General Relativity Aspects of LISA

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High-Precision GW Observing

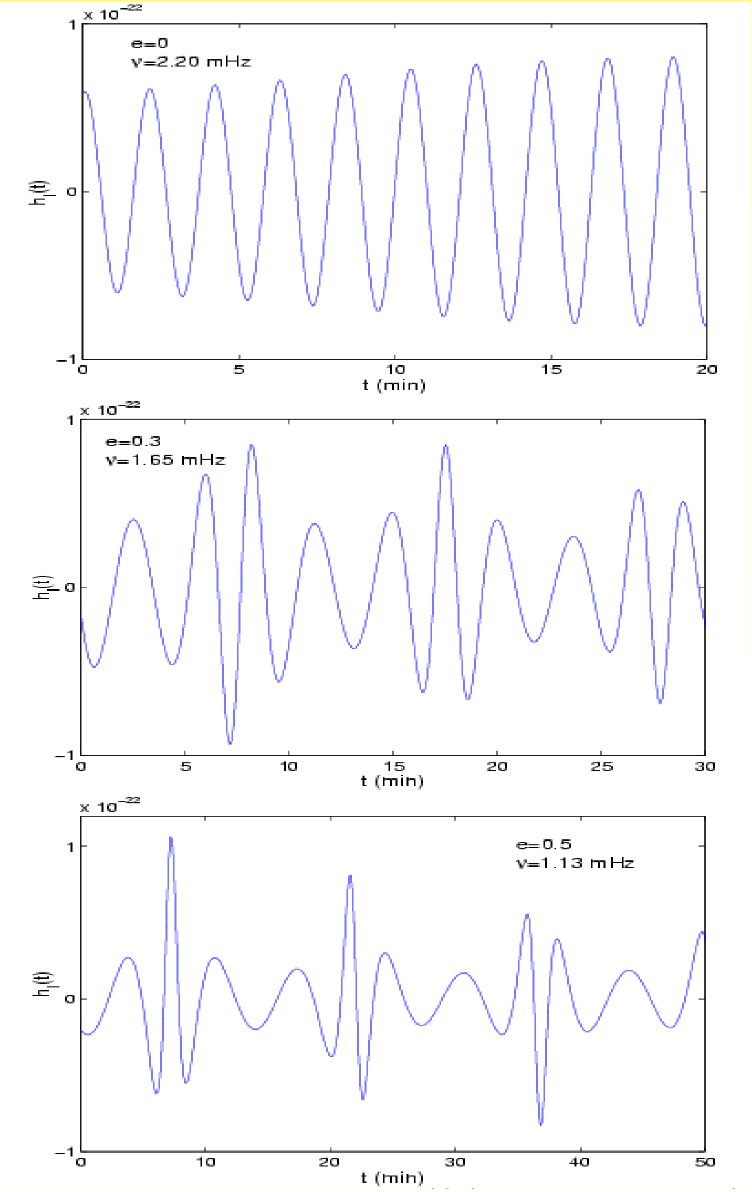
- High S/N, source confusion a serious limit
- Accurate astronomical observations (talk by Vecchio)
- Key measurements in GR:
  - Test BH uniqueness theorems close to horizon
  - Observe strong-field dynamical gravity in BH mergers
  - Compare GR to other theories, limit graviton mass
  - Measure dark energy at high redshifts
- Discovery potential: cosmic strings, other exotic sources
- Parameter extraction, optimal signal analysis needed

Testing BH Uniqueness

- LISA will observe captures of stellar-mass objects by supermassive black holes (SMBH)
- Most will be on plunge orbits as a result of a chance encounter in the cluster of stars around the SMBH
- Some may be formed in accretion discs and spiral in (Levin 2003)
- LISA is likely to observe low-mass main-sequence stars in orbit near the Milky Way's SMBH (Freitag 2003)
- Captured NSs and BHs survive close enough to the SMBH to test our assumptions about the Kerr geometry
- Observations of NS captures onto "small" SMBH (1000 $M_\odot$) can push Brans-Dicke ω beyond 3×10^5 (Will & Yunes 2004)

Finding Capture Signals

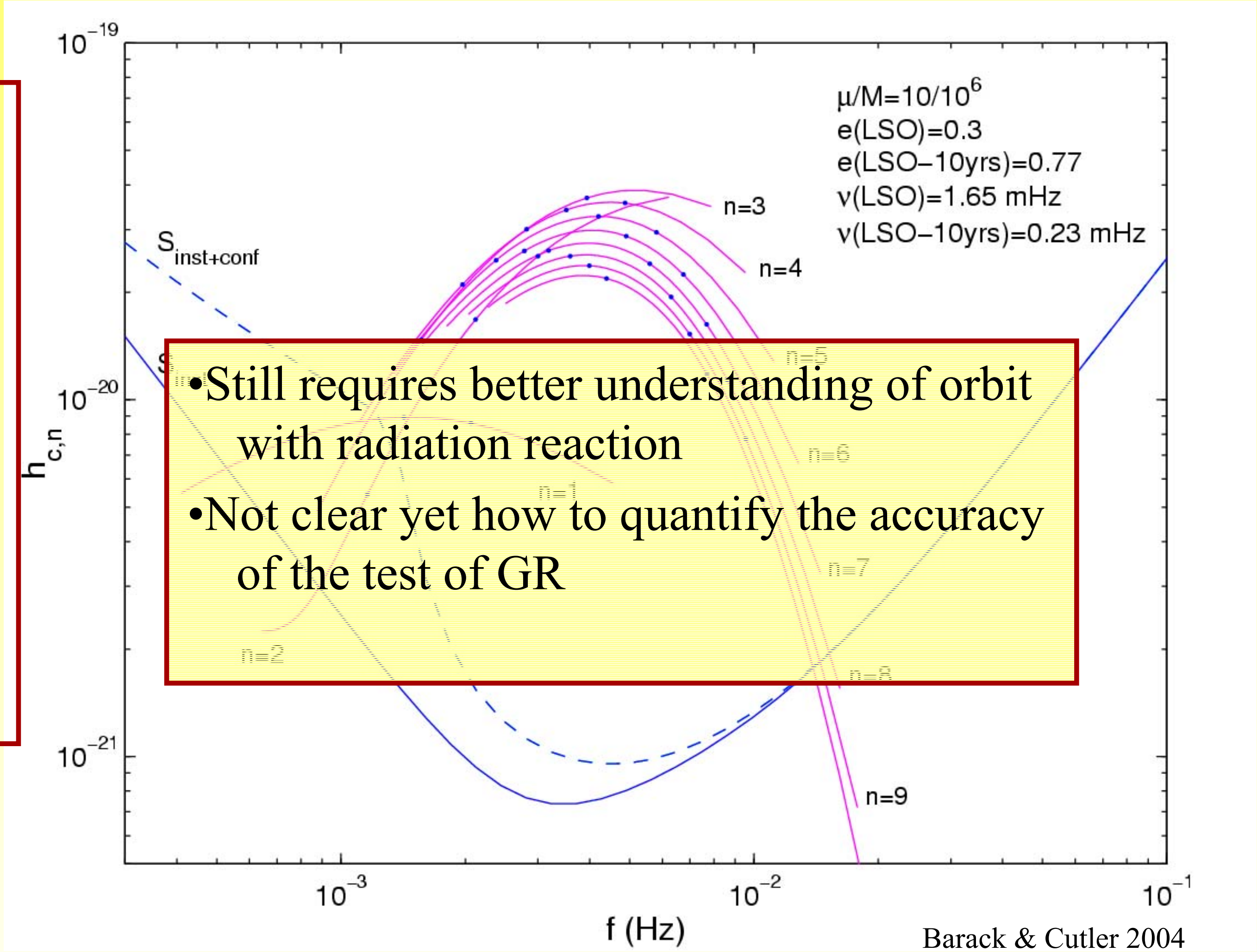
- Barack and Cutler (2004) detailed study of waveforms and parameter space.
- Must match waveform well enough to do filtering over 1-2 year inspiral time, 10^4 orbits.
- Barack and Cutler show this is feasible, and LISA could see 100 such events per year out to z=1.



Barack & Cutler 2004

Capture Event Signal-to-Noise Ratio

"Best case"
- $10\,M_\odot$ BH into $10^6\,M_\odot$ SMBH
- 1 Gpc ($z\sim0.25$)
- Highly eccentric orbit

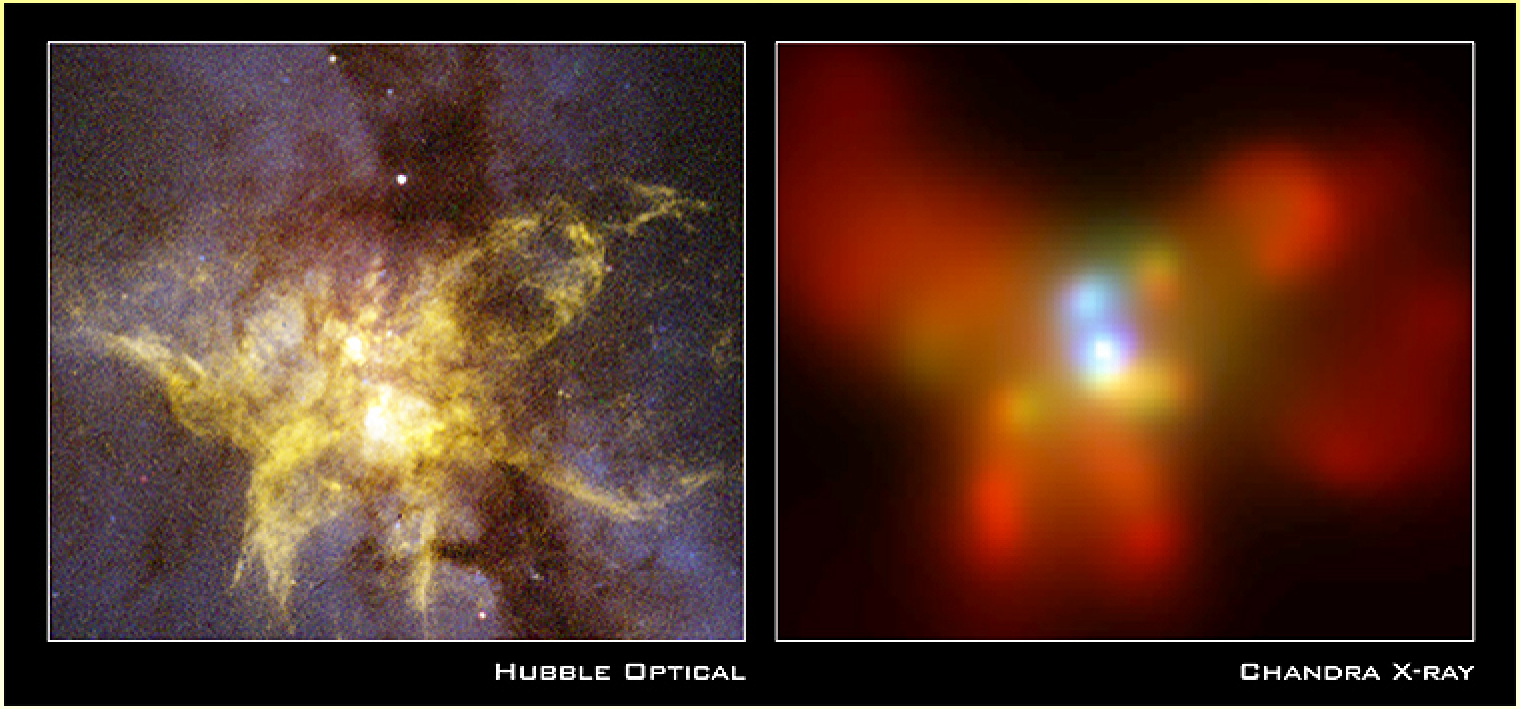


- Still requires better understanding of orbit with radiation reaction
- Not clear yet how to quantify the accuracy of the test of GR

Barack & Cutler 2004

SMBH Binary Mergers

- LISA's most dramatic sources
- Mergers may be happening even at $z \sim$ 7-10.



NGC6240

Comparison with Numerical Waveforms

- The ultimate strong-field dynamical system
- After long, well-understood inspiral phase, holes reach last stable orbit (ISCO) and plunge together to merger
- Inspiral waveform may pin down masses and spins, leaving few parameters free for the merger phase radiation
- Cleanest GR system: pure gravity

- Important physical questions for numerical simulations:
  - Signal removal
  - Energy radiated
  - Angular momentum of merger product
  - Recoil: linear momentum radiated for unequal-mass coalescence (probably peaks at mass ratio 2-3)

Current numerical simulation issues

- Grid meshes large but fine: horizon must be well resolved, but "source" speed v << c ⇒ wavelength long, boundary several wavelengths away. Mesh resolution 0.01M and size 100M (1.5λ) needs 10^{12} grid points.
- Large memory: 50-80 variables per grid point.
- Nonlinear, highly coupled equations.
- Long run times: Courant condition v = c set by wave speed, so orbital timescale several thousand time-steps.
- Singularity at center of hole(s) must be avoided or cut out.
- Boundary condition not understood at finite boundaries, even analytically.
- Equations: 4 constraint (elliptical) + six dynamical (hyperbolic) ⇒ infinite number of ways to formulate an initial-value scheme and identify dynamical variables. Big effect on stability of codes.
- Coordinate freedom: can also affect stability. Slicing into time+space arbitrary.
- Interpretation of output: finding horizons, discovering causes of instabilities, visualization.
- Collaboration, computational complexity
- Initial value problem – see later!

Quality assurance: validity of results

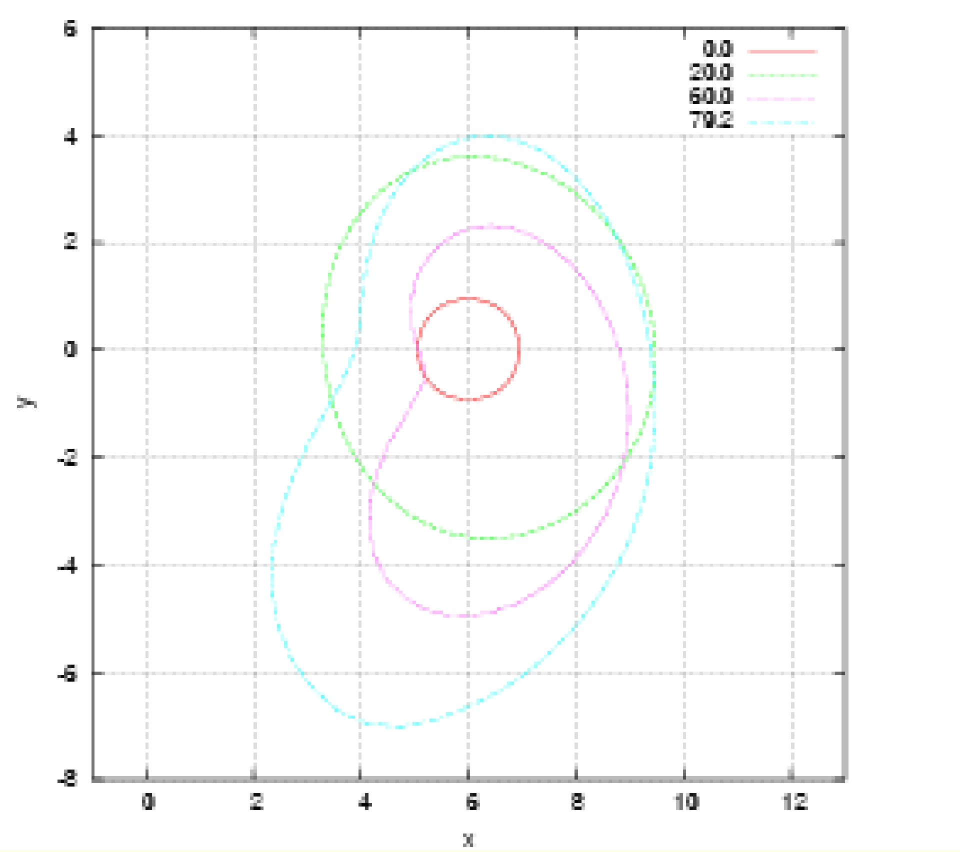
- Convergence – main test used by the groups
- Comparisons with known results, such as 2D simulations.
  - Comparison with *exact solutions* usually difficult: different coordinate systems, variables, boundary conditions.

- **Apples with Apples**: agreement of groups to compare with each other (only vacuum GR, not hydro) –
  - Albert Einstein Institute, Germany
  - Brownsville, USA
  - Goddard, USA
  - ICN-UNAM, Mexico
  - LSU Physics, USA
  - Penn State, USA
  - PITT, USA
  - RIKEN, Japan
  - Southampton, UK
  - UIB, Spain

http://www.appleswithapples.org/

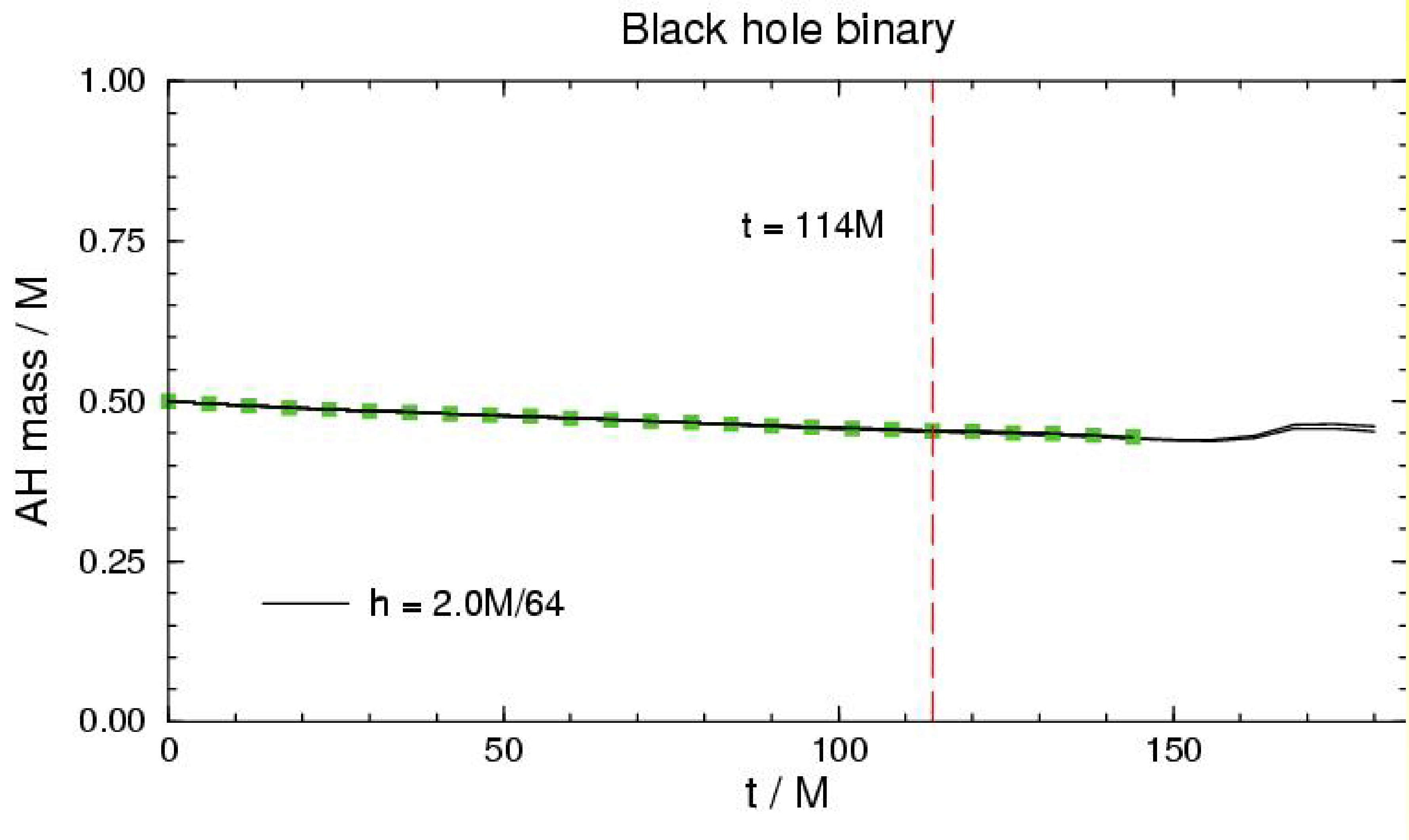
Key Problem: Initial Data

- Where is last stable orbit? What orbital velocity do holes have there?
- Post-Newtonian approx for inspiral does not converge, uses different coordinates and variables. Not an easy solution!
- "Best" data so far probably from Meudon group (Bonazzola et al), but not yet available for unequal masses. AEI intensively testing data.
- Other groups looking for ISCO with neutron-star binary simulations, much more stable.

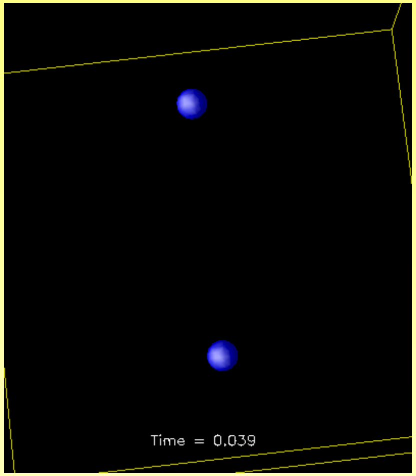


Growth and distortion of apparent horizon from Meudon initial data

State of the art: stability



Detailed Horizon Merger



Simulation and horizon-finding by the AEI numerical relativity group

Cosmology: Measuring Dark Energy

- SMBH mergers are standard candles: observations give luminosity distance D_L to the binary.
- If the host galaxy can be identified, its redshift combined with the distance measurement measures *H(t)*.
- Identifying host: increasing evidence that active galaxies are turned on by binary SMBH "stirring".
- Procedure: (1) Find D_L using LISA's relatively poor position error box. (2) Identify merger host as active galaxy in 3D error box. Resolve ambiguities by reference to known cosmology.
- Post-identification D_L accuracy limited to 10^{-3} by weak lensing (Holz & Hughes)
- With sources at $z > 5$, strong constraints on time-dependence of dark energy are possible. Depends on mass spectrum of mergers.

Constraining the Graviton's mass

- In GR, $m_g = 0$, but one might imagine that a quantum theory could lead to a non-zero mass.
- Gravitational waves would then travel slower than light and suffer dispersion in frequency.
- Solar system gravity bounds Compton wavelength > radius of Pluto's orbit.
- Cutler, Hiscock & Larson (2003): in Galactic binaries the GW signal would be out of phase with the optically observed binary orbit. This could improve the bound by factor of 50.
- Will & Yunes: SMBH binary coalescence signal would be distorted by dispersion. This could potentially improve the bound by a factor of $\sim 10^4$.

Expect the Unexpected

- With such good sensitivity, LISA can be expected to observe signals that are not expected on standard models.
- Some might have implications for GR or fundamental physics:
  - Discovering cosmic strings: their cusps and kinks (Damour & Vilenkin) have characteristic time-evolution, easily distinguished from other expected signals.
  - Seeing a cosmological background, especially one peaked in frequency, would have implications for electroweak transition (strongly first-order).
  - Observing SMBH binary coalescence at very high z could challenge dark matter theories.
- And more?? 90% of the universe is dark, but it all exerts gravitation!