Astrometry in Gaia DR1

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on behalf of the AGIS team and the rest of DPAC

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Outline of talk

The standard astrometric model
  ➔ kinematic and astrometric parameters

Use of priors in Gaia DR1
  ➔ TGAS and the secondary solution

Overview of the astrometric content of Gaia DR1
  ➔ parameters, uncertainties, excess noise

Limitations of Gaia DR1
  ➔ known or suspected biases

What can be expected from Gaia DR2?
The standard astrometric model for “stars”

In the standard astrometric model the Gaia source is assumed

- to be a point source (more precisely: have a well-defined photocentre), and
- to move through space at constant velocity relative to the Solar System Barycentre

This is probably a good approximation for >80% of unresolved Gaia sources beyond the Solar System (and for many resolved sources)
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In Gaia DR1 the standard astrometric model is used for all non-solar system objects (stars, binaries, AGNs, ...)

Deviations are indicated by the astrometric excess noise (explained later)

(Future DRs will use more specialised models for some sources - not discussed here)
Kinematic and astrometric parameters

Point source $\Rightarrow$ well-defined barycentric position vector $r(t)$
Uniform velocity $\Rightarrow r(t) = r_0 + (t - t_0)v$

**Kinematic model:**
- reference time $t_0$ and six kinematic parameters $x_0, y_0, z_0, \nu_x, \nu_y, \nu_z$

**Astrometric model:**
- reference epoch $t_{ep}$ and six astrometric parameters $\alpha, \delta, \varpi, \mu_\alpha^*, \mu_\delta, \nu_r$

The two sets of parameters are *in principle* equivalent, but:

- trivial and always possible - note: $t_{ep} = t_0 + |r_0|c^{-1}$
- difficult, and not always possible (e.g. when $\varpi \leq 0$)
Why use astrometric parameters?

Observations of Solar System can be modelled directly in barycentric coordinates $r(t)$

For stars and more distant objects the astrometric parameters are preferred:

- they can always be fitted to astrometric observations
- resulting errors are approximately Gaussian
- they work even for sources at “infinite” distance
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Example

Astrometric parameters in Gaia DR1 for the quasar 3C273 (HIP 60936):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{ep}$</td>
<td>2015.0 (chosen)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$187.277915798^\circ \pm 0.312$ mas*</td>
</tr>
<tr>
<td>$\mu\alpha^*$</td>
<td>$-0.384 \pm 0.443$ mas/yr</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$+2.052388638^\circ \pm 0.216$ mas</td>
</tr>
<tr>
<td>$\mu\delta$</td>
<td>$+0.111 \pm 0.288$ mas/yr</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$-0.140 \pm 0.377$ mas</td>
</tr>
<tr>
<td>$\nu_r$</td>
<td>0 (assumed)</td>
</tr>
</tbody>
</table>
Three remarks on the astrometric parameters

1. The sixth parameter, radial velocity \( v_r \) (or radial proper motion \( \mu_r = \varpi v_r / A \)), is ignored in Gaia DR1 (assumed = 0)
   - important for a small number of nearby, high-velocity stars (not in DR1 anyway)
   - gives a quadratic variation of position (perspective acceleration)
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2. The astrometric parameters describe the instantaneous motion at the specified reference epoch $t_{\text{ep}}$ (= 2015.0 for Gaia DR1)
   - especially the position parameters ($\alpha$, $\delta$) depend on $t_{\text{ep}}$ due to proper motion
   - the parameters can be transformed to any desired epoch (see documentation)
   - future releases will use a different reference epoch than 2015.0
   - “epoch” not to be confused with “equinox” (e.g. J2000.0 = ICRS)
   - the “equinox” is always the same: ICRS
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3. The asterisk signifies that a differential quantity in $\alpha$ is a “true arc”:
   
   $$
   \mu_{\alpha*} = \frac{d\alpha}{dt} \cos \delta, \quad \sigma_{\alpha*} = \sigma_{\alpha} \cos \delta, \quad \Delta\alpha* = \Delta\alpha \cos \delta
   $$
\[ \Delta \alpha^* = \Delta \alpha \cos \delta \]
Use of priors in Gaia DR1

Four different kinds of “prior information” are used in Gaia DR1:

For all sources:

sources have no radial motion ($v_r = 0$)

→ this is usually an acceptable approximation (except for nearby high-$\mu$ stars)

For the primary (TGAS) solution:

positions at epoch 1991.25 are known from Hipparcos or Tycho-2

→ provides useful proper motions and parallaxes with only ~1 year of data

For the secondary solution:

parallaxes and proper motions are small for most stars (“Galactic prior”)

→ gives positions at 2015.0 with realistic uncertainties

For the auxiliary quasar solution:

quasars have negligible proper motion

→ accurate quasar positions for alignment with the VLBI reference frame (ICRF)
Number of sources and parameters in Gaia DR1

<table>
<thead>
<tr>
<th>Solution</th>
<th>No. of sources</th>
<th>Param.</th>
<th>Prior used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary (TGAS) sources</td>
<td>2 057 050</td>
<td>5</td>
<td>positions at 1991.25</td>
</tr>
<tr>
<td>- of which Hipparcos</td>
<td>93 635</td>
<td>5</td>
<td>- Hipparcos positions</td>
</tr>
<tr>
<td>- of which Tycho-2 (excl Hipp)</td>
<td>1 963 415</td>
<td>5</td>
<td>- Tycho-2 positions</td>
</tr>
<tr>
<td>Secondary sources</td>
<td>1 140 622 719</td>
<td>2</td>
<td>(\varpi, \mu_\alpha, \mu_\delta = 0 \pm \text{few mas/yr})</td>
</tr>
<tr>
<td>ICRF sources (*)</td>
<td>2 191</td>
<td>2</td>
<td>(\mu_\alpha, \mu_\delta = 0 \pm 0.01 \text{mas/yr})</td>
</tr>
<tr>
<td>Total</td>
<td>1 142 679 880</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*) 2080 of the ICRF sources are also secondary sources (with slightly different positions)

References:
Michalik et al. 2015, A&A 574, A115 (TGAS)
Michalik et al. 2015, A&A 583, A68 (secondary solution)
Lindegren et al. 2016, arXiv:1609.04303 (Gaia DR1 astrometry in general)
Magnitude distributions of Gaia DR1
Primary (TGAS) sources

2.06 M sources, mainly $G < 11.5$
  - this is about 80% of the Hipparcos & Tycho-2 catalogues

Missing sources:
  - brights stars ($G < 6$)
  - high-proper motion stars ($\mu > 3.5$ $$/yr$)
  - some 20% of Hip + Tycho-2 with too few observations
    (quasi-random but with large variations over the sky)

Median **position** uncertainty: 0.23 mas at 2015.0

Median **parallax** uncertainty: 0.32 mas

Median **proper motion** uncertainty:
  - 0.07 mas/yr (Hipparcos subset)
  - 1.2 mas/yr (Tycho-2 subset)

Note difference!
TGAS: Sky coverage (equatorial map)

Mean density per pixel (~1 deg$^2$)
TGAS: Standard uncertainty in proper motion (semi-major axis of error ellipse) - All sources

Median uncertainty per pixel (~1 deg$^2$)

Overall median = 1.32 mas/yr
TGAS: Standard uncertainty in proper motion (semi-major axis of error ellipse) - Hipparcos

Median uncertainty per pixel (~16 deg$^2$)

Overall median = 0.07 mas/yr
TGAS: Standard uncertainty in parallax

Median uncertainty per pixel (~1 deg$^2$)

Overall median = 0.32 mas
Can TGAS parallaxes be trusted?

- TGAS and Hipparcos parallaxes are independent!
- Comparison confirms global quality of Hipparcos and Gaia
- Analysis provides realistic error estimates
- Realistic errors are published in Gaia DR1

![Comparison of TGAS and Hipparcos parallaxes](image)
Improved distances to nearby stars

Hipparcos

Gaia DR1 (TGAS)
More stars within parallax horizon ($\varpi/\sigma_\varpi > 5$)
Astrometric quantities in Gaia DR1

Important:

- source_id
- ref_epoch (always = 2015.0 in DR1)
- ra, dec
- ra_error, dec_error
- astrometric_excess_noise

- hip, tycho2_id
- parallax, pmra, pmdec
- parallax_error, pmra_error, pmdec_error

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Less important, but still very useful:
- correlations (ra_dec_corr, etc)
- astrometric_delta_q

\[ \{ \text{HIP subset of TGAS only} \} \]
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**For specialists:**
- number of observations (astrometric_n_*)
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Astrometric excess noise: Background

The astrometric solution can be formulated as a chi-square minimisation problem

$$\arg\min \ X^2(s,a,c) = \sum_{\text{sources } i} \sum_{\text{obs } j \in i} \left( \frac{R_{ij}}{\sigma_{ij}} \right)^2$$

where $s$, $a$, $c$ are the source, attitude, calibration parameters, $R_{ij}$ the residuals of source $i$ in observation $j$, and $\sigma_{ij}$ the formal uncertainty of the observation.

If the model is correct, we expect $X^2_{\text{min}} \sim \chi^2_n$, so $X^2_{\text{min}} / n \simeq 1$

where $n = \text{degrees of freedom}$.

In practice the model is never correct, at least not for all sources, so typically we find $X^2_{\text{min}} / n \gg 1$, too much weight are given to bad sources, and the uncertainties of $s$, $a$, $c$ are underestimated.
Astrometric excess noise: Definition

The problem is instead formulated as

\[ \arg \min \ X^2(s, a, c) = \sum_{\text{sources } i} \sum_{\text{obs } j \in i} \frac{R_{ij}^2}{\sigma_{ij}^2 + \varepsilon_i^2} \]

For every source, the excess source noise \( \varepsilon_i \) is set to the smallest value for which

\[ \sum_{\text{obs } j \in i} \frac{R_{ij}^2}{\sigma_{ij}^2 + \varepsilon_i^2} \leq n_i \]

where \( n_i \) is the number of degrees of freedom for source \( i \)

Remarks:

• The excess noise is an angle (in mas)
• Binaries and other badly fitting sources should get large values of \( \varepsilon_i \)
• Unfortunately, attitude and instrument modelling errors also increase \( \varepsilon_i \)
Excess noise versus magnitude (TGAS)
Excess noise versus magnitude (TGAS)

modelling errors for very bright sources

G magnitude [mag]

Astrometric excess noise [mas]
Excess noise versus colour index (TGAS)
Excess noise versus colour index (TGAS)

modelling errors for sources of non-central colour (chromaticity)
Excess noise distribution (TGAS/Hip)

The fraction of problematic sources increases with the excess noise.
Binary sequence in HR diagram
Binary sequence in HR diagram
Distribution of excess noise for sample S and B

Excess noise > 1 mas is twice as common in sample B as in S
Systematic errors (bias) in Gaia DR1

There are systematic errors in Gaia DR1!
Systematic errors (bias) in Gaia DR1

There are systematic errors in Gaia DR1!

They are complicated (and largely unknown) functions of many things:
  position, magnitude, colour, number of observations, prior used, ...
Systematic errors (bias) in TGAS parallaxes: - Comparison with Hipparcos (FvL 2007)

Colour & position dependent systematics on the level ±0.1 mas

Lines connect median values in 50 colour bins
Systematic errors (bias) in TGAS parallaxes: Comparing solutions from split FoV

“Late” minus “early” data
Systematic errors (bias) in TGAS parallaxes: Comparing solutions with and w/o colour terms

“w/o” minus “with” data

Parallax difference [mas]
Systematics in Gaia DR1 parallaxes

Due to known limitations in the astrometric processing
- a global offset of ±0.1 mas may be present
- there are colour dependent, spatially correlated errors of ±0.2 mas
- over large spatial scales, parallax zero point errors reach ±0.3 mas
- in a few small areas even ±1 mas

Parallax uncertainties should be quoted as
\[ \varpi \pm \sigma_{\varpi} \text{ (random)} \pm 0.3 \text{ mas (syst.)} \]

Averaging parallaxes e.g. in a cluster does not reduce the systematics!
Reference frame from observations of quasars

Gaia DR1 is aligned with the International Celestial Reference Frame through Gaia’s observations of ~2000 faint (17-20 mag) quasars with accurate VLBI positions.

Gaia’s observations show:

1. Excellent agreement between radio and optical positions (RMS < 1 mas)

2. That the Hipparcos reference frame rotates wrt QSOs by 0.24 mas/yr
Secondary solution: Reality check on new sources (overlay on HST image - in Baade’s Window)

Yellow = IGSL (input list)
Blue = new
What can be expected from Gaia DR2?

- Will be completely independent of Hipp/Tycho-2
- Based on a longer stretch of data (22 versus 14 months)
- Improved attitude and instrument models will reduce the modelling errors and hence both random and systematic errors in results
- Parallax accuracies of about 50 μas can be reached for sources down to G ~ 15 mag, larger errors for fainter sources
The diagram illustrates the distribution of stars in the Gaia DR1 dataset, showing a comparison with the Hipparcos catalog (van Leeuwen 2007). The plot displays the magnitude on the x-axis and the logarithm of the position error (log10(σ_x)) on the y-axis. The Gaia DR1 data is marked with a green region, while the Hipparcos data is shown in black. The Gaia 5-year mission is indicated by a red line that extends to a magnitude of approximately 20.0.
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• Proper motions of about 100 μas yr⁻¹ (comparable to the Hipparcos subset of TGAS) down to G ~ 15 mag
• This will be obtained for many tens of millions of sources
• Improved and more photometry (G, BP, RP) will enhance the scientific usefulness enormously
• Gaia DR1 is a good training set to get prepared for the real thing!