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The Cepheid PL Zero-Point from Hipparcos Trigonometrical Parallaxes ⋆

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ABSTRACT
Hipparcos trigonometrical parallaxes of Cepheid variables are used to derive a zero-point for the PL relation. Adopting a slope from the LMC, the relation is found to be, $< M_V >= -2.81 logP - 1.43$.
The standard error of the zero-point is 0.10 mag. Together with metallicity corrections this corresponds to distance moduli of 18.70 ± 0.10 for the LMC and 24.77 ± 0.11 for M31. Some implications of these results are discussed. Estimates of the Hubble constant (H_0) which are based on Cepheid observations together with an adopted LMC distance modulus of 18.50 will on average now need decreasing by ~ 10 percent. However metallicity corrections, which have frequently been ignored, will result in the actual percentage change varying with the sample of galaxies studied. Calibration of RR Lyrae absolute magnitudes using the LMC and M31 Cepheid distances implies an age for the oldest Galactic globular clusters of $\sim$ 11Gyr. The parallax data show that the period of Polaris corresponds to first overtone pulsation.

Key words: Cepheids – distance scale.

1 INTRODUCTION AND PROCEDURE

Since the discovery of the period-luminosity (PL) relation for Cepheids (Leavitt 1908, 1912) the establishment of its zero-point has been a major goal, of primary importance for both galactic and extragalactic distance scales. The observations made by the Hipparcos astrometry satellite (ESA 1997) now enable one to derive this zero-point with good accuracy, directly from trigonometrical parallaxes of Cepheids themselves.

In the first release of Hipparcos data on Cepheids there are trigonometrical parallaxes (π) for 223 Galactic (classical) Cepheids. The mean standard error of a single parallax is $\sigma_\pi = 1.5$ milliarcsec (mas). Most of these parallaxes are of course very small and of little individual value. The full data set will be listed in a future paper dealing mainly with the Hipparcos proper motions of these stars. However the 26 Cepheids which contribute most of the weight in the present discussion are listed in Table 1.

In the present paper we are concerned primarily with the zero-point (ρ) of the PL relation at V:

$$< M_V >= \delta logP + \rho. \quad (1)$$

If $< V >$ is the visual magnitude of a Cepheid and $< V_0 >$ † its reddening corrected value, then given the parallax (π) in milliarcsecs, the function $10^{0.2\rho}$ can be derived from the equation:

$$10^{0.2\rho} = 0.01\pi 10^{0.2(<V_0> - \delta logP)}. \quad (2)$$

The necessary reddening corrections can be obtained from multicolour photometry (e.g. by the BVI method (Dean,Warren & Cousins 1978, Caldwell and Coulson 1985) or from a single colour using a period-colour relation such as:

$$< B >_0 - < V >_0 = \tau logP + \phi. \quad (3)$$

Both equations 1 and 3 are approximations to a period-luminosity-colour (PLC) relation:

$$< M_V >= \alpha logP + \beta(< B >_0 - < V >_0) + \gamma. \quad (4)$$

There is thus intrinsic scatter in equations 1 and 3. If these two equations refer to mean relations, it will be seen from equation 4 that at a given period a Cepheid whose intrinsic

⋆ Based on data from the Hipparcos astrometry satellite.

† Angle brackets refer to intensity-mean magnitudes.

Table 1. The 26 Stars with the Greatest Weight

Name	π	σ_π	$\log P_0$	$< V >$	$< B > - < V >$	p_A	p_B	Note
SU Cas	2.31	0.58	0.440	5.970	0.703	72	69	o
SZ Tau	3.12	0.82	0.651	6.530	0.852	15	14	o
β Dor	3.14	0.59	0.993	3.756	0.799	86	81	
RT Aur	2.09	0.89	0.571	5.446	0.595	22	21	
ζ Gem	2.79	0.81	1.006	3.918	0.798	37	36	L
AH Vel	2.23	0.55	0.782	5.695	0.579	19	18	o
BG Vel	1.33	0.65	0.840	7.635	1.175	11	11	L
l Car	2.16	0.47	1.551	3.735	1.260	66	64	
T Cru	0.86	0.62	0.828	6.566	0.922	16	16	L
R Mus	1.69	0.59	0.876	6.298	0.757	11	11	
S Cru	1.34	0.71	0.671	6.600	0.761	13	13	
R TrA	0.43	0.71	0.530	6.660	0.722	19	19	
X Sgr	3.03	0.94	0.846	4.549	0.739	24	23	
Y Oph	1.14	0.80	1.234	6.150	1.385	12	12	L
W Sgr	1.57	0.93	0.881	4.668	0.746	20	19	
Y Sgr	2.52	0.93	0.761	5.744	0.856	16	16	
U Sgr	0.27	0.92	0.829	6.685	1.091	11	11	
FF Aql	1.32	0.72	0.806	5.372	0.756	23	23	o
U Aql	2.05	0.93	0.847	6.446	1.024	10	10	
U Vul	0.59	0.77	0.903	7.128	1.275	13	13	
η Aql	2.78	0.91	0.856	3.897	0.789	52	51	
S Sge	0.76	0.73	0.923	5.622	0.805	13	13	
T Vul	1.95	0.60	0.647	5.754	0.635	30	30	
DT Cyg	1.72	0.62	0.549	5.774	0.538	30	30	o
δ Cep	3.32	0.58	0.730	3.954	0.657	132	124	
αUMi	7.56	0.48	0.754	1.982	0.598	909	596	o

colour is $\Delta(B-V)$ greater than the mean will have an absolute magnitude ($< M_V >$) which is fainter than that predicted by equation 1 by $\beta\,\Delta(B-V)$. However if the reddening is derived from equation 3, the adopted reddening corrected magnitude ($< V_0 >$ will be too bright by R $\Delta(B-V)$, where R is the ratio of total to selective absorption ($A_V/E_{(B-V)}$). Hence for a Cepheid of known distance, the derived absolute magnitude will be brighter than the mean at that period by $(R-\beta)\Delta(B-V)$. Since $\beta \sim 2.5$ and $R \sim 3.3$ (e.g. Feast and Walker 1987), the scatter of individual Cepheids about the mean PL(V) relation is reduced by a factor of more than three from that which results if true individual reddenings are used. Using equation 3 to estimate reddening corrections is therefore an effective way of reducing intrinsic scatter amongst individual values of $10^{0.2\rho}$ derived from equation 2. (The same advantage is of course gained in using equation 1 (with a known value of ρ) together with equation 3 to derive distances of Cepheids.)

In addition to this, the instability strip narrows at fainter absolute magnitudes (shorter periods) (e.g. Figure 12 of Chiosi et al. 1993) reducing the scatter about equations 1 and 3, whilst reddenings derived from BVI data become less accurate due to the decreasing angle between the intrinsic and reddening lines for hotter (shorter period) Cepheids (Caldwell and Coulson 1985). This is an important consideration for the present work since the nearer Cepheids tend to be of relatively short period.

In the present application values of τ and ϕ have been adopted from Laney and Stobie (1994). These give:

$$< B >_0 - < V >_0 = 0.416 logP + 0.314. \tag{5}$$

This is derived from Galactic Cepheids with BVI reddenings (or space reddenings on the same system) and with some weight in the slope to data on Magellanic Cloud Cepheids. This relation is negligibly different from that derived by Caldwell and Coulson (1986) for Galactic Cepheids:

$$< B >_0 - < V >_0 = 0.412 logP + 0.310 \tag{6}$$

or (in the period range of interest) from that given by Caldwell and Coulson (1987). These latter workers included a term for a radial variation of the relation in the Galaxy. Since the Cepheids of relevance in the present work are relatively close to the sun, this term may be put equal to zero. In deriving the visual extinction (A_V) the relation derived by Laney and Stobie (1993);

$$R = 3.07 + 0.28(B-V)_0 + 0.04E_{(B-V)} \tag{7}$$

was used.

The slope (δ) of the PL(V) relation cannot usefully be derived from the present data and the value found by Caldwell and Laney (1991) from 88 Cepheids in the LMC ($\delta = -2.81 \pm 0.06$) was adopted. Laney and Stobie (1994) derived a closely similar value ($\delta = -2.87 \pm 0.07$) from LMC, SMC and Galactic Cepheids in clusters and associations. This is less suitable for the present purpose since it partly depends on an adopted cluster and association distance scale. Values of $< V >, < B > - < V >$, and log P were taken from the electronic catalogue of Fernie et al. (see Fernie et al. 1995) unless these data were in Laney and Stobie (1993).

Of the 223 type I Cepheids in the initial Hipparcos release of data, the following were omitted from the present analysis; DP Vel because the photometric data are too sparse; AW Per and AX Cir because they are in binary systems and the photometry might be seriously affected by the companions in these cases. The Hipparcos data set includes a number of Cepheids which have been identified as first overtone pulsators (Poretti 1994 and references there). These are ; BY Cas, SU Cas, SZ Tau, AH Vel, GI Car, AZ Cen, BB Cen, FF Aql, DT Cyg. In addition αUMi (Polaris) is included in the analysis as a first overtone pulsator and this, and the general question of overtone pulsators, is discussed later. Alcock et al. (1995) derive the following relation for Galactic Cepheids:

$$P_1/P_0 = 0.720 - 0.027 logP_0 \tag{8}$$

where P_0 and P_1 are, respectively, the fundamental and first overtone periods. With sufficient precision for the present purpose this may be written:

$$P_1/P_0 = 0.716 - 0.027 logP_1 : \tag{9}$$

and this relation has been used to derive P_0 for the overtone Cepheids. In the case of α UMi a solution is given later for the possibility that it is a second overtone pulsator. In that case a ratio, $P_2/P_0 = 0.55$, was used, derived from the above and from data on second overtone pulsators in the LMC (Alcock et al. 1995).

In deriving a mean value of $10^{0.2\rho}$ from the data, individual values were weighted by the reciprocal of the square of the standard error of the right hand side of equation 2. This standard error takes into account the standard error of the parallax (σ_π), which is listed in the Hipparcos catalogue (ESA 1997), and the error in $10^{0.2(<V_0>-\delta \log P)}$. Caldwell and Laney (1991) found the scatter about a PL(V) relation for the LMC to be $\sigma_{PL} = 0.21$ with reddenings derived from BVI data (or on the BVI system).Most of this scatter is intrinsic. In view of the discussion above regarding the combined use of equations 1 and 3, the effective scatter in the present case is reduced to $\sigma_H = 0.21(R/\beta - 1)$ or ~ 0.07. Since the instability strip narrows at shorter periods the value of σ_{PL} appropriate to the present work should be somewhat less than that found for the LMC sample. Two solutions are included in Table 2. One with $\sigma_H = 0$ (solution A) and the other with $\sigma_H = 0.1$ (solution B). The relevant individual weights (p_A and p_B) are given in table 1 ‡. The second solution makes some allowance for photometric error and the two solutions should bracket the best value. As Table 2 shows the differences between the two solutions are negligible. In principle a mean value of $10^{0.2\rho}$ derived as above can be used to obtain directly the parallaxes of Cepheids. However since the scatter about a PL(V) relation is relatively small (especially if reddenings are derived from equation 3) there is negligible bias in deriving distance moduli from equation 1 with a value of ρ obtained directly from the mean value of $10^{0.2\rho}$ and it is convenient to use this value of ρ in the following.

In Table 2, solution 1 is for all the Cepheids in the present sample. The total weight, as defined above is given. The weight of α UMi is much greater than any other star in the sample (909 in solution A, 596 in solution B). Solution 2 therefore leaves out this star. The change in the value of ρ is negligible, although its standard error is naturely increased.

The EROS observations of LMC Cepheids (Beaulieu et al. 1995) show rather clearly that, as expected, overtone Cepheids obey the normal PLC relation (at their fundamental periods), but are in the mean brighter than a PL relation (and also bluer than a PC relation). Because of the present method of analysis, using equations 1 and 3 together with values of P_0, such a star will on the average give an estimate of the PL zero-point (ρ) with is too positive (i.e. too faint). For the present sample of stars the effect will be small. A comparison of reddenings from BVI photometry (or equivalent) from Caldwell and Coulson (1987) and the present ones for the ten overtone Cepheids in the present sample shows that the weighted mean difference, $\Delta E_{(B-V)}$, in the sense, BVI reddening minus PC reddening is only +0.001 (or +0.026 without αUMi). One would therefore expect that the error in deriving ρ from these stars would be negligible. In fact leaving out these stars as in solution 3 of Table 2 shows that the value of ρ remains essentially unchanged. Note that if any stars in the present discussion are unrecognized overtone pulsators, then they will tend to give too bright an estimate of the zero-point.

Leaving aside α UMi, ~ 75 percent of the total weight is in the 25 cepheids listed, together with α UMi, in Table 1. Solution 4 is for these 25 stars alone and solution 6 for these stars plus α UMi. Whilst the mean value of ρ is not significantly changed, its standard error is reduced compared with the solutions containing the bulk of the stars. This almost certainly implies that these general solutions include many stars with very small true parallaxes and the inclusion of such stars simply adds noise to the solutions. Solutions 4 and 6 thus appear to be preferred. Notice that the stars in these solutions were chose on the basis of weight and not on the basis of observed π. The weight depends on σ_π and is independent of π in the case of solution A and only weakly dependent on π in the case of solution B. Thus statistical bias should not affect the resulting solutions §.

Amongst the 25 Cepheids in solution 4 there are five (indicate by "o" in Table 1) which have been treated as overtone pulsators. Omitting these stars gives solution 5. In addition there are four stars (indicated by "L" in Table 1) which have relatively small amplitudes ($\Delta V \leq 0.5$mag). If these are actually overtone pulsators, solutions 7 and 8

‡ In compiling Tables 1 and 2 the weights have been rounded off to whole numbers which accounts for any discrepancy in totals.

§ A referee has raised the question of statistical bias of the Lutz-Kelker type. One reason for analysing the data in the way described was to minimize such effects. The method used may be thought of as accurately reducing all the Cepheids to the same, unknown, common distance. The (reduced) parallaxes then give estimates of this distance. This has much in common with combining the parallax determinations of individual stars in a star cluster to determine the mean parallax of the cluster with an error much smaller than that of the individual stars. Any bias is then related to the standard error of the final result (in the present case a 5 percent error in the distance). On the model adopted by Lutz and Kelker (1973) this implies that the Cepheid zero-point adopted in the present paper is too faint by about 0.02 mag. However even this small bias will be offset by a bias of about the same amount in the opposite direction. This arises because of the small, but finite (see text), spread in the Cepheid absolute magnitudes about the PL relation when the present method of reduction is employed. Malmquist-type bias indicates that for a uniform space distribution, the Cepheids in the sample will be about 0.02 mag too bright for their periods compared with an unbiased sample.

Table 2. Zero-point Solutions

Solution	N	Description	Solution A ρ	Solution A weight	Solution B ρ	Solution B weight
1	220	Whole Sample	-1.40 ± 0.11	1937	-1.40 ± 0.12	1598
2	219	Whole Sample minus αUMi	-1.39 ± 0.16	1029	-1.40 ± 0.16	1003
3	210	Whole Sample minus overtones	-1.42 ± 0.17	867	-1.43 ± 0.17	845
4	25	High Weight minus αUMi	-1.44 ± 0.13	776	-1.44 ± 0.13	751
5	20	High Weight minus Overtones	-1.49 ± 0.13	616	-1.50 ± 0.14	596
6	26	High Weight plus αUMi	-1.42 ± 0.09	1685	-1.43 ± 0.10	1347
7	25	High Weight (4 more as overtones)	-1.39 ± 0.14	741	-1.40 ± 0.14	716
8	26	Solution 7 plus αUMi	-1.40 ± 0.09	1649	-1.40 ± 0.10	1321
9		αUMi (fundamental)	-2.06 ± 0.14			
10		αUMi (first overtone)	-1.41 ± 0.14			
11		αUMi (second overtone)	-0.97 ± 0.14			

result. Despite its very low pulsation amplitude (e.g. Fernie et al. 1993) α UMi has often in the past been considered as a fundamental pulsator. Table 2 shows the values of ρ that result from considering it as pulsating in the fundamental, or the first or second overtone. Comparison with the solutions for the other Cepheids demonstrates rather clearly that the star is a first overtone pulsator and it has been used as such in the adopted solution. α UMi is the only Cepheid in the sample that gives a very useful individual zero-point. The star of next highest weight (δ Cep) yields $\rho = -1.51 \pm 0.37$.

The various solutions in Table 2 indicate that the mean value of ρ is rather insensitive to the precise solution adopted. On the basis of these results and giving high weight to solution 6, a value of $\rho = -1.43 \pm 0.10$ is adopted as the best value from the present data. The weighted mean $\log P_0$ for solution 6 is 0.80. The final result appears to be rather insensitive to the method of weighting adopted since the unweighted mean for the stars in either solutions 4 or 6 is $\rho = -1.44$.

2 DISCUSSION

The reddenings adopted in the present paper have a zero-point based on the reddenings of Cepheids in open clusters. Distances derived using the present PL(V) relation are independent of any error in this adopted reddening zero-point provided that reddenings with the same zero-point are used for any programme stars.

Until the present Hipparcos results, the most accurate zero-point for the PL relation came from Cepheids in open clusters and associations. A recent discussion of the results of this method has been given by Laney and Stobie (1994), based primarily on the compilation of Feast and Walker (1987). Fitting the 21 stars in their analysis to equation 1 above, and adopting a cluster distance scale based on a preliminary Hipparcos distance modulus to the Pleiades (5.67)(Penston 1994) gives $\rho = -1.37$. Feast (1993) estimated an uncertainty of ~ 0.1 for a determination of this type. There is obviously good agreement with the parallax results. However there is some uncertainty in the cluster and association results due to questions of Cepheid membership and the possibility of differential reddening in clusters. In addition, unlike the parallax results, the cluster determinations are sensitive to the adopted zero-point of the reddening scale. This is due to the sensitivity of the main sequence fitting procedure to reddening. For a consistently applied reddening scale a zero-point error in this of $\Delta E_{(B-V)}$ leads to an error in the distance modulus of a programme Cepheid of $\sim 2\Delta E_{(B-V)}$ (e.g. Feast 1991). It would be useful to reinvestigate this reddening zero-point in detail. However the discussion in Dean, Warren and Cousins (1978) suggests that $\Delta E_{(B-V)}$ might be several hundredths of a magnitude. Thus a systematic error in the Cepheid distance scale, based on clusters, of $\sim$ 0.05mag or perhaps more, seems possible. A comparison of the present PL(V) zero-point with that just derived from clusters shows that at least there is no very large error in the reddening zero-point, since if all the difference between the parallax and cluster zero-points is due to an error in the reddening zero-point this is $\Delta E \sim 0.03 \pm 0.07$.

In view of the fact that the Cepheid V data in the LMC is more extensive than at other colours (e.g. I) and that the most extensive photometric data on the parallax Cepheids is in B and V, it seems best to use the PL(V) relation derived above as the basic relation and to derive PL relations at other colours from it, using period-colour relations such as the $(<V>_0 - <K>_0)$ - log P relation for Galactic Cepheids derived by Laney and Stobie (1994). In cases where observations in two colours are made (e.g. in B,V or in V,I, as in the HST programme on the Hubble constant, H_0) there would seem to be a good deal to recommend the use of a period-colour relation to determine reddening and the PL(V) relation for distance. In the case of V,I data, Caldwell and Coulson (1987) have derived a $(V-I)_0$ - logP relation based on extensive data for Galactic Cepheids and this would appear to be useful for reddening determinations. (Note that their relation refers to magnitude means not intensity means.)

The above discussion assumes that the parallax Cepheids, which are all relatively close to the sun, are sufficiently similar in chemical composition that metallicity effects can be ignored. In applying the present results to Cepheids of different compositions, corrections need to be applied to both PL and PC relations. The demonstration that the mean temperature of Cepheids of a given period varies with metallicity (Laney and Stobie 1986, 1994) implies that the PL(V) relation has a greater sensitivity to metallicity than was at one time thought. Estimates of these corrections (Laney and Stobie 1994) range from +0.014 to +0.042 for the LMC with a metal deficiency of a factor 1.4 and from 0.036 to 0.082 for the SMC with a metal deficiency of a factor 4. In each case Laney and Stobie prefer the higher value. Adopting this higher value in the case of the LMC and using the adopted PL(V) zero-point (-1.43) together with the LMC $< V >_0$ - log P relation (Caldwell and Laney 1991) yield a true LMC distance modulus of 18.70 ± 0.10.

This value may be compared with the result derived from the ring round SN1987A. On the assumption that the supernova was 500pc in front of the LMC centre, Gould (1995) found an upper limit to the LMC distance modulus of 18.37 ± 0.04. This is disquietingly different from the Cepheid distance based on parallaxes. However Sonneborn et al. (1996) give revised data on the supernova which, together with the adopted relative distances of the SN and the LMC centre lead to an LMC modulus of 18.46 ± 0.10. They regard their quoted standard error as more realistic than Gould's but suggest that there might also be significant systematic errors. Since these workers indicate that the above figures should be regarded as preliminary, a further discussion seems unwarranted. van Leeuwen et al. (1996) have recently obtained a zero-point for the Mira PL relation from Hipparcos parallaxes of Galactic Miras and from this they derive an LMC modulus of 18.60 or 18.47 depending on whether or not the PL relation at K or m_{bol} is used. They suggest that the higher value is to be preferred. The uncertainty of this modulus was estimated to be less than 0.2 mag.

Some of the work on the HST key project to determine H_0, is based on the assumption that the Cepheids in the LMC are at a mean distance modulus of 18.5 (e.g. Freedman et al. 1994). Thus, other things being equal, an increase in the LMC modulus by 0.2 as derived in the present paper would decrease the value of H_0 by 10 percent. In fact this conclusion does not automatically follow since there may be differences between the true reddenings of the LMC Cepheids and that adopted in the HST work. In addition metallicity differences between the LMC and the target galaxies may result in significant metallicity corrections to the derived moduli (Beaulieu et al. 1996) especially since the colour used to derive reddenings is metallicity sensitive. Re-evaluation of the distances of galaxies based on Cepheids needs to take into account these problems and the revised PL relation derived in the present paper. However it is worth noting that if one accepts the relative distance moduli of M31 and the LMC (ΔMod $= 6.07 \pm 0.05$) as derived by Gould (1994) from Cepheids, taking into account estimated metallicity corrections, then on the basis of the present results, the M31 modulus is 24.77 ± 0.11. This is a 17 percent increase in distance over the widely used value of Freedman and Madore (1990).

One can calibrate the absolute magnitudes of the RR Lyrae variables using the Cepheid distance to the LMC and the data of Walker (1992) on RR Lyraes in LMC globular clusters. In this way one obtains $M_V(RR) = 0.25$mag at a mean metallicity of $[Fe/H] = -1.9$. Similarly the data on M31 globular clusters from Fusi Pecci et al. (1996) together with the M31 Cepheid modulus just derived implies $M_V(RR) = 0.36$mag at the same metallicity. Recently Chaboyer et al (1996) have derived an age for the oldest Galactic globular clusters (mean $[Fe/H] = -1.9$) of 14.56 Gyr based on $M_V(RR) = 0.6$mag at that metallicity. The mean of the LMC and M31 results is 0.3mag brighter than this, and using the relation between RR Lyrae absolute magnitude and age (Renzini 1991) lead to a revised age of the oldest Galactic globular clusters of $\sim$ 11Gyr. This is even less than that given in a recent discussion (Feast 1996) where the different methods of estimating $M_V(RR)$ are discussed in more detail, together with their uncertainties.

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