Phasing of gravitational waves from inspiralling eccentric binaries

with Thibault Damour & Bala R. Iyer

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Outline

- The Introduction.
- The 'ready to use' inspiral templates for compact binaries in eccentric orbits via 'the phasing'.
- A formalism to do the 'phasing' & explicit 2.5PN accurate 'phasing'.
- The domain of validity of our approach & future plans.

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INTRODUCTION.

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- Gravitational waves (GW) from inspiralling compact binaries (ICB) should be detectable by **LIGO, VIRGO** & proposed **LISA**.
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- A post-Newtonian (PN) approximation gives corrections to Newtonian gravitational theory in terms of a small parameter $\nu \sim (v/c)^2 \sim (G\,m\,/c^2\,r)$, where m, v & r are the total mass, orbital velocity and the separation of the binary.

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- However, ICBs in eccentric orbits are probably most promising sources of GWs for **LISA**.
  Plausible sources for **LIGO & VIRGO** too.

The ‘ready to use’ templates

- The widely used gravitational wave templates, to detect gravitational waves from compact binaries in *quasi-circular orbits*.
- They consists of PN accurate expressions for h_+ & $h_\times$, supplemented by expressions giving *adiabatic* time evolution for the orbital phase and frequency.

  Blanchet, Damour, Iyer, Will & Wiseman 1995; Blanchet Damour & Iyer 1995; Blanchet, Iyer, Will & Wiseman 1996; Will & Wiseman 1996
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- **Currently, it is available to 2.5PN order. (Arun. *et. al.*, 2004)**
- Very recently, the time evolution for the orbital phase and frequency to 3PN & hence to 3.5PN order was achieved. (Blanchet, Damour, Esposito-Farèse, & Iyer, 2004)

Templates for eccentric binaries

- Constructing PN accurate 'ready to use' search templates for compact binaries moving in **inspiralling eccentric orbits** is more involved & non-trivial.

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- **We need to combine consistently *three* times scales, without treating the radiation-reaction in an adiabatic manner.**

- The times scales linked to the orbital motion, precession of periastron & radiation reaction.

- **We adapt the mathematical formulation, which resulted in an accurate 'timing formula' for binary pulsars, Damour 1983, 1985, to describe the orbital dynamics. We also have PN accurate amplitude corrections to h_+ & $h_\times$.**

Phasing: I

- **GW phasing: An accurate mathematical modeling of the continuous time evolution of the gravitational wave polarization states h_+ & $h_\times$.**
-

$$h_+ = \frac{1}{2}\left(p_i\,p_j - q_i\,q_j\right)h_{ij}^{TT}\,, \qquad h_\times = \frac{1}{2}\left(p_i\,q_j + p_j\,q_i\right)h_{ij}^{TT}$$

- h_{ij}^{TT}, the transverse-traceless (TT) part of the radiation field. $\mathbf{p}$ & $\mathbf{q}$ are two orthogonal unit vectors in the plane of the sky.
- h_{ij}^{TT} & hence $h_{\times,+}$ **are PN accurate quantities.**

Phasing: II

- **To the leading approximation** $h^0_\times(r, \phi, \dot{r}, \dot{\phi}) = -2\frac{G\,m\,\eta\,C}{c^4\,R'}\left\{\left(\frac{G\,m}{r} + r^2\,\dot{\phi}^2 - \dot{r}^2\right)\sin 2\phi - 2\dot{r}\,r\,\dot{\phi}\,\cos 2\phi\right\}$ where $C = \cos i$.

- To construct 'search templates', we require PN accurate $h_{+,\times}$ supplemented by explicit expressions describing the temporal evolution of the PN accurate relative motion, *i.e.* describing the explicit time dependences $r(t)$, $\phi(t)$, $\dot{r}(t)$, and $\dot{\phi}(t)$.

- **The description for the temporal evolution of PN accurate relative motion will be *coordinate dependent*.** ***We want to describe the system almost analytically*** .

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- We employ an improved method of variation of constants to incorporate the radiation reaction which leads to the above 'phasing relations'.

Phasing: III

- Let the relative acceleration of the compact binary be $\mathcal{A} = \mathcal{A}_0 + \mathcal{A}'$.

- **$\mathcal{A}_0$ is the 'conservative' (integrable) part & $\mathcal{A}'$ is the reactive perturbative part.**

- The method first constructs the solution to the 'unperturbed' system, whose dynamics is governed by $\mathcal{A}_0$ & in this talk, the conservative part is restricted to the second post-Newtonian (2PN) order.

- **The solution to the binary dynamics, governed by $\mathcal{A}$, is obtained by *varying the constants* in the generic solution to the unperturbed system.**

Phasing: IV

- **For 2PN accurate dynamics, in the COM frame, there are 4 first integrals. The 2PN accurate energy and angular momentum of the binary, denoted by c_1 & c_2^i:**

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\begin{aligned}
c_1 &= \mathcal{E}(\mathbf{x_1}, \mathbf{x_2}, \mathbf{v_1}, \mathbf{v_2})|_{\text{2PN CM}} \,, \\
c_2^i &= \mathcal{J}_i(\mathbf{x_1}, \mathbf{x_2}, \mathbf{v_1}, \mathbf{v_2})|_{\text{2PN CM}} \,,
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- **This is true even when radiation reaction is present.** **Damour 1983** & we can introduce polar coordinates in the plane of the orbit.

Phasing: V

- **The functional form for the solution to the unperturbed (2PN accurate) equations of motion**

$$r = S(l; c_1, c_2) \qquad ; \qquad \dot{r} = n \frac{\partial S}{\partial l}(l; c_1, c_2)\,,$$

$$\phi = \lambda + W(l; c_1, c_2) \qquad ; \qquad \dot{\phi} = (1+k)n + n \frac{\partial W}{\partial l}(l; c_1, c_2)\,,$$

The basic angles l and λ are given by

$$l = n(t - t_0) + c_l\,, \quad \lambda = (1+k)n(t - t_0) + c_\lambda$$

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- t_0 **is some initial instant and the constants** c_l **&** c_λ**, the corresponding values for** l **&** λ**.**

Phasing: VI

- The generalized quasi-Keplerian orbital representation gives fully 2PN parametric solution to 2PN accurate EOM: Damour, Schäfer & Wex 1988, 1993

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r &= a_r \left(1 - e_r \cos u\right) \,, \\
l \equiv n\left(t - t_0\right) &= u - e_t \sin u + \left(\frac{g_{4t}}{c^4}\right) \left(v - u\right) + \left(\frac{f_{4t}}{c^4}\right) \sin v \,, \\
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- Rest of the quantities are PN accurate expressions in E & J.
  The expressions for n & Φ are gauge invariant.

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- **Also, we have following definitions for l & λ**
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- **The four variables $\{c_1, c_2, c_l, c_\lambda\}$ replace the original four dynamical variables r, $\dot{r}$, ϕ & $\dot{\phi}$ and $\{c_\alpha\}$ satisfies first order evolution equations.**

Phasing: VIII

- **The explicit expressions for $\{dc_\alpha/dt\}$** **Damour 1985**

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\begin{aligned}
\frac{dc_1}{dt} &= \frac{\partial c_1(\mathbf{x},\mathbf{v})}{\partial v^i}\,\mathcal{A}'^i\,,\\
\frac{dc_2}{dt} &= \frac{\partial c_2(\mathbf{x},\mathbf{v})}{\partial v^j}\,\mathcal{A}'^j\,,\\
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- The evolution of Eqs. for c_l & c_λ follow from the fact that we have same functional form for $\dot{r}$ & $\dot{\phi}$ in unperturbed & perturbed cases.

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- It can be shown that to $\mathcal{O}(c^{-10})$,
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- **This periodicity along with that fact that G_α is of $\mathcal{O}(c^{-5})$ allow us to introduce a two-scale decomposition for $c_\alpha(l)$.**
- $c_\alpha(l) = \bar{c}_\alpha(l) + \tilde{c}_\alpha(l)$
  $\bar{c}_\alpha(l)$ **represents a slow secular drift.**
  $\tilde{c}_\alpha(l)$ **represents periodic fast oscillations.**

Phasing: X

- **The evolution Eqs. for $\bar{c}_\alpha$ & $\tilde{c}_\alpha$ are**

$$\frac{d\bar{c}_\alpha}{dl} = \bar{G}_\alpha(\bar{c}_a) \equiv \frac{1}{2\pi}\int_0^{2\pi} dl\, G(l, c_a)$$

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- **After the integration, c_a replaced by the slowly drifting solution of the averaged system, namely $\bar{c}_l(l)$.**

Phasing: XI

- To explicitly perform 'GW phasing', we have solved evolution Eqs. for $\{\bar{c}_\alpha, \tilde{c}_\alpha\}$ to the leading $\mathcal{O}(c^{-5})$ on the 2PN accurate description for the dynamical variables r, $\dot{r}$, ϕ & $\dot{\phi}$ entering the expressions for $h_\times$ & h_+ (Newtonian accurate in amplitude).

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- **We have chosen to employ $\{n, e_t, c_l, c_\lambda\}$ as time dependent variables to describe the orbit.**
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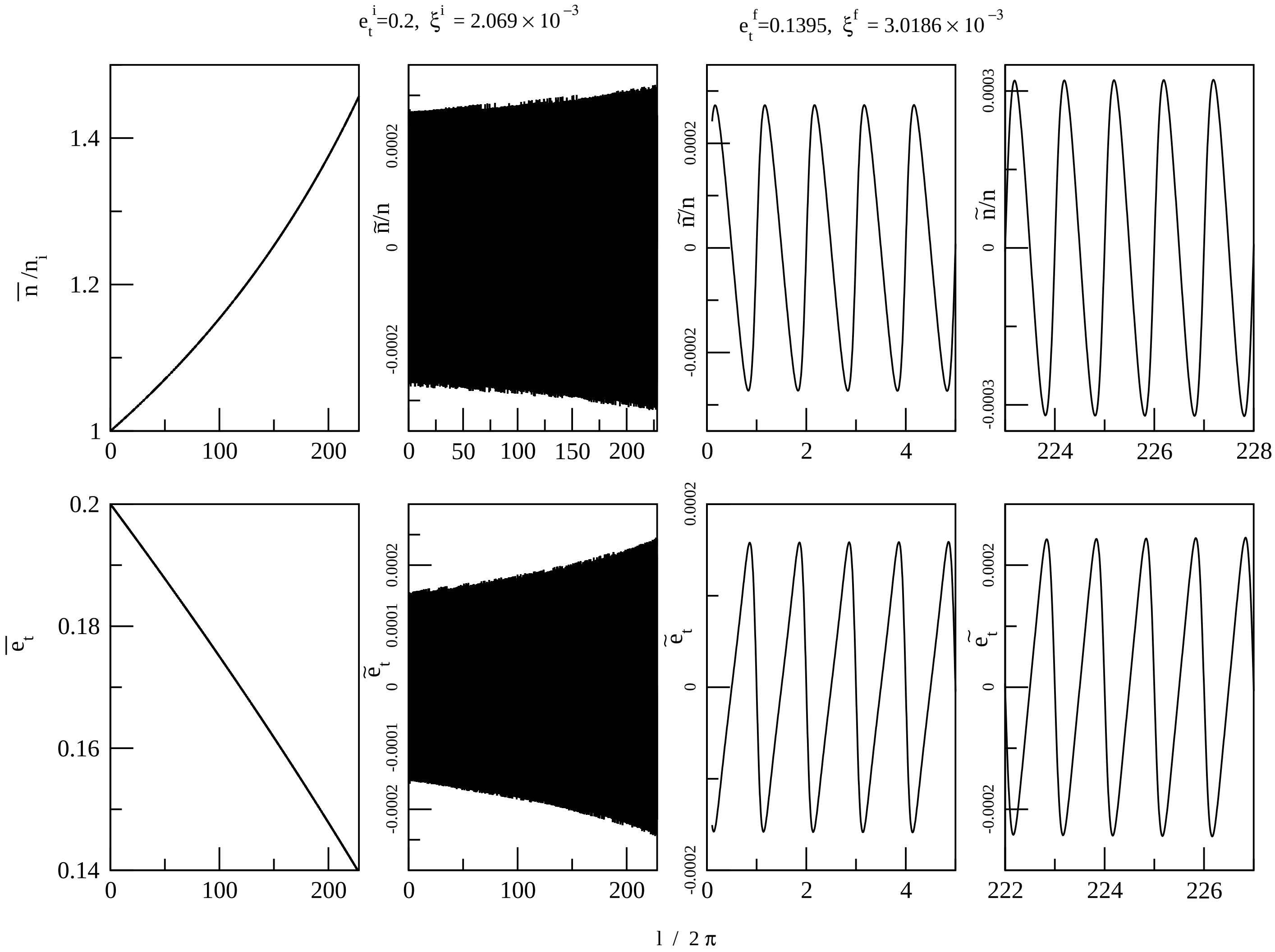
- **It turns out that $\bar{c}_l = \bar{c}_\lambda \equiv 0$ to $\mathcal{O}(c^{-10})$. The expression for $d\bar{n}/dt$ & $d\bar{e}_t/dt$ are in agreement with results based on ‘balance arguments’ Peters 1964.**

PICTORIAL PRESENTATION OF THE RESULTS

$\{\bar{n}, \bar{e}_t, \tilde{n}, \tilde{e}_t\}$ **Vs # of orbital cycles.**



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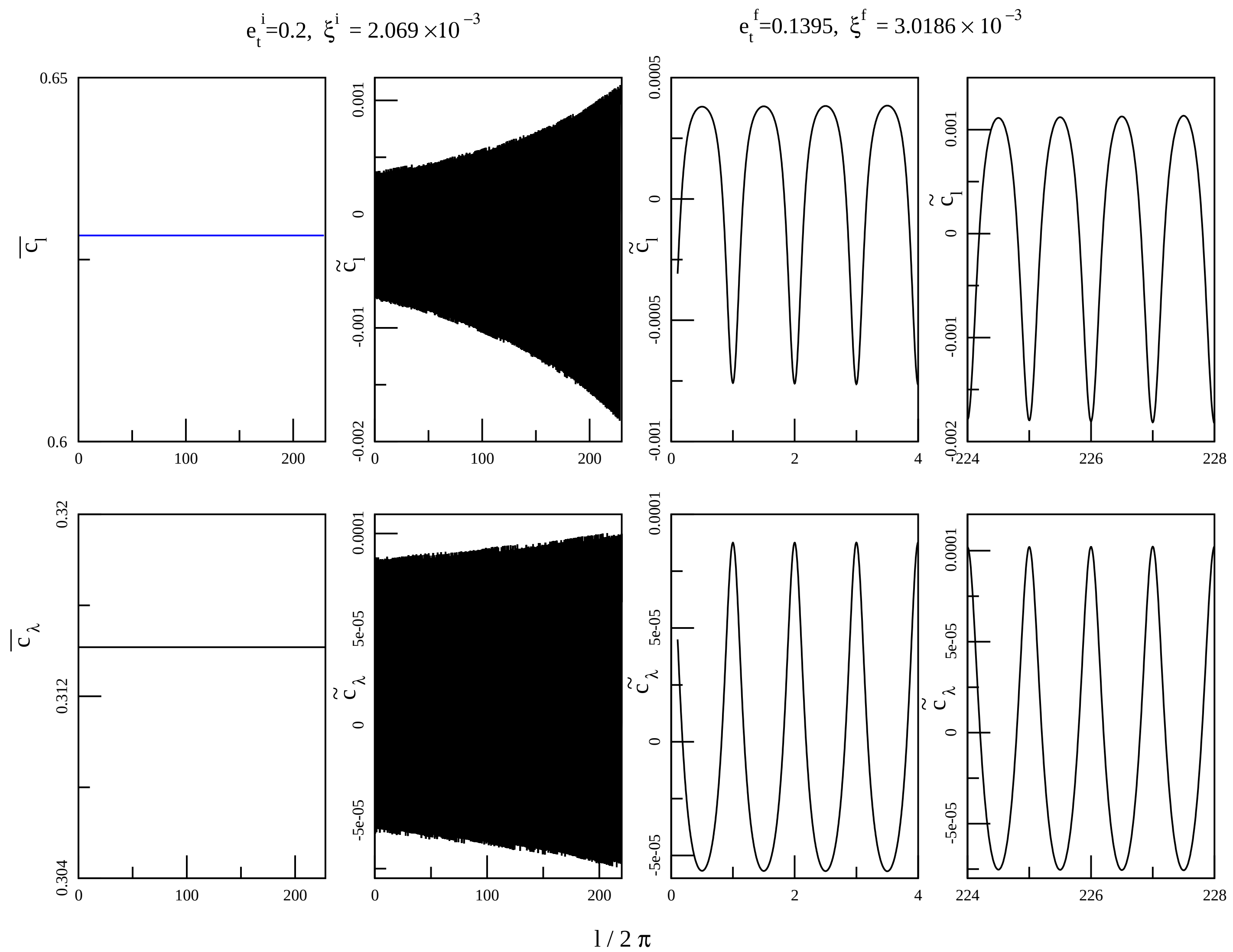
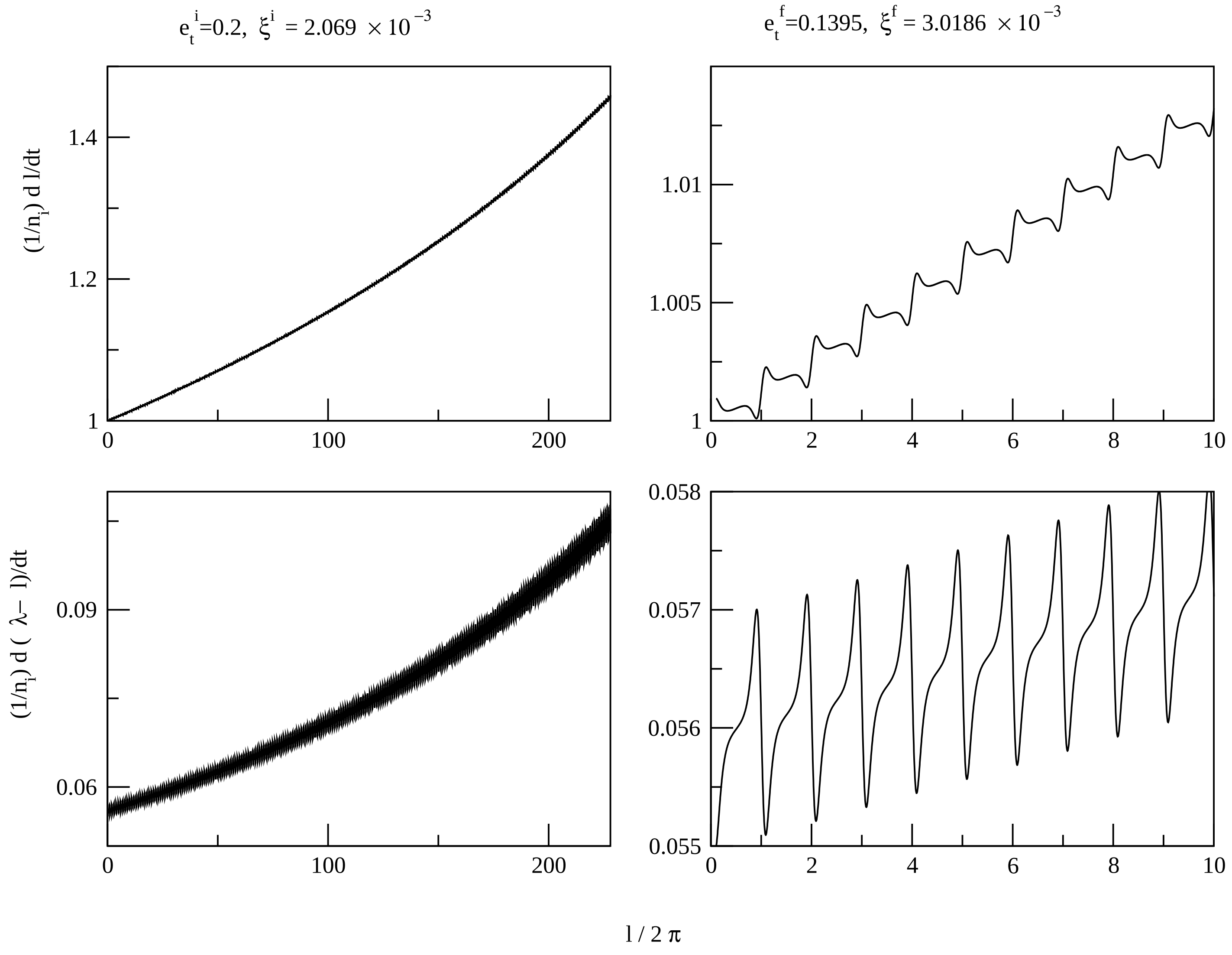
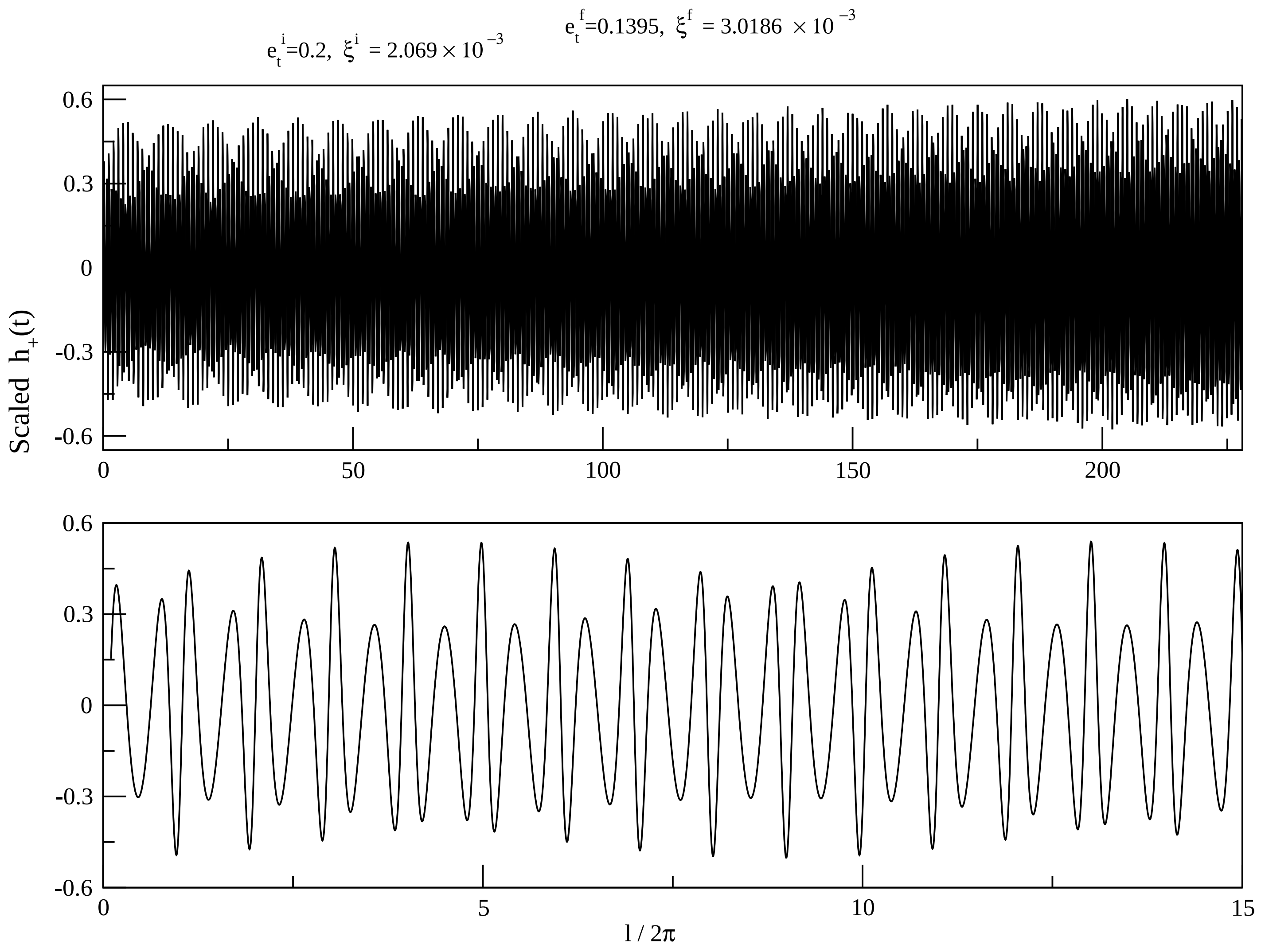


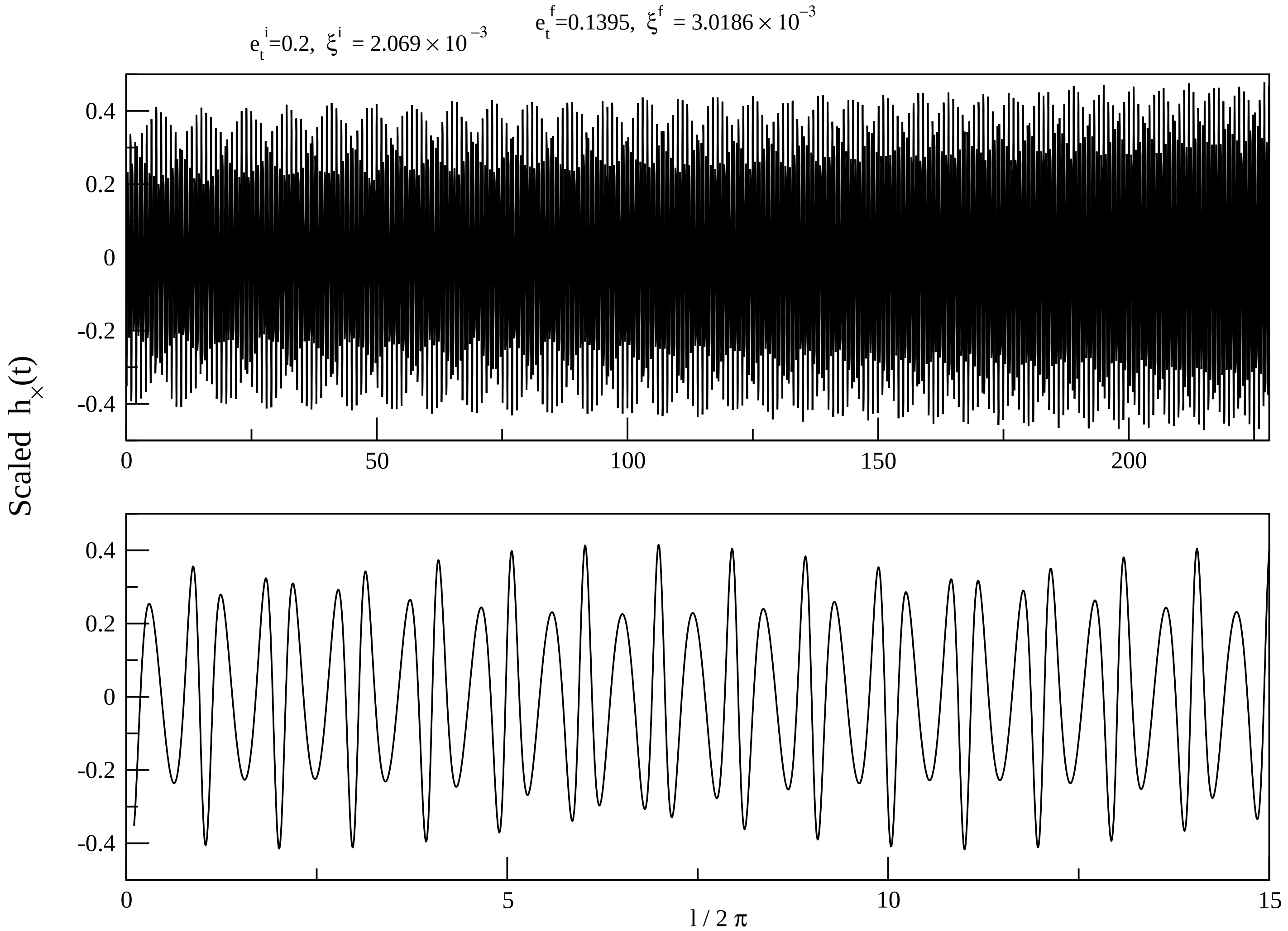
Scaled time derivative of l & λ Vs # of orbital cycles.



Scaled h_+ Vs # of orbital cycles for $\eta = 0.24$



Scaled $h_\times$ Vs # of orbital cycles for $\eta = 0.24$



Remarks on figures

- In Figs., if we choose $m = 2.8\,M_\odot$, the evolution occurs for ~ 8 sec, during which orbital frequency changes from 150 Hz to ~ 217 Hz.
  Inspiral relevant for LIGO

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Remarks on figures

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- **We, for the first time, exhibit $h_+(t)$ and $h_\times(t)$ with EXACTLY 2.5PN accurate orbital motion.**

The domain of validity: I

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- For a test particle, in Schwarzchild spacetime, bound orbits are for $E < 1$ & $j^2 = 12$.
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- **Near LSO, the orbit of a test particle executes ‘zoom-whirl’ motion, which contain a rapidly precessing quasi-circular motion. Glampedakis & Kennefick 2003**

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- Since the EOB representation, qualitatively, indicates that the orbital motion of a comparable mass binary is rather close to the test mass case, $j^2 = 48$ is quite conservative limit.

What we have done

- $$\boxed{h_{+,\times}(r^{\mathrm{harm}}, \phi^{\mathrm{harm}}, \dot{r}^{\mathrm{harm}}, \dot{\phi}^{\mathrm{harm}})}$$

$$\Downarrow$$

$$\boxed{h_{+,\times}(r^{\mathrm{ADM}}, \phi^{\mathrm{ADM}}, \dot{r}^{\mathrm{ADM}}, \dot{\phi}^{\mathrm{ADM}})}$$

$$\Downarrow$$

$$\boxed{h_{+,\times}(n, e_t, l, \lambda)}$$

$$\Downarrow$$

$$\boxed{h_{+,\times}(\bar{n} + \tilde{n}, \bar{e}_t + \tilde{e}_t, \bar{l} + \tilde{l}, \bar{\lambda} + \tilde{\lambda})}$$

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- Efforts are underway to address data analysis issues using our $h_{+,\times}$.