous equations in considering the distribution $F(M,\pi,l,b)$. If $M_A = M - A_V(\pi,l,b)$ is the absolute magnitude corrected from absorption, $F(M_A,\pi)$ is given by:

$$F(M_A,\pi) = \int\int F(M(M_A,\pi,l,b),\pi,l,b)dldb \quad (8)$$

## 5. CONCLUSION

In this work we have examined the bias due to selections and errors on parallaxes, and described a possible way to take it into account in the determination of the luminosity function. We haven't mentioned double and multiple stars systems which are an other important source of uncertainty. The effect of unresolved systems on the luminosity function will be examined in a future work.

Thanks to the high precision of Hipparcos data, the luminosity function can be computed simply making star counts if we restrict our study to a sphere of 150 pc. However to fully exploit the content of the Survey, an appropriate statistical treatment should be used to take bias into account.

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