Candidate LISA Frequency (Modulation) Plan

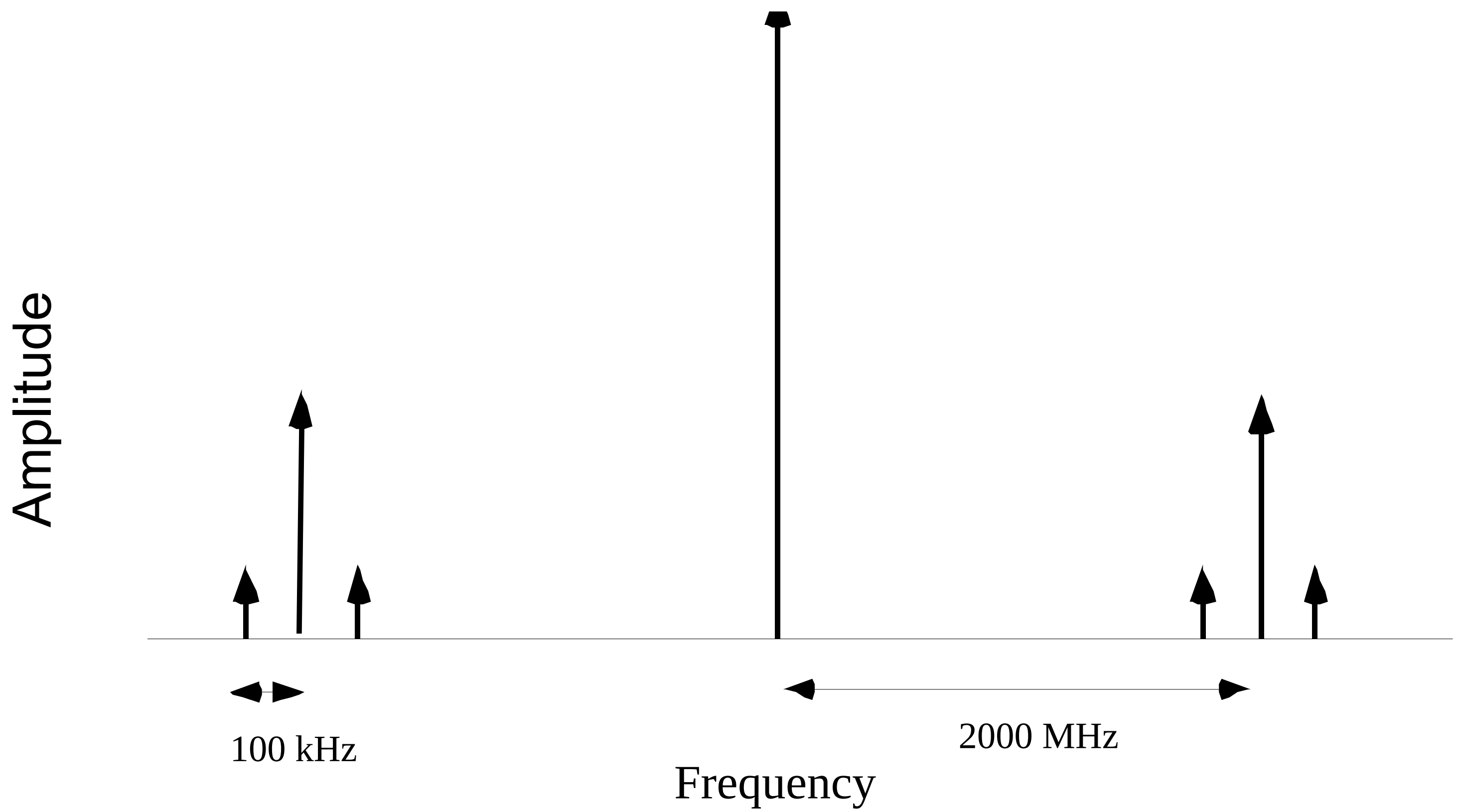
W. Folkner
D. Shaddock, M. Tinto, R. Spero,
R. Schilling, O. Jennrich

USO Frequency Noise Calibration

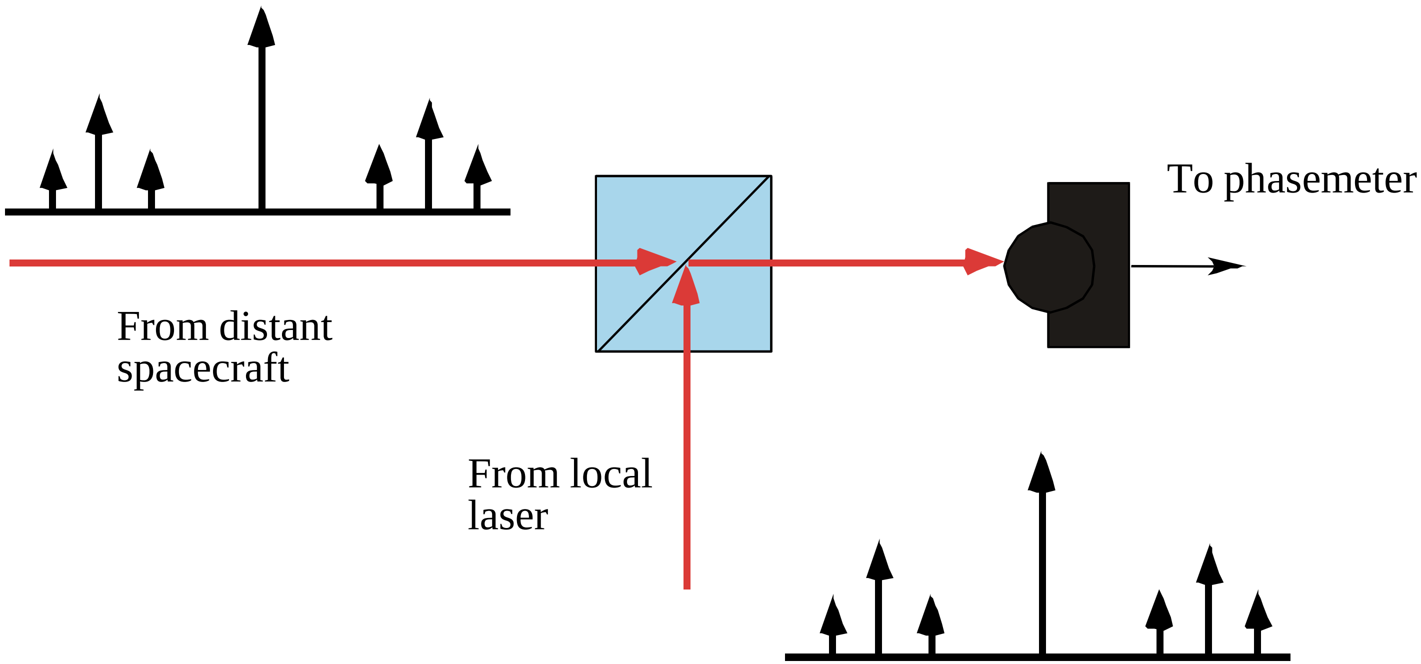
- In the scheme first proposed by Hellings, Danzmann et al. (Optics comm., 124, 313, 1996) , in addition to the six main laser signals of frequencies n_i, n^*_i, a second laser signal is superimposed on each beam by either modulating it at the frequency f_i of its USO (two main side-bands), or by combining each beam with a coherent second laser signal at $n_i + f_i$ or $n^*_i + f_i$
- **(R. Hellings, G. Giampieri, L. Maleki, M. Tinto, K. Danzmann, J. Hough, and D. Robertson, Optics Communications, 124, 313, (1996))**
- The received signals are mixed against the local signals
  - Carrier-carrier beat note is primary science observable
  - Carrier-sideband or sideband-sideband beat notes used to calibrate USO frequency noise
- In addition, a second sideband frequency is nominally used to determine the distance between spacecraft and synchronize clocks

Nominal Signal Spectrum

- Nominal spectrum of laser power transmitted from each spacecraft includes carrier, upper and lower clock sidebands, with ranging sidebands about the clock sidebands



Photodetector Output Signals



- At each bench incoming frequencies interfere with local spectrum
  - s_{ij} denotes carrier-carrier beat notes at front (telescope) end.
- Assume for this presentation that lasers are phase locked to master.

Clock Measurement Noise Terms

- Gravitational-wave signasl are extracted from TDI combinations of signals measured at various optical benches
  - Include carrier-carrier beat notes and clock calibration beat notes
  - Each beat note includes (independent) shot-noise-induced phase noise
- Desired contribution from clock correction is 1/10 of carrier-carrier shot noise limit

[illegible]

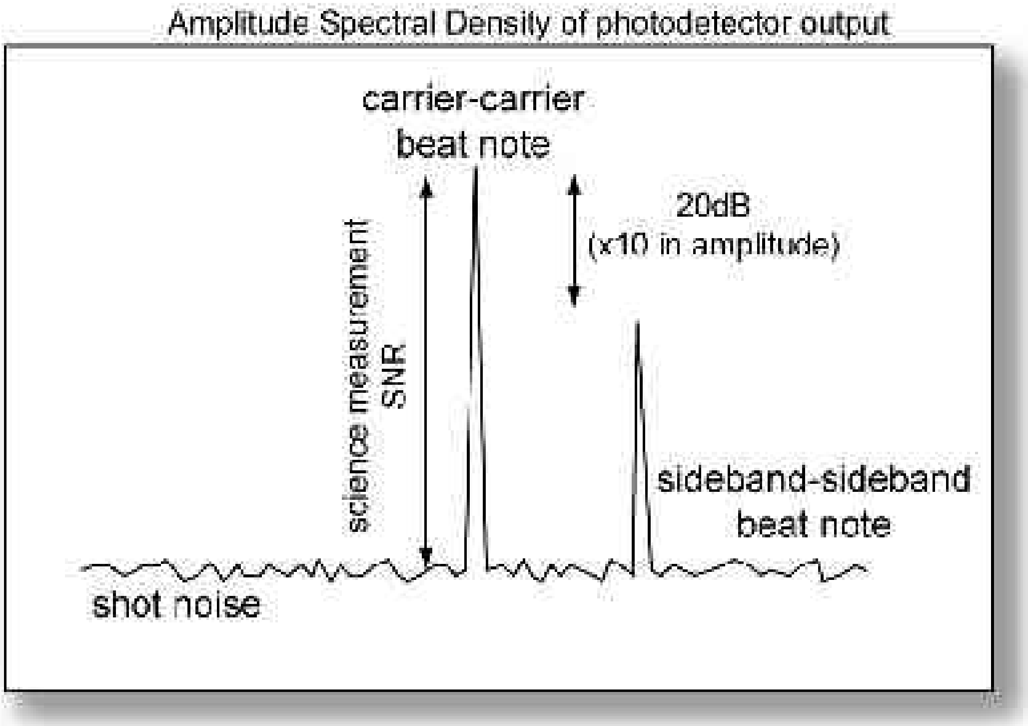
- a_{ij} measurement of the clock noise.
- r_{ij} scaling factor for clock noise measurement ~ f_h/f_{USO}

Frequency Selection Criteria

- In order for clock calibration noise to be 1/10 of the carrier-carrier shot-noise limit, the frequency of the measured clock calibration signal needs to be ~1/100 of the clock signal modulated onto the laser beam
  - Assuming a sideband-sideband beat signal is used for calibration
- If clock modulations are constant, then the measured clock frequency will get as large as ~20 MHz due to doppler shift
  - Implies that modulation frequency needs to be ~ 2GHz
- Ranging modulation frequency needs to be at a wavelength large enough to encompass the uncertainty in distance between spacecraft
  - To avoid problem with cycle ambiguity
    - (Could use a series of frequencies to resolve ambiguity)
  - 100 kHz ranging signal has ambiguity length of 3 km
    - Range measured to 1/1000 of cycle gives accuracy of ~ 3m

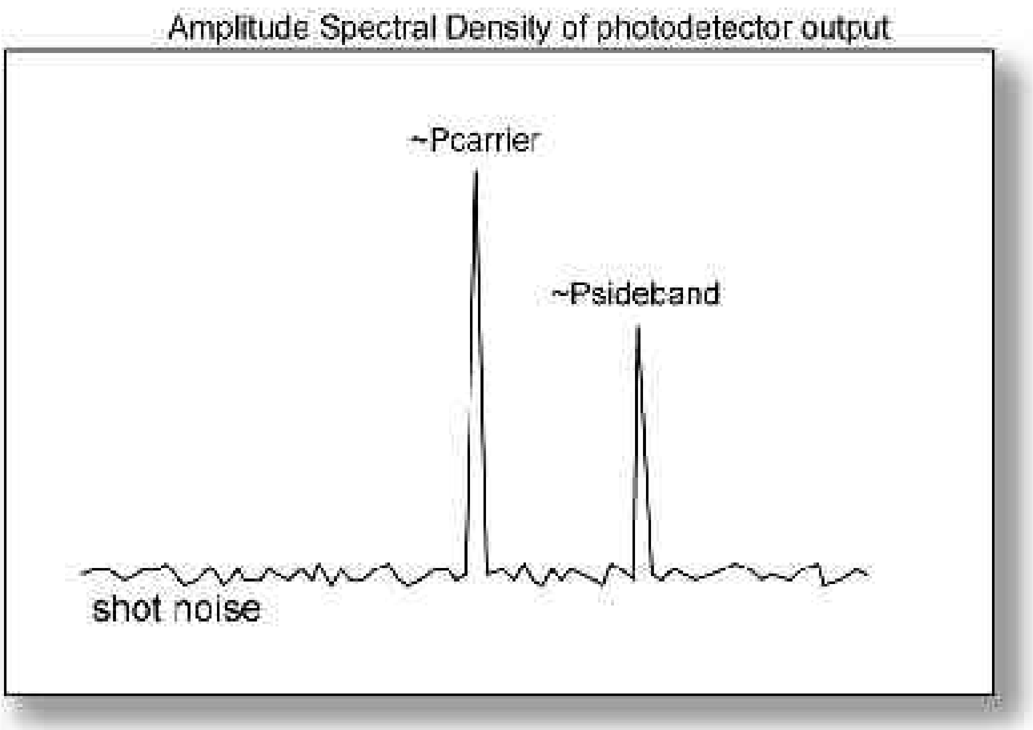
Shot noise limit

- Shot noise level determined by carrier power (local oscillator).



Shot noise limit

•Shot noise level determined by carrier power (local oscillator).



•Need 10% of power in sidebands.

Shot-Noise Limited Phase Noise

Assume that optical power on photodiode E^*E is

$$P \quad E_1^2 \quad E_2^2 \quad 2E_1E_2\cos(\ _1 \quad _2)t$$

the current (zero-peak) amplitude for the beat note is

$$I_s \quad 2RE_1E_2$$

where R is the responsivity of the photodiode in (A/

The shot noise (single-sided) poser spectral density (PI

$$S_i \quad 2q_eI \quad 2q_eRE_1^2 \quad (A^2/Hz)$$

The noise in the measurement of the phase of the beat note is given by the ratio of the signal to the shot noise

$$S \quad \frac{S_i}{I_s^2} \quad \frac{q_e}{2RP_2} \quad \mathrm{rad}^2/\mathrm{H}$$

With P_2=60 pW, R=0.7 A/W (and $q_e=1.6\times10^{-19}$ C), the phase noise RPSD is 7 μcycles/√Hz.

Main Beam and Side Tone

Assume two distant and two local signals fall on photodiode

$$E_1 e^{i \;\, _1 t} \quad 0.3 E_1 e^{i(\;\, _1 \;\; 2 \, f_{c1})t}$$
$$E_2 e^{i \;\, _2 t} \quad 0.3 E_2 e^{i(\;\, _2 \;\; 2 \, f_{c2})t}$$

so that power in clock frequency side band is ~1/10 of main beam

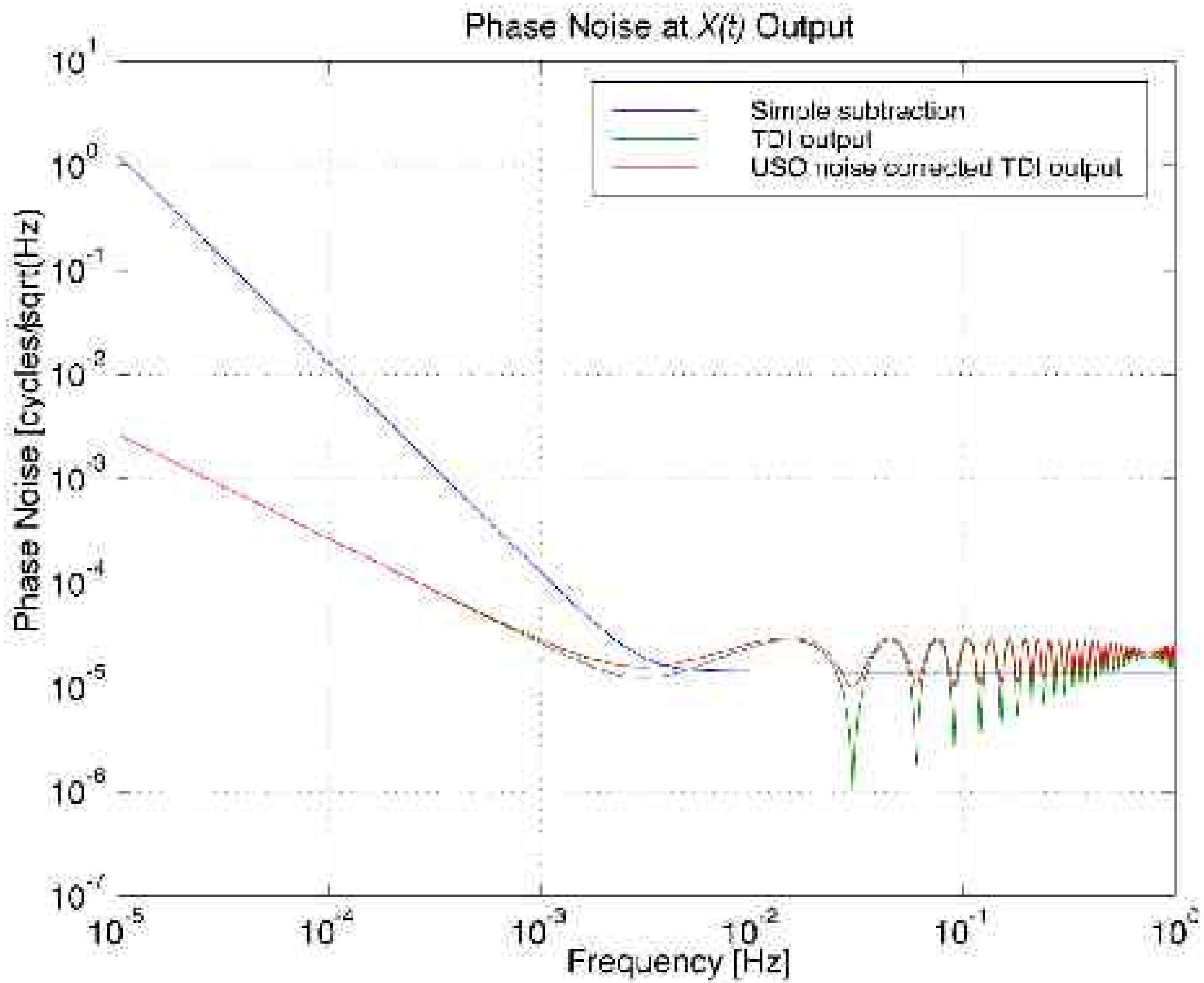
The signal for the sideband-sideband beat signal is smaller than for the carrier-carrier beat note, but the shot noise is the same. The phase noise on the sideband-sideband signal is thus

$$S \;_c \quad S_i / I_{sc}^2 \quad 2 q_e R a E_1^2 / (2R \; 0.3 E_1 \; 0.3 E_2)^2 \quad 100 q_e / 2 R P_2 \;\; (Hz) \quad 100 \;\; S$$

The square-root of clock phase noise PSD ($\sqrt{S_{\phi c}}$) is ~10 times higher than for main signal,
And ~3 times higher than for sideband-carrier signal

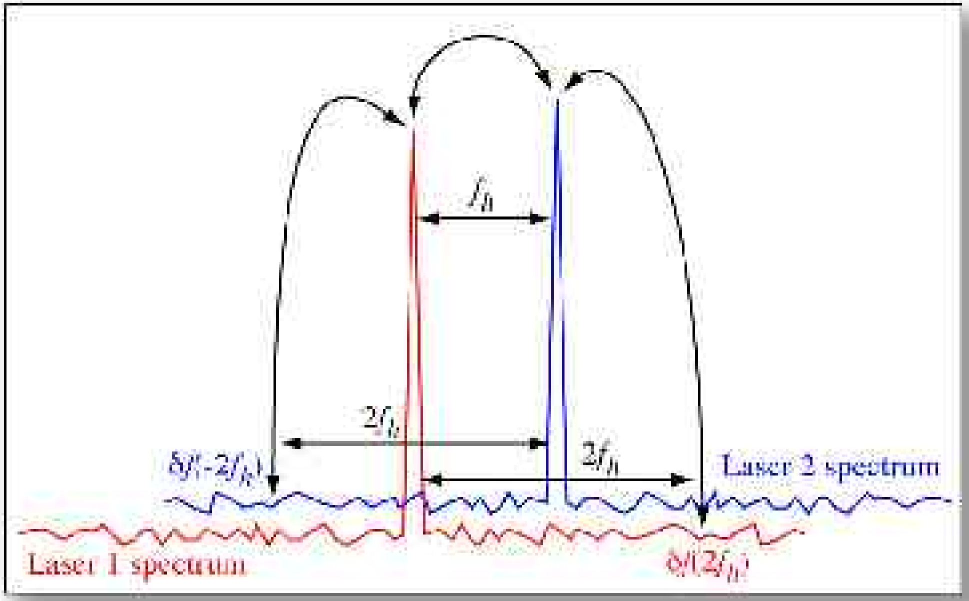
Increased Noise

$$X \approx s_{21} - s_{31} + s_{23} - s_{32} + s_{13} - s_{12} \ldots r(1-q) \ldots r(1-q)$$



Minimum Frequency Constraint

- Photodiode can not distinguish between positive and negative frequencies, signal phase measurements include laser frequency noise at both the measurement frequency and at the image frequency.



- Need to have all beat frequencies higher than a value where the laser frequency noise becomes insignificant
  - Assume for now that 1 MHz meets this constraint. (Need a laboratory measurement to determine minimum frequency)

Candidate Frequency Scheme

1

Master Laser

12 13

Carrier: $f_{12} = 3 \times 10^{14}$ Hζ
Χλοχκ ϕ_{12} + 2000 MHζ (10% οφ ποωερ)
Ρανγε: ϕ_{12} + 2000.1 MHζ (1% οφ ποωερ)

$\phi_{13} = \phi_{12}$ + 1 MHζ
ϕ_{13} + 2000 MHζ
ϕ_{13} + 2000.1 MHζ

$\phi_{21} = \phi_{12} \times (1+ \varpi_{12}/\chi)$ + 1 MHζ
ϕ_{21} + 2004 MHζ
ϕ_{21} + 2004.3 MHζ

$\phi_{31} = \phi_{13} \times (1+ \varpi_{12}/\chi)$ – 1 MHζ
ϕ_{31} + 2002 MHζ
ϕ_{31} + 2002.2 MHζ

21 31

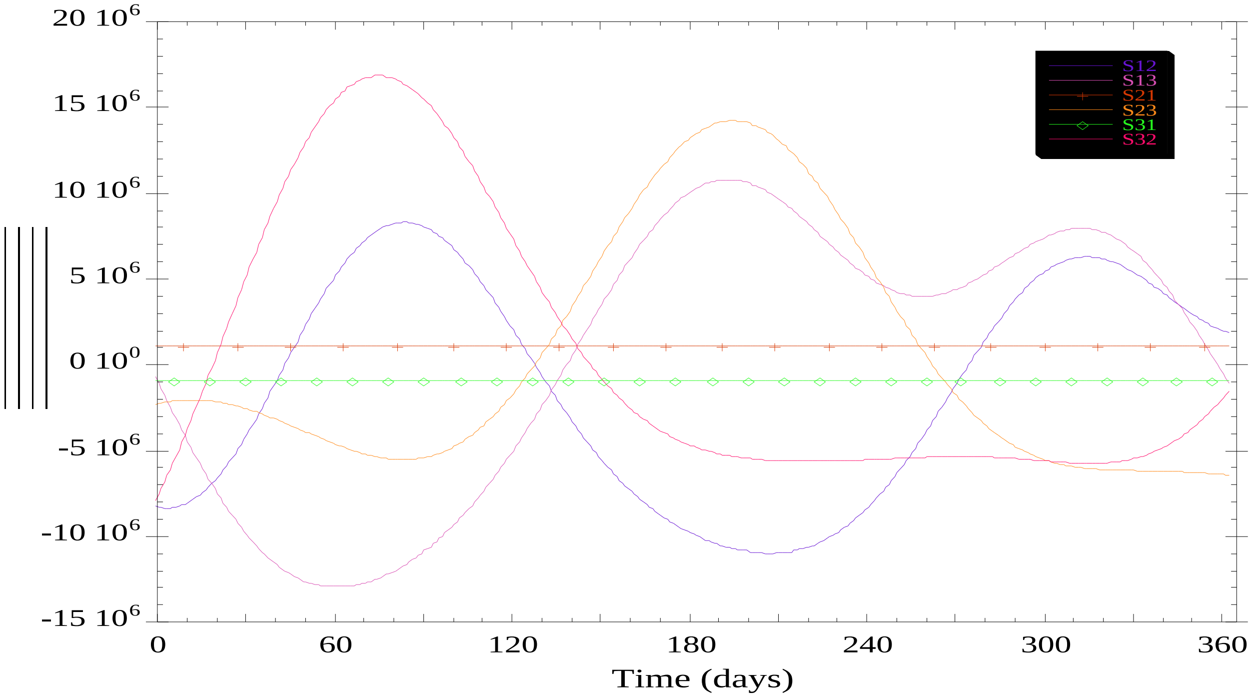
2 23 32 3

$\phi_{23} = \phi_{23}$ + 1 MHζ
ϕ_{23} + 2004 MHζ
ϕ_{23} + 2004.3 MHζ

$\phi_{32} = \phi_{31}$ + 1 MHζ
ϕ_{32} + 2002 MHζ
ϕ_{32} + 2002.2 MHζ

Main Beat Note Variation

- With fixed offset frequencies, main beat frequencies will vary
  - Shown below for first year of 'nominal' orbits
- Clock, range frequencies will have nearly fixed offsets from main beats
- At ~ 10 times per year signals move through zero frequency

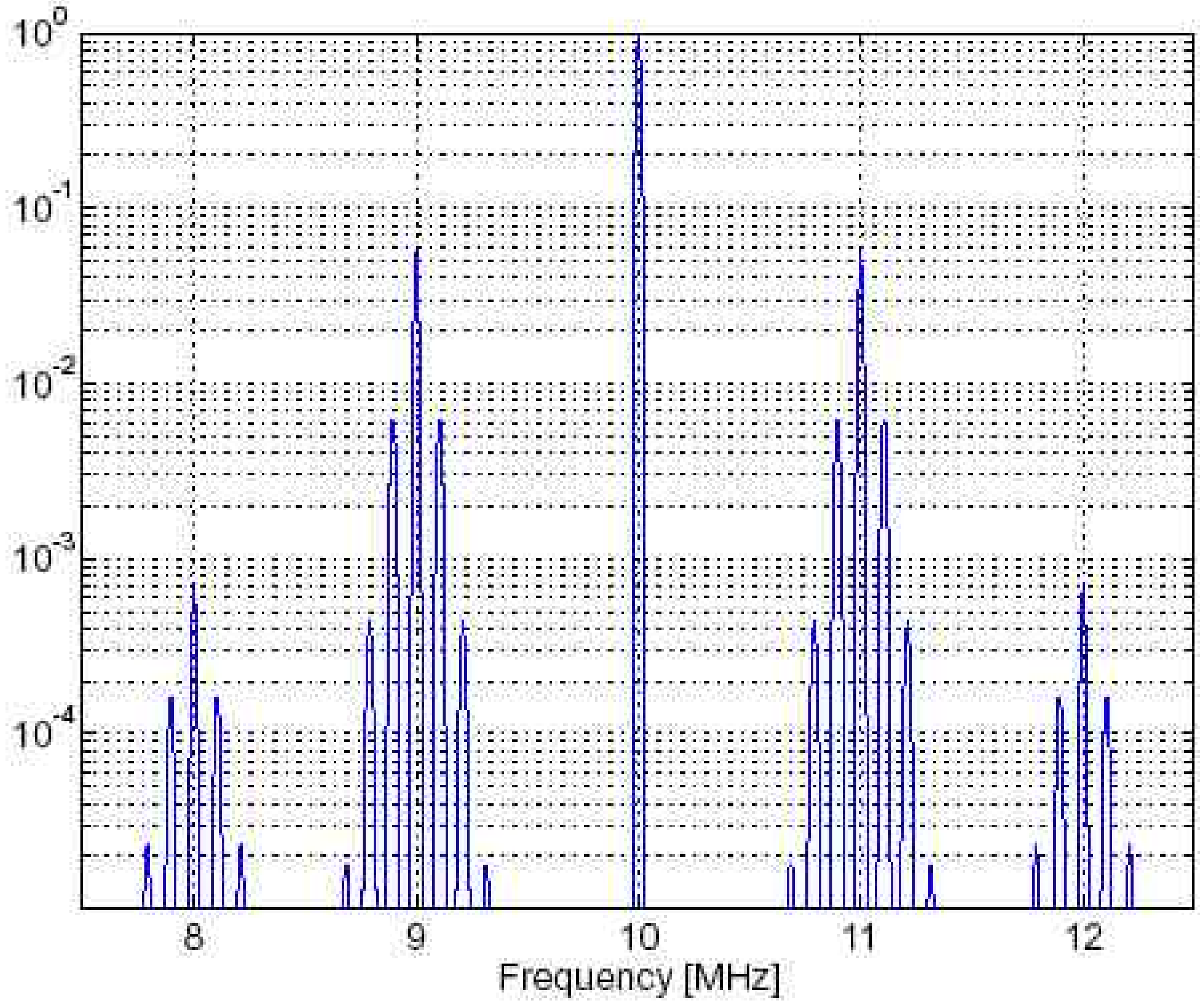


Dealing With Doppler Variation

- With fixed frequency assignment, signal frequencies will pass through zero with higher noise at ~ 10 times per year
- If noise is acceptable for f > 100 kHz, excess noise will exist for only ~0.5 day near each zero crossing
- Excess noise duration can be shortened by introducing changes in offset frequency selection at those times

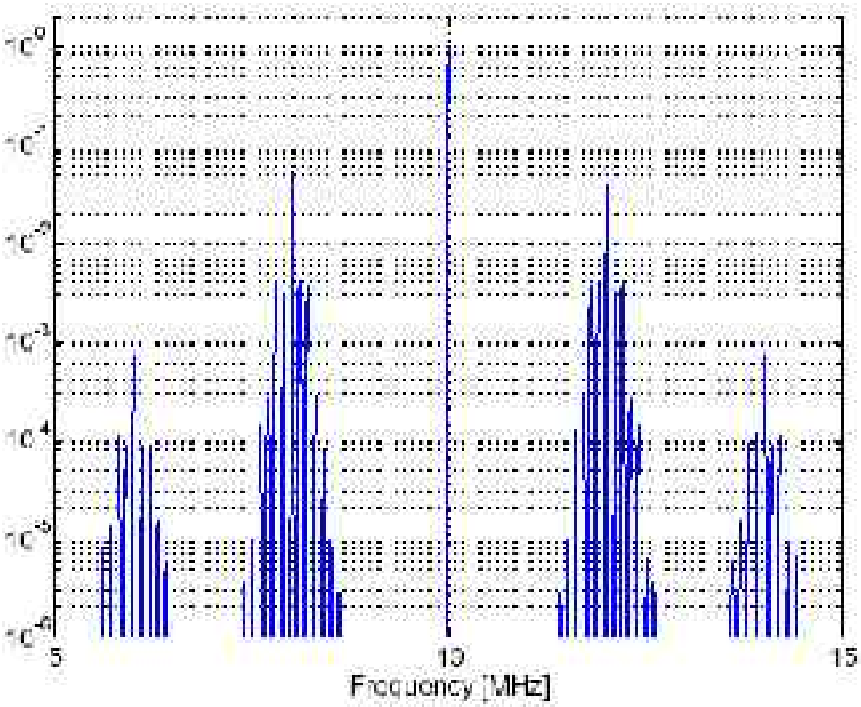
Realistic Modulation Spectrum

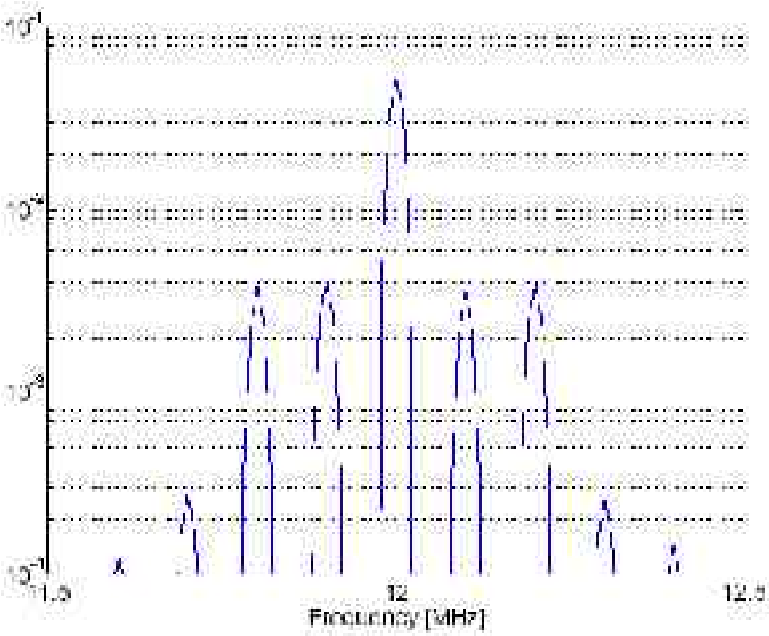
- A real phase modulation will have harmonics of the ranging tones about the sidetones and harmonics of the side-tones



Realistic Interference Spectrum

- Interference of realistic spectra have multiple beat notes
  - No clock or ranging tones are near main carrier-carrier signal
  - Side tones and harmonics are spaced by more than ~10 kHz
    - Are easily distinguishable





Future Work

- Check that 2 GHz modulation preserves USO stability
- Examine sideband beat-note spectra at all spacecraft over full orbit
  - Using any laser as Master
- Check that upper/lower sidebands can be combined
  - Or else pay penalty of $\sqrt{2}$
- Check that signals can be deconvolved with adequate resolution
  - I.e. interference from harmonics introduces tolerable error
- Investigate inclusion of telemetry