Forest Ray Moulton and his plans for a new lunar theory

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Early years

At the University of Chicago

- 1892: University of Chicago founded
- 1895: Begins as a graduate student
- Astronomy Dept. = G.E. Hale, T.J.J. See and K. Laves
- Attends courses in Astronomy and in Mathematics (E.H. Moore, O. Bolza and H. Maschke)
- 1896: See departs, Moulton begins lecturing in Astronomy
- 1899: PhD *summe cum laude*: 'Periodic Oscillating Satellites'
- 1900: Instructor; 1903: Assistant professor; 1908: Associate professor; 1912: Full professor

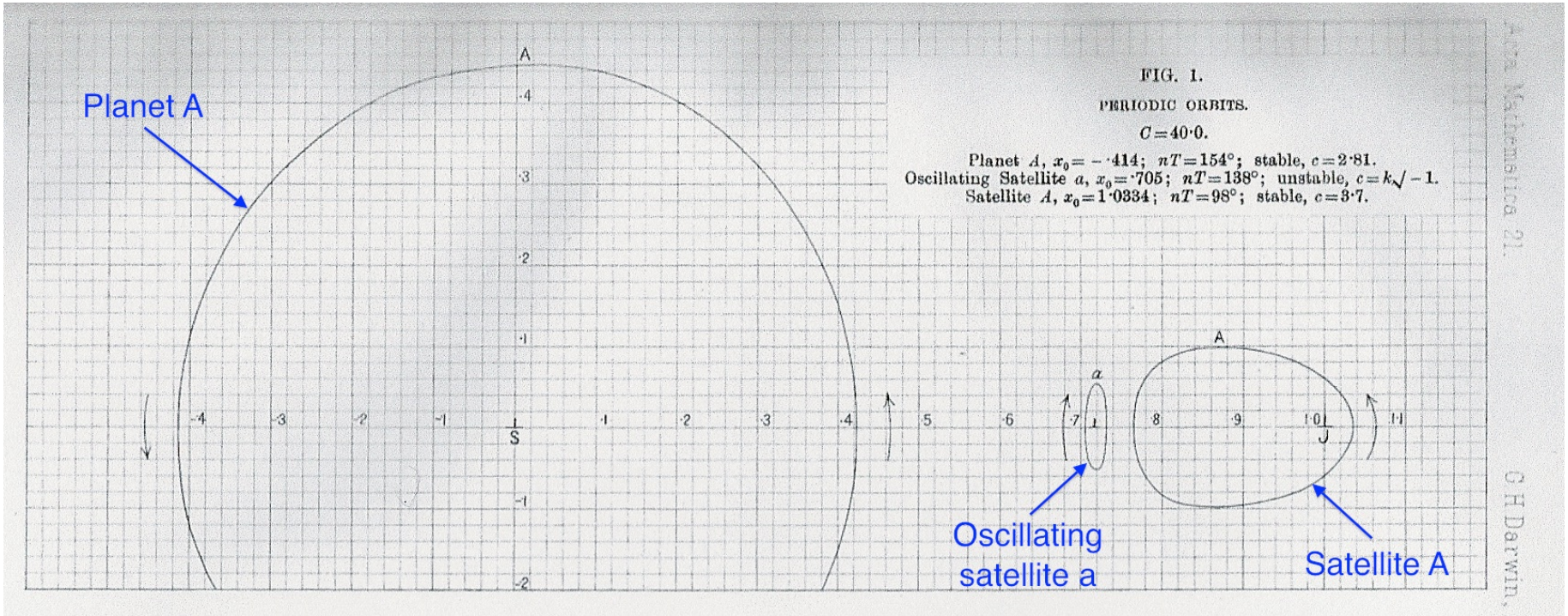
The Goal of Celestial Mechanics

- Goal of celestial mechanics is verification of Newton's law of universal gravitation [Poincaré, *Les Méthodes nouvelles*, 1892]
- Lunar theory is best-known example of three-body problem
- G.W. Hill [1877] wanted to see whether Newton's law could fully account for motion of lunar perigee

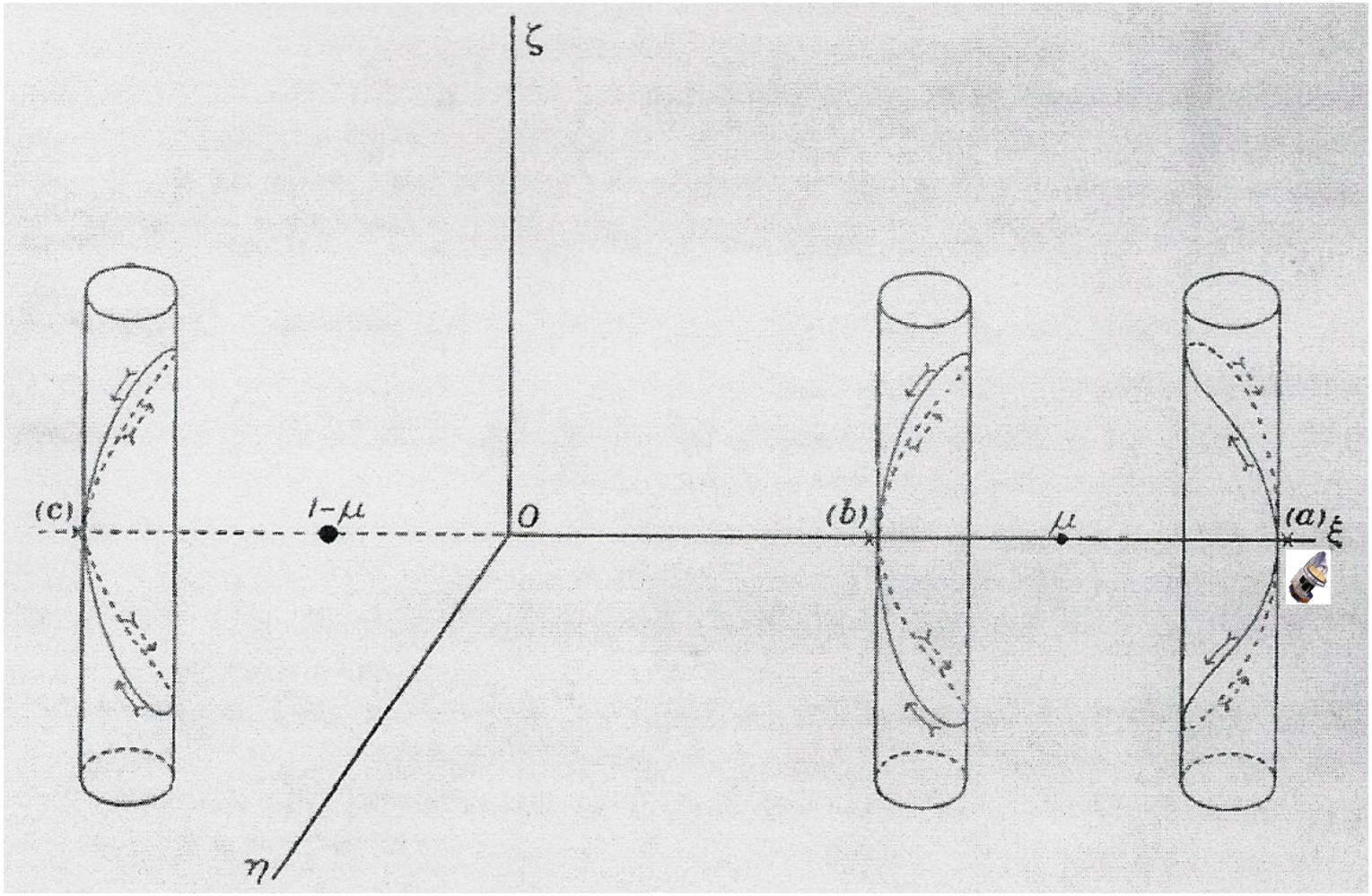
The Study of Periodic Orbits (POs)

- Hill's variational orbit [1878]:
  - First particular solution to be discovered since [Lagrange, 1772]
  - New family of POs
- Importance of POs stressed by Poincaré
  "No subject held his attention longer" [Hadamard, *L'œuvre mathématique de Poincaré*]
- Non-existence theorems:
  - Jacobi [1843]: reduced 3-body problem to order 6
  - Bruns [1887]: no new algebraic integrals (rectangular coords.)
  - Poincaré [1892]: no new uniform analytic integrals (Delaunay variables, ...)
- "Mr. Moulton seems to me to have marked ability and promise as research-man ... he is expecting to go into the field opened up by Poincaré" (Moore to Hale, May 1899)

Darwin's examples of Periodic Orbits



Oscillating satellites



1902: Early interest in a rigorous lunar theory

- Gives lecture course on lunar theory
- Writes 10-page review (Bulletin of AMS) of Brown's treatise
- Presents paper at AAAS meeting: *A representation of the coördinates of the moon in power series which are proved to converge for a finite interval of time*. Two methods:
  - Application of Mittag-Leffler's generalized power series
  - Expands coordinates as power series of certain parameters
- Publishes *On certain rigorous methods of treating problems in celestial mechanics* (University's Decennial Publications)

Moulton's Lunar Theory

- Aug. 1906: Moulton applies to Carnegie Institution of Washington for financial assistance with computations
- Moulton's lunar theory:
  - Use of rigorous mathematical methods
  - Use of power series instead of Fourier series
  - Parameters:
    - ★ Keep: longitude, longitude of node and longitude of perigee
    - ★ mean distance $\rightarrow$ mean angular motion
      inclination $\rightarrow$ mean motion of line of nodes
      eccentricity $\rightarrow$ mean motion of perigee
    - ★ Values determined more easily from observations

Moulton's Lunar Theory (cont.)

That is, an exact periodic orbit can be constructed in which the distribution of masses is the same as in the earth-moon-sun system, in which the mean motions of the bodies, the mean motions of the nodes and of the apsides, as well as the phase parameters, are all given by the observations. It is hoped and believed that it can be shown theoretically that the moon's motion may be represented within any assigned limits for any assigned time by such a periodic orbit.

Moulton to Woodward, 30 Aug. 1906, Carnegie Institution of Washington.

Periodic orbit conjecture (Poincaré)

Given equations of the form defined in §13 [i.e. a conservative Hamiltonian system with a Hamiltonian function of a certain form] and any particular solution of these equations, one can always find a periodic solution (the period of which, it is true, may be very long) such that the difference between the two solutions is as small as one wishes, for as long a time as one wishes.

[Poincaré, *Les Méthodes nouvelles*, 1892]

First step in Moulton's lunar theory

- *A Class of Periodic Solutions of the Problem of Three Bodies with Application to the Lunar Theory* [Moulton, 1906]
  - Provides starting point of his lunar theory
  - Finds literal series in parameter $m = \frac{n'}{n-n'} = \frac{\text{synodic month}}{\text{sidereal year}}$
  - Includes variational terms [Hill, 1878] and parallactic terms [Brown, 1892]

Moulton's method of finding periodic orbits

1. Prove existence using *Poincaré's method* of analytic continuation
   - Given a system of analytic differential equations

$$\frac{dx_i}{dt} = X_i(x_j; \mu), \quad 1 \leq i, j \leq n, \tag{1}$$

     and a T-periodic *generating solution* $x_i = \phi_i(t)$ for $\mu = 0$.

   - Analytically continue solution to obtain power series $x_i = p_i(\beta_j; \mu; t)$ in $\beta_j \equiv x_j(0) - \phi_j(0)$ and μ.

   - Impose periodicity conditions:

$$\psi_i(\beta_j; \mu; \tau) \equiv p_i(\beta_j; \mu; T + \tau) - p_i(\beta_j; \mu; 0) = 0. \tag{2}$$

     Cauchy-Poincaré theorem guarantees convergence of $\psi_i(\beta_j; \mu; \tau)$ in $0 \leq t \leq T_1$, where $T_1 > T + \tau$, for sufficiently small β_j, μ.

   - Solve (2) for the n unknowns $\beta_1, \ldots, \beta_{n-1}$ and τ as power series in μ ($\beta_n = 0$).

Moulton's method (cont.) / Extension of lunar theory

2. Construct solution using *method of undetermined coefficients*
   - Substitute solution of assumed form (as found from Step 1) into differential equations (1)
   - Equate coefficients of the various powers and solve *seriatim*

- Lunar theory had been extended (unpublished)
  - Inclusion of solar eccentricity
  - Inclusion of lunar inclination (motion of line of nodes)
  - Inclusion of lunar eccentricity (motion of line of apsides)
- Step 2 produces linear differential equations of the form:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} \left[a_{ij} + \sum_{k=1}^{\infty} \theta_{ij}^{(k)}(t)\lambda^k \right] x_j, \tag{3}$$

  where the a_{ij} are constants and the $\theta_{ij}^{(k)}$ are 2π-periodic in t.
- Resulted in: *On the Solutions of Certain Types of Linear Differential Equations with Periodic Coefficients* [Moulton and MacMillan, 1911]

The grant application

- Application (September 1906):
  - Six people, each calculating 18 hr./week
  - 3 years @ $2200/yr.
  - Rejected on 19 Sep 1906
- April 1907 (Moulton to E. Frost[1]):
  - Work on lunar theory progressing steadily
  - Results ready in 2–3 months
- June 1907 (Moulton to E. Frost):
  - Delayed due to presence of a numerical error

[1] Director of Yerkes Observatory: 1905–1932.

The birth of *Periodic Orbits*

- June 1908: Plans to publish a book on Periodic Orbits in 1909
  - To contain an entirely new lunar theory
  - Will give them undisputed leadership
- January 1909:
  - Becomes a Research Associate of Carnegie Institution of Washington
    - ★ Grant of \$2000/yr.
    - ★ Relinquishes half of his teaching duties
  - Decides to split the book into two volumes
    - ★ Volume 1: *Periodic Orbits*
    - ★ Volume 2: Application to the lunar theory
- June 1909:
  - Volume 1, *Periodic Orbits*, is finished ($\sim$550 pages)
  - But would like to add a couple of extra chapters

The completion of *Periodic Orbits*

- Sep 1920: *Periodic Orbits* is published!
- 1909–1912:
  - Periodic orbits of superior planets (Ch.12)
  - Closed orbits of ejection (Ch.15)
  - Oscillating satellites when the finite masses describe elliptical orbits (Ch.7)
- 1911–1916:
  - Synthesis of periodic orbits in the restricted problem of three bodies (Ch.16)
    - ★ Abandoned quest for absolute rigour
    - ★ Complemented analysis with numerical computations
    - ★ "[T]he discussion remains in certain respects incomplete" [Moulton, Periodic Orbits, p.v]

Final delays / Book's reception

- 1917–1920 (final delays):
  - Death (meningitis) of 3-year old son Forest Ray Jr. (Jan 1917)
  - Service in army (Mar 1918 – Apr 1919)
  - Periods of illness
- Moulton's *Periodic Orbits* generated little interest:
  - General theory of relativity (1915) established (perihelion of Mercury, deflection of light)
  - Sundman's (1912) general solution of 3-body problem:
    - ★ "Killed interest in this line of inquiry" [Saari, 1990]
    - ★ Estimated $10^{8,000,000}$ terms needed to match observational accuracy [Belorizky, 1931]
  - Publication (1919) of Hill-Brown lunar theory tables (0.01″ accuracy)
- Jan 1927: Moulton resigns from University of Chicago[2]

[2] 1927–37: Financial Director (Utilities Power and Light Corporation of Chicago); 1937–46: Permanent secretary (AAAS).