Exploring uncertainties in fireball modelling using estimators

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Introduction

Camera networks dedicated to observing fireball phenomena allow the bright flight trajectory of meteoroids to be triangulated. The evolution of a meteoroid throughout its flight can be modelled by a set of simple dynamic equations (after [1]):

$$\begin{aligned} \frac{dm}{dt} &= -\frac{1}{2}\kappa\sigma\rho_a v^3 m^{2/3} \\ \frac{dv}{dt} &= -\frac{1}{2}\frac{\kappa\rho_a v^2}{m^{1/3}} + g\sin\gamma_e \end{aligned} \tag{1}$$

where ρ_a, g and γ_e are the local atmospheric density, gravity and flight angle from horizontal respectively, the shape density coefficient $\kappa = \frac{c_d A_0}{\rho_m^{2/3}}$ (c_d being the drag coefficient and A_0 the shape parameter) and the ablation coefficient $\sigma = \frac{c_h}{H^* c_d}$ (c_h is the coefficient of heat and H^* the enthalpy of vaporisation).

In order to gain an understanding of the unknown variables, typical methods perform a least squares analysis and residuals are used as an indicator of overall model errors (eg. [1]). A more robust understanding of errors introduced by the model itself (1) as well as errors in observations can be examined by using tracking algorithms. The estimators to be discussed include the Extended Kalman Filter (EKF) as originally proposed by Sansom et al. [2]; the Unscented Kalman filter (UKF) and its inclusion in an Interactive Multiple Model estimator (IMM); and Sequential Importance Sampling Particle Filter (SISPF).

Tracking Algorithms

The state of a meteoroid at any discrete time step, k, may be represented by a state vector $\mathbf{x}_k = [position\,(l),\, velocity\,(v),\, mass\,(m)]$ and an associated covariance matrix, $\mathbf{P_k}$. Although brightness has not been incorporated at this stage, it can simply be included as an additional state parameter.

Tracking algorithms typically perform a *prediction* at time k using the system equations and includes a process noise $\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$. This is followed by an *update* where the observations (including observation noise $\mathbf{n}_k \sim \mathcal{N}(0, \mathbf{R}_k)$) are compared to the model prediction.

The non-linear system (1) requires non-linear estimations algorithms. An EKF predicts the future state covariance, $\mathbf{P}_{k+1}$, by using an approximate, linearised form of (1) for the state transition matrix [2]. An UKF uses a set of sample points to represent the mean state and covariance of a Gaussian distribution. These are individually propagated through (1) and the mean state and covariance recalculated. Although fragmentation is not explicitly included in the model, sudden increases in mass loss are incorporated by the process noise covariance, $\mathbf{Q_k}$, to a certain degree. By running two simultaneous UKFs in an IMM, with different values for mass in $\mathbf{Q_k}$, fragmentation events can be identified. All Kalman Filters require initial values for state parameters, κ and σ. This requires a preceding optimisation step using the least squares method (eg. [2]).

A statistical analysis that includes determination of likely starting parameters can be performed using the iterative Monte Carlo approach of a SISPF. A set of particles are initiated with a range of values for mass and velocity as well as for κ and σ (which are included as state parameters in $\mathbf{x}_k$). Each particle is propagated using (1) and its likelihood calculated based on observation values. A new set of particles are resampled from this pool resulting in a robust final estimate.

Conclusion

This presentation will outline the contrasting results of these different tracking methodologies using the flight trajectory of the Bunburra Rockhole meteoroid and assess the advantages and disadvantages of each.

References

[1] Ceplecha, Z. and ReVelle, D. O., MAPS, Vol. 40, pp. 35-54, 2005.

[2] Sansom, E. K., Bland, P. A., Paxman, J., Towner, M., MAPS, Vol 50, pp. 1423-1435, 2015.