LISA Pathfinder: TT-gauge and geodesy

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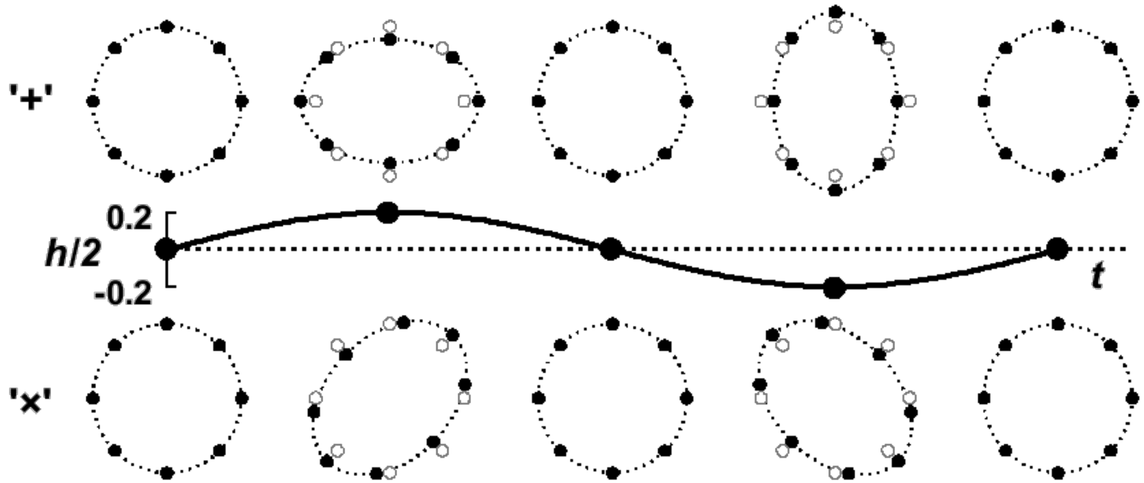
(11 days to LHC first beam)

"entia non multiplicanda sunt praeter necessitatem"

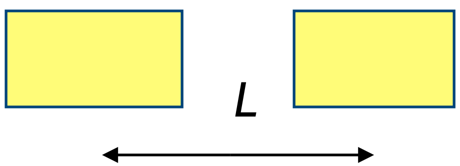
- GW: simplest thread connecting the pieces!
  - Curvature/pure-Tensor gravity (GR)
  - Gauge invariance/Symmetries
  - Causality/Finite speed of signal propagation
  - Necessity of wave viewpoint
  - Quantum remnants (spin 2 field)
- But! Only indirect proofs until now (quite stringent thought!)
  - PSR1913+16 (ω_{GW}~70μHz, $h_{GW} \sim 7\times10^{-23}$)

GW distort space-time

- Ring of objects



- Free test particles



$$\frac{\delta L}{L} \sim \frac{h}{2}$$

Geodetic motion is the key!

- As detectable gravity is the tide...

$$\Phi(x) = \Phi(0) - \sum_j g_j x_j + \boxed{\sum_{i,j} R_{i0j0} x_i x_j} + \dots$$

- Einstein's view of tides ⇨ (nearly) free falling observers (1) as frame markers (TT-gauge)
- Difference of acceleration (a/m~Φ''~R) is a true detectable effect ⇨ gauge-invariant marker of proper distance (2)
- Minimize disturbances ⇨ high sensitivity (3) and "assisted flight" for observers (drag-free)

1. Particles in free-fall as frame markers

$$\frac{\mathrm{d}^2 x^\alpha}{\mathrm{d}\tau^2} + \Gamma^\mu{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0$$

$$h_{\mu 0} = 0\,, \qquad \Gamma^i{}_{00} = \Gamma^0{}_{00} = \Gamma^0{}_{0j} = 0\,,$$

$$\eta_{ij}h^{ij} = h_i{}^i = 0\,,$$

$$h_{ij;j} \simeq h_{ij,j} = 0$$

$$\boxed{\begin{aligned}\Gamma^0{}_{jk} &= -\frac{1}{2}h_{jk,0}\,.\\ \Gamma^i{}_{0j} &= \frac{1}{2}h_j{}^i{}_{,0}\end{aligned}}$$

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}t^2} = \cancel{\left(-2\Gamma^i{}_{0j} - \Gamma^i{}_{jk}v^k + \Gamma^0{}_{jk}v^i v^k\right)} v^j$$

$$v^j = 0$$

"TT-gauge" → unique marking of coordinates, stretching of "distances"

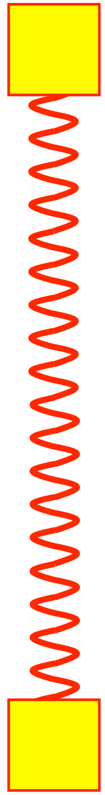
2. Phase of laser light as ruler!

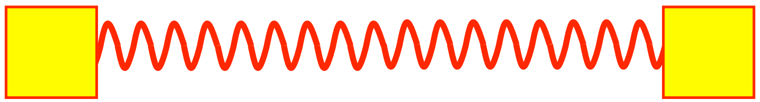
- Light wave vector is directly affected by Riemann $k_\mu k^\mu = 0$
- Interferometer, 2 non-parallel arms (quadrupole)

$$\Delta\phi = \omega\left(\Delta x(2+h_{xx}) - \Delta y(2+h_{yy})\right)$$

$$\boxed{\frac{d^2\Delta\phi}{dt^2} = 2\omega L\left(R_{tyty} - R_{txtx}\right)}$$

$$\frac{\mathrm{d}\,\Delta\theta_{\mathrm{laser}}(t)}{\mathrm{d}\,t} \simeq \frac{\pi c}{\lambda_{\mathrm{laser}}}\left(h(t) - h\left(t - \frac{2L}{c}\right)\right)$$

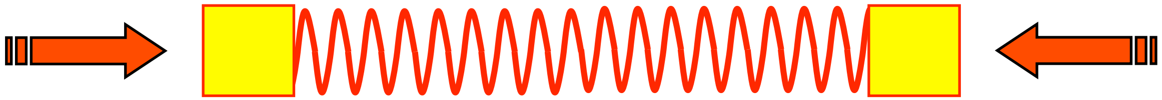




3. Detection and noise

$$\frac{\Delta\omega(t)}{\omega_0} \simeq \frac{1}{2}\left(h(t) - h\left(t - \frac{2L}{c}\right)\right)$$

$$\frac{\Delta\omega(t)}{\omega_0} = \frac{1}{c}\left(v_1(t) - 2v_2\left(t - \frac{L}{c}\right) + v_1\left(t - \frac{2L}{c}\right)\right)$$



Parasitic force fluctuations change distances and mimic gravitational waves!

Commercial time!

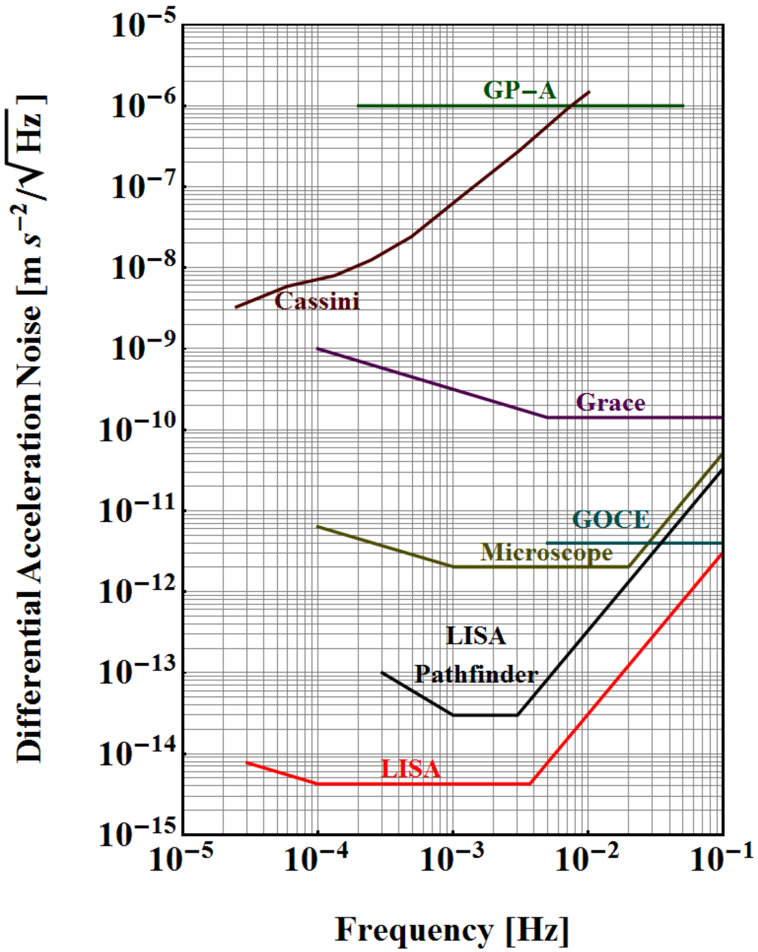
- Google (type “michele armano”)
  http://www.science.unitn.it/~armano/michele_armano_phd_thesis.pdf

- Spires-HEP website (type “find a armano and t pathfinder”)
  http://arXiv.org/pdf/gr-qc/0504062

- Minor references... ;-)
  - Misner, Thorne, Wheeler, “Gravitation”, Freeman 1973
  - Maggiore, “Gravitational Waves”, Oxford 2007

Geodesy: suppression of disturbances

- Stray electromagnetic fields
- Thermal radiation pressure
- Impact of molecules
- Coupling to wrong reference frame
- Other surface phenomena
- Fluctuating gravitation
- ...



$$S^{1/2}_{\Delta F/m,\mathrm{LTP}}(\omega) = 3 \times 10^{-14} \left(1 + \left(\frac{\omega}{2\pi \times 3\,\mathrm{mHz}}\right)^4\right)^{1/2} \mathrm{m}/\mathrm{s}^2\sqrt{\mathrm{Hz}}$$

Experimental runs

6.5 Mission goal: the physical model

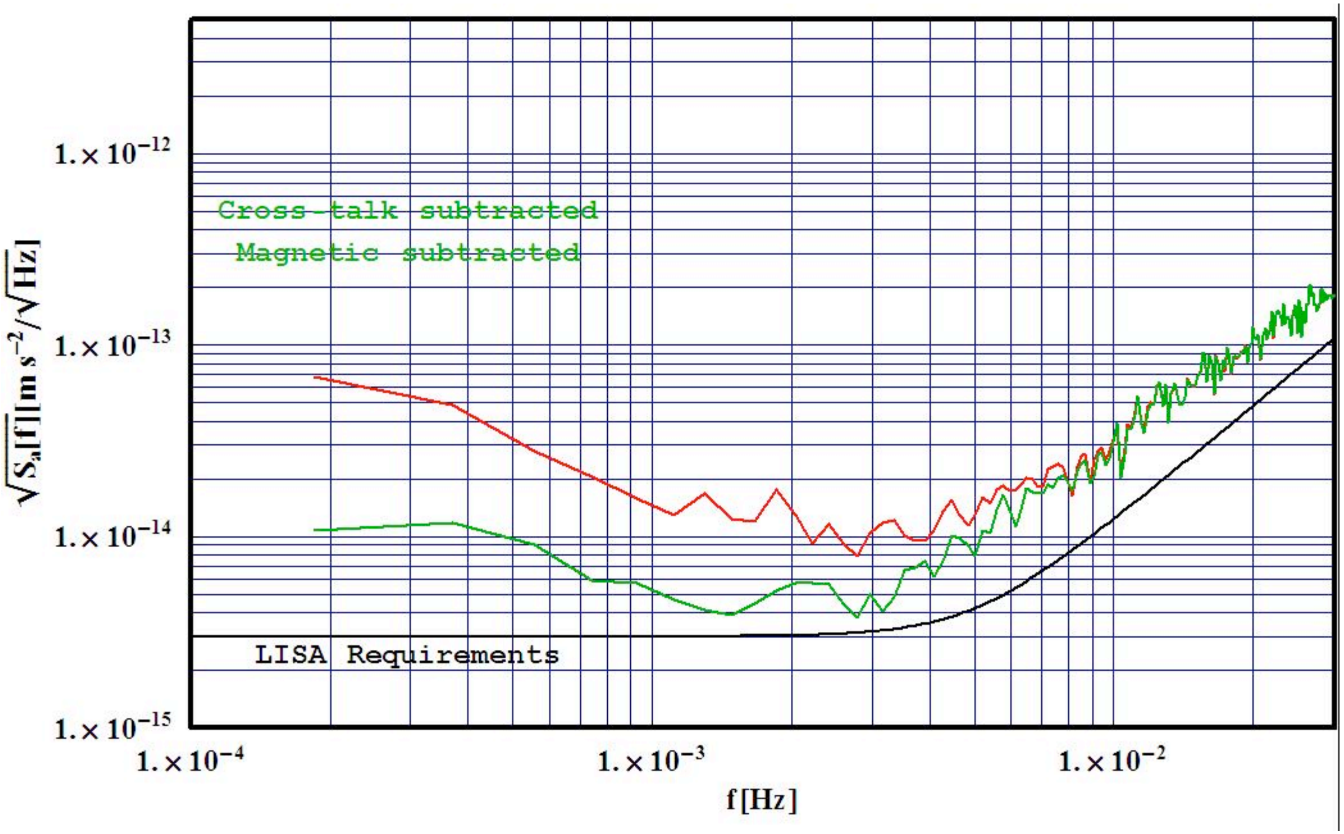
The final objective of LISA Pathfinder is to confirm the overall physical model of the forces that act on a test-mass in interplanetary space. To fulfil this program, the mission is not going to just make a measurement of acceleration but will implement a full menu of measurements:

- Measurement of acceleration noise between 0.0001 and 1 Hz.
- Measurement of dc-forces
- Measurement of force gradients
- Calibration of control loop transfer functions
- Characterization of thrust and thrust noise of micro-thrusters
- Measurement of interferometer performance and interferometer cross-talk
- Measurement of all cross-talk coefficients among different degrees of freedom
- Test of continuous charge measurement
- Test of continuous discharging and of discharging induced noise
- Test of magnetic induced noise
- Test of thermally induced acceleration noise
- Characterization of charging environment.

At the end of this set of measurements, the noise model will be verified down to painstaking detail.

Experiment main goals

1. Demonstrate that total acceleration noise in realistic conditions is not larger than goals
2. Pave the way for LISA:



M.Armano: LTP, the geodesy lab

1st IDSW, 28-29/08/2008

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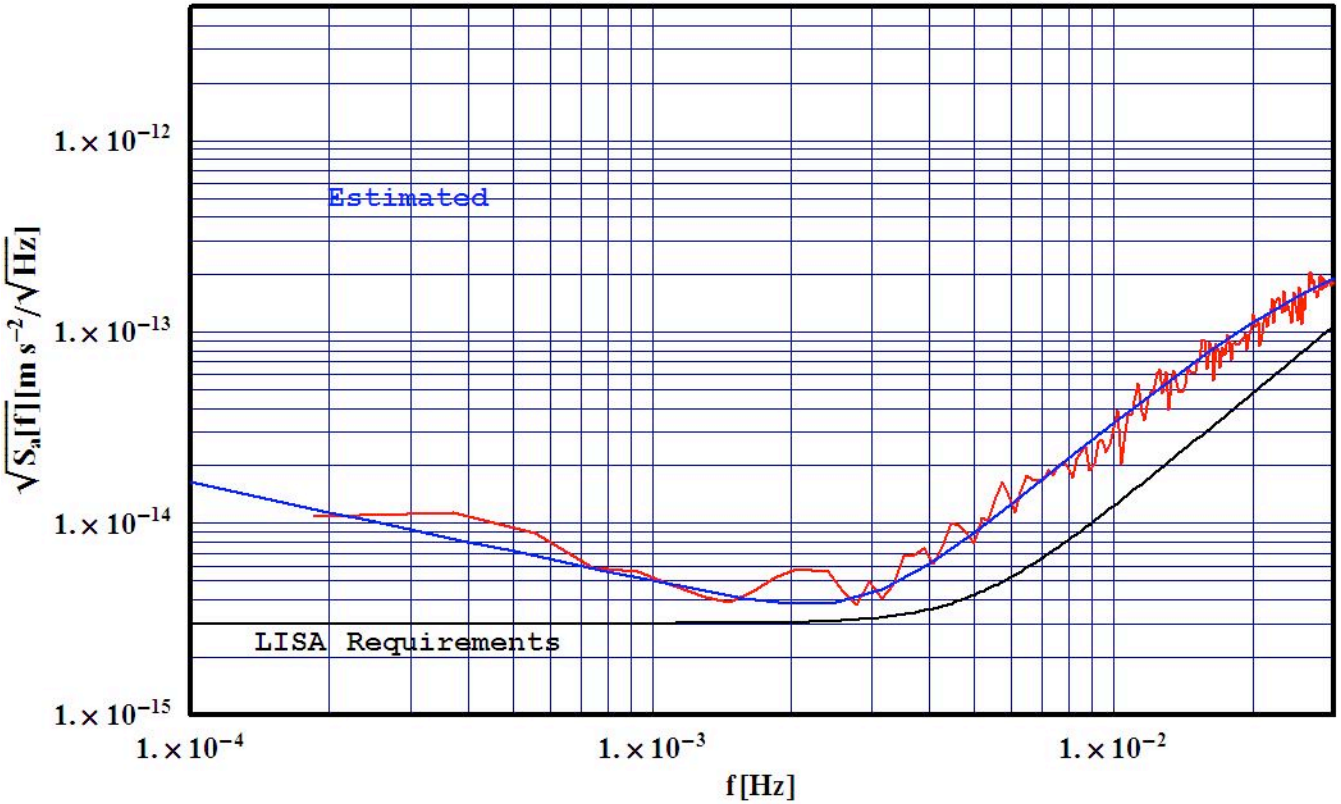

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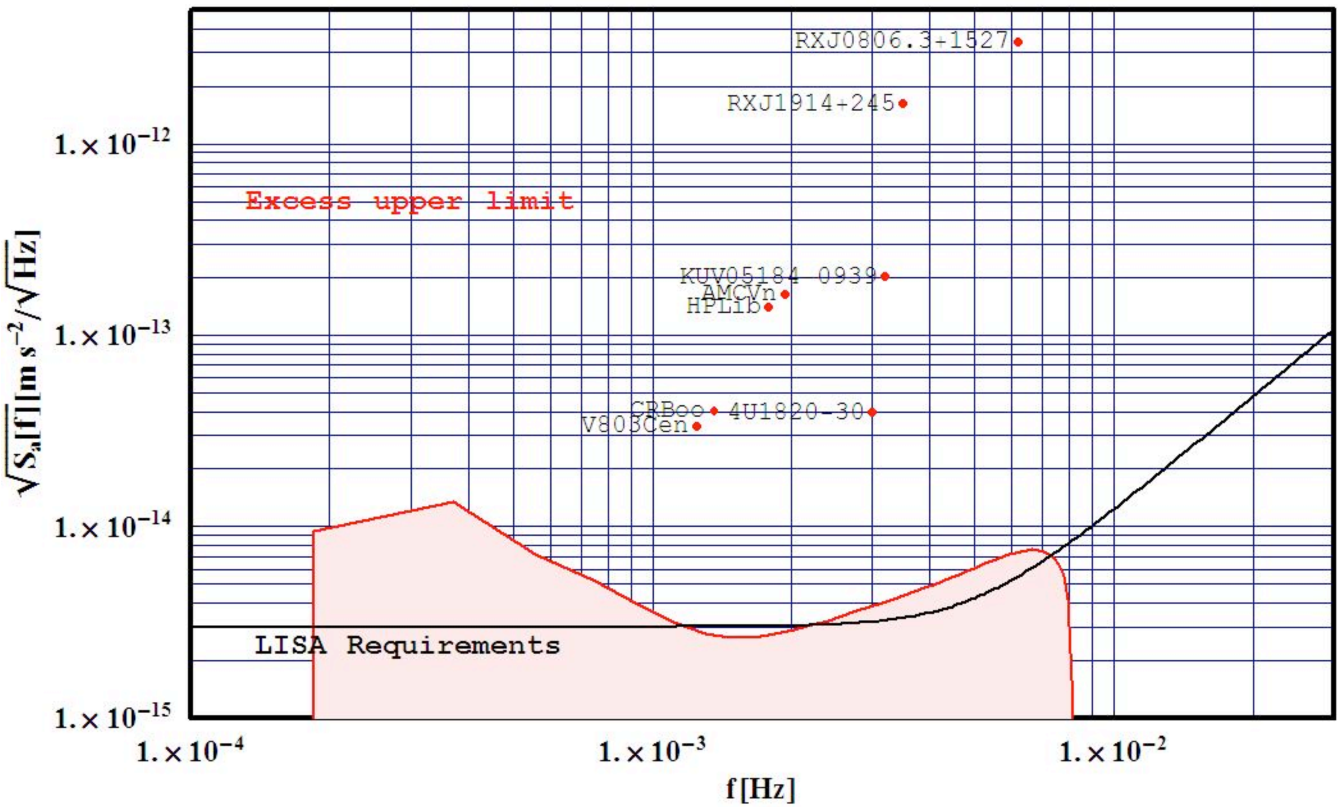


- The binary systems? We can see them (with correct L)!

1. Cross-talk (TT: how slant?)

$$(M s^2 + K)\cdot \boldsymbol{x} = \boldsymbol{f}\,,$$

$$\boldsymbol{f} = I_{\rm aff}\cdot\left(\boldsymbol{f}_0 - \hat{\Lambda}\cdot \boldsymbol{o}\right) + \boldsymbol{f}_n\,,$$

$$\boldsymbol{o} = \Omega\cdot\boldsymbol{x} + \boldsymbol{o}_0 + \boldsymbol{o}_n$$

Stiffness 5E-3	$K \rightarrow K + \delta K\,,$
Signal 1E-4..1E-3	$\Omega \rightarrow \Omega + \delta\Omega\,,$
Actuation 5E-3	$I_{\rm aff} \rightarrow I + \delta A\,,$
DC effects 1E-8 (m/rad s^2)	$\boldsymbol{f}_n \rightarrow \boldsymbol{f}_n + \delta\Lambda_{\rm DC}\cdot\boldsymbol{x}\,,$
Coordinate perturbation	$\boldsymbol{x} \rightarrow \boldsymbol{x}_0 + \boldsymbol{\delta x}\,.$

$$D_0 \equiv M s^2 + K + \hat{\Lambda}\cdot\Omega$$

$$S_{\rm IFO,\ cross\text{-}talk,\ n}(\Delta x) = \sum_j S\left[\Delta\cdot D_0^{-1} - \Omega\cdot D_0^{-1}\delta A\right]_{\Delta x, j} \boxed{S_{g_n,j}} + - \sum_j S\left[\Delta\cdot D_0^{-1}\hat{\Lambda}\right]_{\Delta x, j} \boxed{S_{n,o_j}}$$

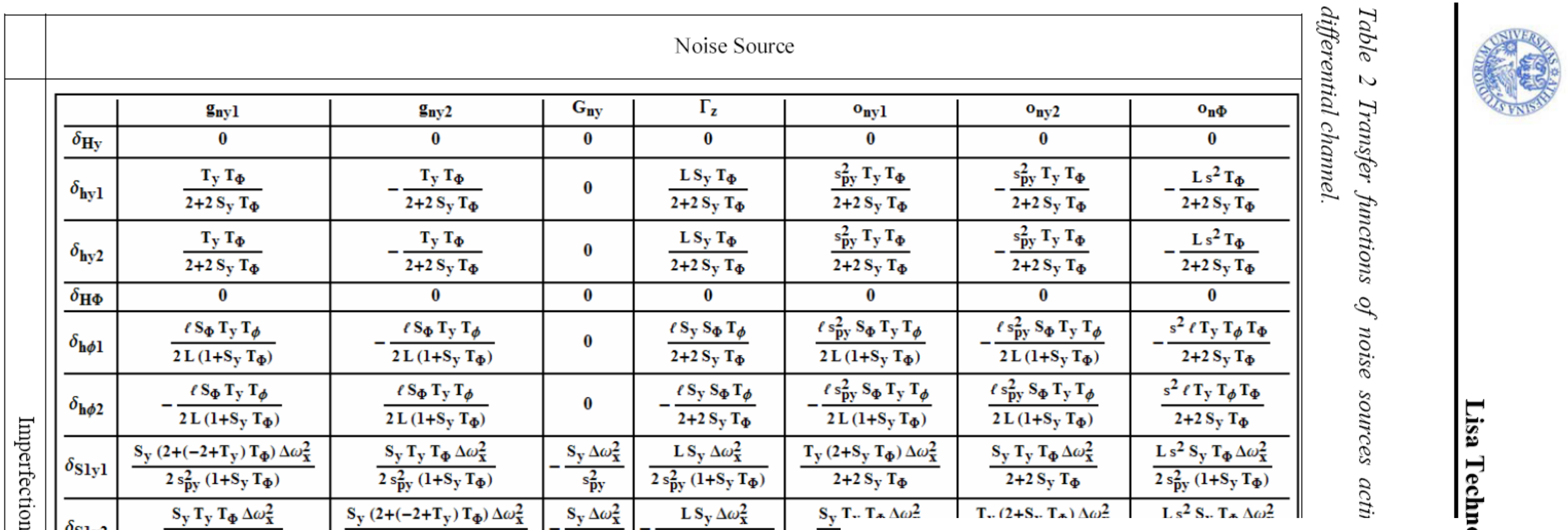
Readout noise
~1E-12 (IFO)
~1E-9 (GRS)
(m/Hz$^{1/2}$)

$$\boxed{S^{1/2}_{a,\mathrm{crosstalk}}(@1\,\mathrm{mHz}) = 6.12\times 10^{-15}\ \mathrm{m/s^2\sqrt{Hz}}}$$

Residual g
~1E-14 (TM1,2)
~1E-8 (SC)
(m/s^{2}Hz$^{1/2}$)

Monkeys riding on an elephant
Bharhut, c. 100 BC
Indian Museum, Calcutta

M.Armano: LTP, the geodesy lab



Lisa Technology Package

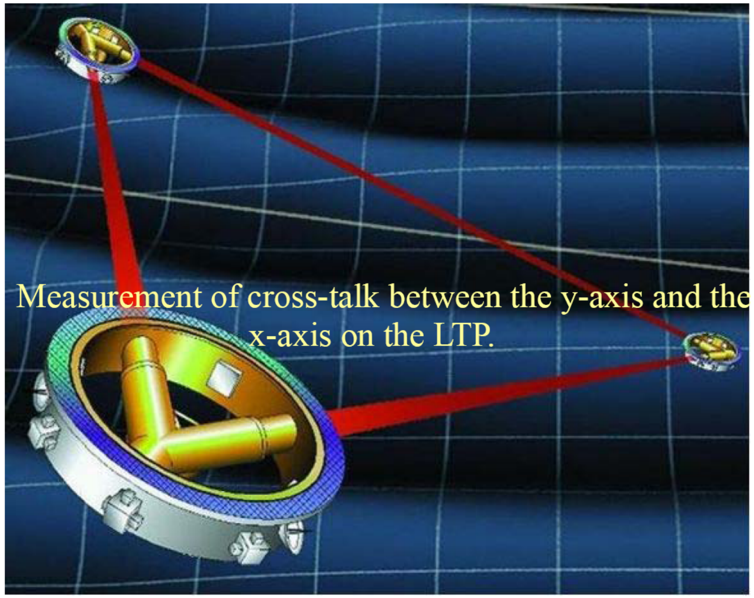
Doc: S2-UTN-TN-3052
Issue/Rev. 1.0
Date: 13-08-2007
Pag.20

Table 2 Transfer functions of noise sources acti
differential channel.

Noise Source

Imperfection coefficient

	g_{ny1}	g_{ny2}	G_{ny}	Γ_z	o_{ny1}	o_{ny2}	$o_{n\Phi}$
δ_{Hy}	0	0	0	0	0	0	0
δ_{hy1}	$\frac{T_y T_\Phi}{2+2 S_y T_\Phi}$	$-\frac{T_y T_\Phi}{2+2 S_y T_\Phi}$	0	$\frac{L S_y T_\Phi}{2+2 S_y T_\Phi}$	$\frac{s_{py}^2 T_y T_\Phi}{2+2 S_y T_\Phi}$	$-\frac{s_{py}^2 T_y T_\Phi}{2+2 S_y T_\Phi}$	$-\frac{L s^2 T_\Phi}{2+2 S_y T_\Phi}$
δ_{hy2}	$\frac{T_y T_\Phi}{2+2 S_y T_\Phi}$	$-\frac{T_y T_\Phi}{2+2 S_y T_\Phi}$	0	$\frac{L S_y T_\Phi}{2+2 S_y T_\Phi}$	$\frac{s_{py}^2 T_y T_\Phi}{2+2 S_y T_\Phi}$	$-\frac{s_{py}^2 T_y T_\Phi}{2+2 S_y T_\Phi}$	$-\frac{L s^2 T_\Phi}{2+2 S_y T_\Phi}$
$\delta_{H\Phi}$	0	0	0	0	0	0	0
$\delta_{h\phi 1}$	$\frac{\ell S_\Phi T_y T_\phi}{2 L (1+S_y T_\Phi)}$	$-\frac{\ell S_\Phi T_y T_\phi}{2 L (1+S_y T_\Phi)}$	0	$\frac{\ell S_y S_\Phi T_\phi}{2+2 S_y T_\Phi}$	$\frac{\ell s_{py}^2 S_\Phi T_y T_\phi}{2 L (1+S_y T_\Phi)}$	$-\frac{\ell s_{py}^2 S_\Phi T_y T_\phi}{2 L (1+S_y T_\Phi)}$	$-\frac{s^2 \ell T_y T_\phi T_\Phi}{2+2 S_y T_\Phi}$
$\delta_{h\phi 2}$	$-\frac{\ell S_\Phi T_y T_\phi}{2 L (1+S_y T_\Phi)}$	$\frac{\ell S_\Phi T_y T_\phi}{2 L (1+S_y T_\Phi)}$	0	$-\frac{\ell S_y S_\Phi T_\phi}{2+2 S_y T_\Phi}$	$-\frac{\ell s_{py}^2 S_\Phi T_y T_\phi}{2 L (1+S_y T_\Phi)}$	$\frac{\ell s_{py}^2 S_\Phi T_y T_\phi}{2 L (1+S_y T_\Phi)}$	$\frac{s^2 \ell T_y T_\phi T_\Phi}{2+2 S_y T_\Phi}$
δ_{S1y1}	$\frac{S_y (2+(-2+T_y) T_\Phi) \Delta\omega_x^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$\frac{S_y T_y T_\Phi \Delta\omega_x^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$-\frac{S_y \Delta\omega_x^2}{s_{py}^2}$	$\frac{L S_y \Delta\omega_x^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$\frac{T_y (2+S_y T_\Phi) \Delta\omega_x^2}{2+2 S_y T_\Phi}$	$\frac{S_y T_y T_\Phi \Delta\omega_x^2}{2+2 S_y T_\Phi}$	$\frac{L s^2 S_y T_\Phi \Delta\omega_x^2}{2 s_{py}^2 (1+S_y T_\Phi)}$
δ_{S1y2}	$\frac{S_y T_y T_\Phi \Delta\omega_x^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$\frac{S_y (2+(-2+T_y) T_\Phi) \Delta\omega_x^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$-\frac{S_y \Delta\omega_x^2}{s_{py}^2}$	$-\frac{L S_y \Delta\omega_x^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	[illegible]	[illegible]	[illegible]
$\delta_{S\Delta y1}$	$-\frac{s_{px2}^2 S_y (2+(-2+T_y) T_\Phi)}{2 s_{py}^2 (1+S_y T_\Phi)}$	$-\frac{s_{px2}^2 S_y T_y T_\Phi}{2 s_{py}^2 (1+S_y T_\Phi)}$	$\frac{s_{px2}^2 S_y}{s_{py}^2}$	$-\frac{L s_{px2}^2 S_y}{2 s_{py}^2 (1+S_y T_\Phi)}$	[illegible]	[illegible]	[illegible]
$\delta_{S\Delta y2}$	$-\frac{s_{px2}^2 S_y T_y T_\Phi}{2 s_{py}^2 (1+S_y T_\Phi)}$	$-\frac{s_{px2}^2 S_y (2+(-2+T_y) T_\Phi)}{2 s_{py}^2 (1+S_y T_\Phi)}$	$\frac{s_{px2}^2 S_y}{s_{py}^2}$	$\frac{L s_{px2}^2 S_y}{2 s_{py}^2 (1+S_y T_\Phi)}$	[illegible]	[illegible]	[illegible]
δ_{xy1}	$\frac{S_y (2+(-2+T_y) T_\Phi) \omega_{py}^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$\frac{S_y T_y T_\Phi \omega_{py}^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$-\frac{S_y \omega_{py}^2}{s_{py}^2}$	$\frac{L S_y \omega_{py}^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	[illegible]	[illegible]	[illegible]
δ_{xy2}	$-\frac{S_y T_y T_\Phi \omega_{py}^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$-\frac{S_y (2+(-2+T_y) T_\Phi) \omega_{py}^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	$\frac{S_y \omega_{py}^2}{s_{py}^2}$	$\frac{L S_y \omega_{py}^2}{2 s_{py}^2 (1+S_y T_\Phi)}$	[illegible]	[illegible]	[illegible]

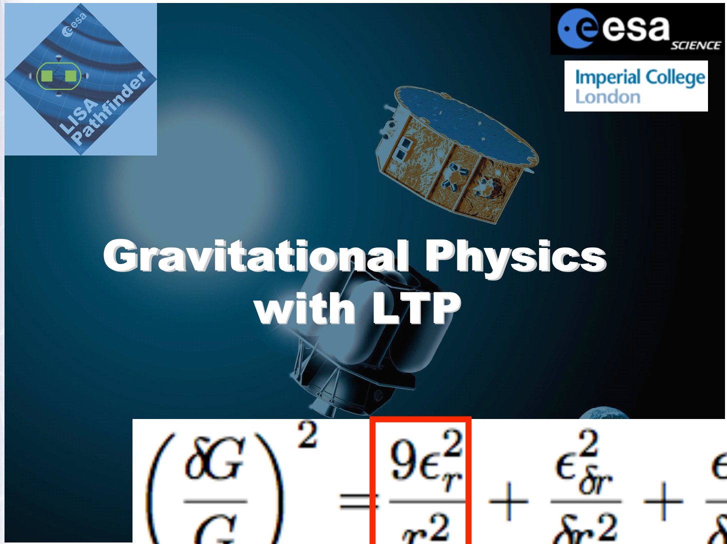


- System identification
- Fisher matrix analysis
- Decoupling of coefficients

S2-UTN-TN-3052

Issue/Rev. 1.0

2. Measure of G (maybe)



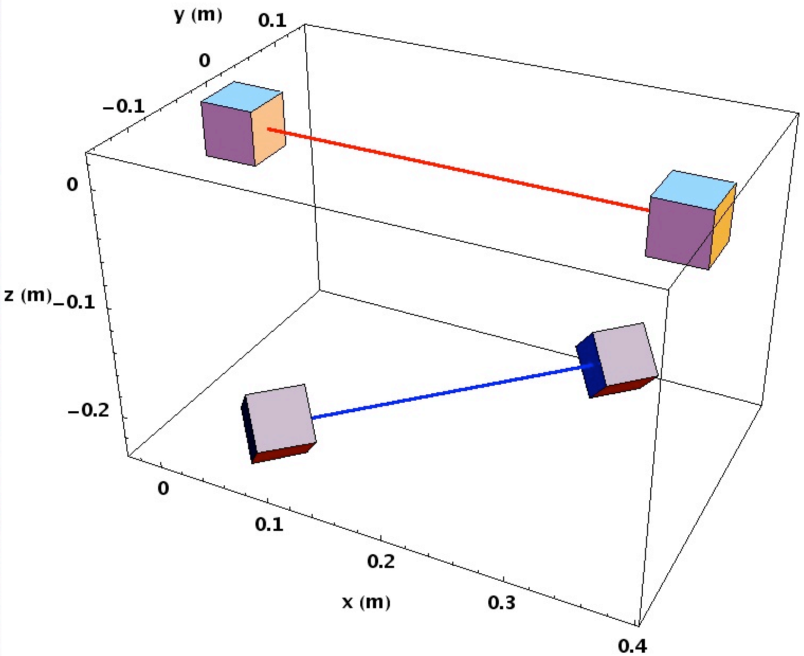
$$G = 6.67\boxed{42} \times 10^{-11}\ \mathrm{m^3/kg\,s^2}$$

$$G = \frac{r^3\left(k_1 - \omega^2 m_1\right)\delta x}{2\Pi(L,r)m_1 m_2 \delta r}$$

$$\left(\frac{\delta G}{G}\right)^2 = \boxed{\frac{9\epsilon_r^2}{r^2}} + \frac{\epsilon_{\delta r}^2}{\delta r^2} + \frac{\epsilon_{\delta x}^2}{\delta x^2} + \frac{\epsilon_\Pi^2}{\Pi^2} + \frac{4\omega^2 m_1^2 \epsilon_\omega^2}{\left(k_1 - \omega^2 m_1\right)^2} + \frac{\epsilon_{k_1}^2}{\left(k_1 - \omega^2 m_1\right)^2} + \frac{k_1^2 \epsilon_{m_1}^2}{m_1^2\left(\omega^2 m_1 - k_1\right)^2} + \boxed{\frac{\epsilon_{m_2}^2}{m_2^2}},$$

Parameter	Nominal value	Relative error	Requirement
m_1	1.95 kg	$6.\times 10^{-4}$	$8.\times 10^{-3}$
m_2	1.95 kg	$6.\times 10^{-4}$	$2.\times 10^{-6}$
r	0.30 m	$3.\times 10^{-4}$	$6.\times 10^{-7}$
δr	$2.\,\mathrm{nm}/\sqrt{\mathrm{Hz}}$	$2.\times 10^{-6}\ 1/\sqrt{\mathrm{Hz}}$	$3.\times 10^{-6}$
δx	$2.\,\mathrm{nm}/\sqrt{\mathrm{Hz}}$	$1.7\times 10^{-6}\ 1/\sqrt{\mathrm{Hz}}$	$3.\times 10^{-6}$
k_1	$1.\times 10^{-6}$	0.1	$2.\times 10^{-2}$

G (2)

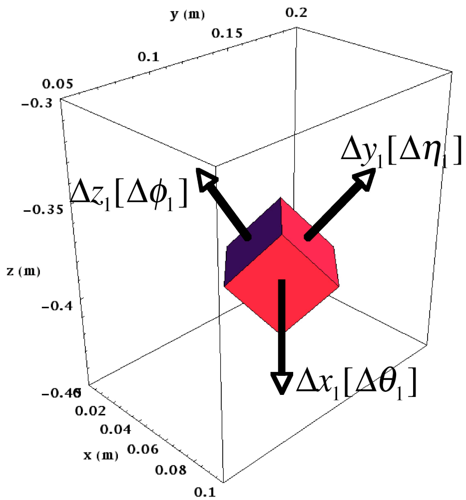


$$\omega_D = 2\pi \times 3 \times 10^{-3}\,\mathrm{Hz}\,,$$
$$A = 200\,\mu\mathrm{m}\,,$$
$$T = 3600\,\mathrm{s}$$
$$\sigma_{\mathrm{LTP},\Delta Fx/m} \simeq 3 \times 10^{-14}\,\mathrm{m/s^2}\sqrt{\mathrm{Hz}}\,,$$

$$\boxed{\Delta F_x \simeq 2.18 \times 10^{-13} m/s^2}$$

$$\boxed{\mathrm{SNR} = \Delta F_x/\sigma_{\mathrm{LTP},\Delta F_x/m} \sim 435}$$

$$G = 6.6\boxed{74}2 \times 10^{-11}\,\mathrm{m^3/kg\,s^2}$$



Conclusions

- LISA Pathfinder is worth flying INDEPENDENTLY of LISA as it will be the first mission in the gravity-quiet L1 environment with the specific goal of demonstrating geodesic motion of a free-particle!
- The main output of LPF is the physics model of the free falling test mass and its environment (the real TT-gauge!)
  - Model of the orthogonality of reference system
  - Role of static gravity, stiffness, magnetics...
- The first measurement of G @ L1 might be performed
- Test of modified gravity (MOND/TeVeS) after official mission time?


THANKS!