COMPUTING THE TILT OF THE VELOCITY ELLIPSOID FOR A GALAXY MASS MODEL

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ABSTRACT

Starting with a mass model of the Galaxy, we apply a scheme developed by Schwarzschild (1979) in order to determine a phase-space distribution function of stars that is characteristic of the mass model. In a first step, we limit our investigations to the solar neighbourhood, that is, among the large amount of stellar orbits required by Schwarzschild's procedure we only consider nearly circular orbits that occur in the vicinity of the Sun. Thus, it allows us to compute a rough estimation of the tilt of the velocity ellipsoid in the solar neigbourhood.

Key words: Galactic dynamics; distribution function; velocity ellipsoid.

1. OBJECTIVE

Our aim is to constrain parameters of a Galactic mass model with the help of new observational data and especially Hipparcos data. So we have first to compute the theoretical quantities for the model to be compared to these observations. This led us to work out the tilt of the velocity ellipsoid in the vicinity of the Sun, for a given Galaxy model, in addition to other classical quantities such as the rotation curve, local density, escape velocity, and so on.

2. MASS MODEL

For this first run, we adopted a slightly modified model of Ostriker & Caldwell (1979), whose components are:

- a central mass;
- a flattened disk whose surfaces of equal density are oblate spheroids
- a spherical bulge-halo;
- a spherical dark halo.

Ostriker & Caldwell's surface disk has been replaced by a flattened disk in order to make numerical orbit computations easier. It stands to reason that we chose for that component a density law that reproduces by projection on the Galactic plane Ostriker & Caldwell's surface density. Note that we suppose the Galaxy to be axisymmetric.

3. DENSITY LAWS

3.1. Flattened Disk

The adopted density is given by:

$$\rho_d(m) = \rho_d \frac{e^{-m}}{\sqrt{m}} \tag{1}$$

where $m = \sqrt{(\frac{R}{a})^2 + (\frac{z}{c})^2}$.

3.2. Spherical Bulge-Halo

$$\rho_{bh}(r) = \begin{cases} \frac{\Sigma_0}{\pi n} \left\{ \frac{u+3}{\sqrt{u}} \mathrm{arctg}\left(\sqrt{u}\right) - 3 \right\} \frac{1}{u^2} & \text{if } r > n \\ \frac{4\Sigma_0}{15\pi n} & \text{if } r = n \\ \frac{\Sigma_0}{\pi\, n} \left\{ \frac{3-u}{\sqrt{u}} \ln\left(\frac{1+\sqrt{u}}{\sqrt{1-u}} \right) - 3 \right\} \frac{1}{u^2} & \text{if } r < n \end{cases} \tag{2}$$

where $u = |(\frac{r}{n})^2 - 1|$.

3.3. Spherical Dark Halo

The density law reads:

$$\rho_{dh}(r) = \frac{\rho_{dh}}{1 + (\frac{r}{r_{dh}})^2} \tag{3}$$

4. ROTATION CURVE

Before dealing with the velocity ellipsoid, we adjusted the contribution of each component in order to obtain the best agreement with the observed rotation curve.

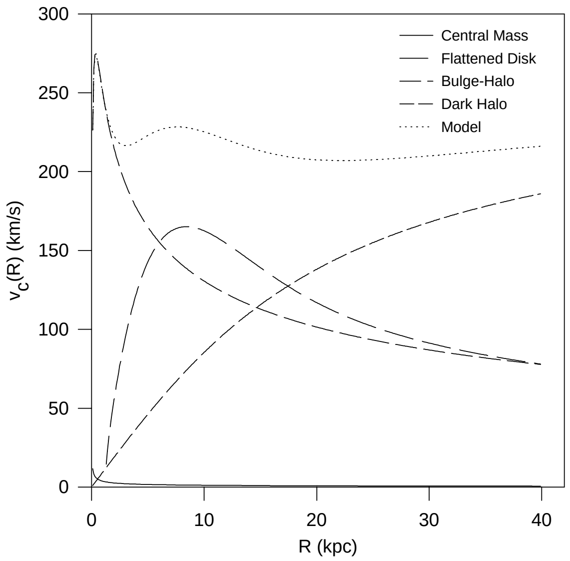

Figure 1. Contributions to the Rotation Curve.

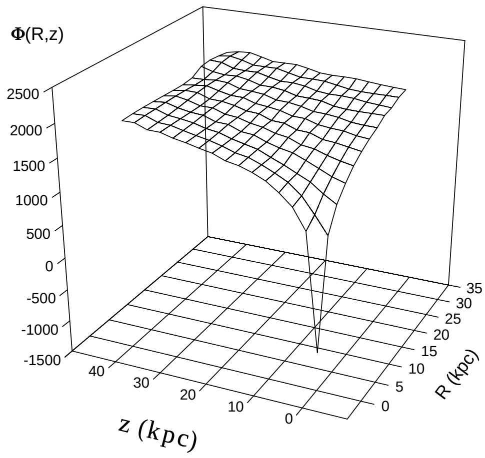

Figure 3. The Potential as a function of cylindrical coordinates.

Other parameters in the density laws were left with their values taken from Ostriker & Caldwell's model. The observational data for the rotation curve we used are those derived by Allen & Santillan (1991).

Results as contributions to the rotation curve are shown in Figure 1.

5. THE POTENTIAL

The potential of each component is calculated numerically from the Poisson's equation $\Delta\Phi = 4\pi G\rho$. The isopotential contours in the meridional plane and the potential as a function of cylindric coordinates R and z are shown hereafter.

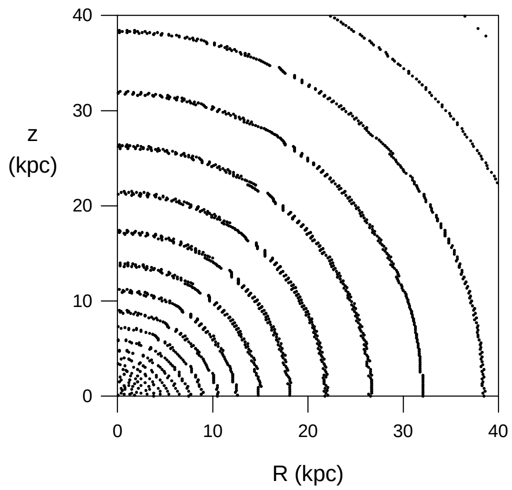

Figure 2. Isopotential contours in the meridional plane.

6. THE VELOCITY ELLIPSOID

In order to compute the velocity ellipsoid, we applied a scheme developed by Schwarzschild (1979): first, we divide physical space into N cells.. Then, we integrate a large number M of orbits numerically and store the fraction of time t_{ij} our i^{th} orbit spends in the j^{th} cell. Finally, we try to reproduce the model density by assigning each orbit a non-negative number called *occupation number* C_i, that satisfies:

$$\sum_{i=1}^{M} C_i t_{ij} = m_j, \quad (1 \leq j \leq N) \tag{4}$$

where m_j designates the mass held in cell number j. Indeed, one may say that the above equation corresponds to the equality between the density and the integral over all velocities of the distribution function of the model.

In our run, we adopted square cells of 1 kpc^2 in the meridional plane, we computed approximately 1300 orbits, and took into account the cells they crossed. As the above linear system was underdetermined, we chose to maximize a linear function of the occupation numbers to select a solution (simplex method was applied). Note that as we wanted to compute the tilt of the velocity in the solar neighbourhood, the maximization function was constructed to give greater importance to orbits that were confined in the vicinity of the Sun.

Unfortunately, the mean radial and vertical velocities we found are not as small as they should be. This is certainly due to the choice of the orbits' sample, and to the adopted size and shape for the cells.

We show nevertheless in the following tables the radial and vertical velocity dispersions just as the tilt of the velocity ellipsoid in the cells located next to the Sun.

Table 1. Radial Velocity Dispersion σ_R.

R	z 0-1 kpc	1-2 kpc	2-3 kpc
6-7 kpc	4.1	33.1	11.8
7-8 kpc	1.7	31.8	42.9
8-9 kpc	2.1	27.6	52.9
9-10 kpc	1.9	21.4	57.5
10-11 kpc	3.7	27.4	46.7

Table 2. Vertical Velocity Dispersion σ_z.

R	z 0-1 kpc	1-2 kpc	2-3 kpc
6-7 kpc	16.1	33.1	8.7
7-8 kpc	7.6	19.8	19.2
8-9 kpc	2.1	11.7	12.8
9-10 kpc	1.9	4.5	13
10-11 kpc	6.9	15.7	12

Table 3. Tilt of the Velocity Ellipsoid σ^2_{Rz}.

R	z 0-1 kpc	1-2 kpc	2-3 kpc
6-7 kpc	- 5.2	270	83.5
7-8 kpc	8.9	-240	393
8-9 kpc	3.1	16	- 48.8
9-10 kpc	- 3.1	- 15.4	-478
10-11 kpc	-11.1	-206	-392

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