Center for Gravitational Wave Astronomy

Extreme Mass-Ratio Inspirals

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based on

Barack and Cutler, PRD 2004

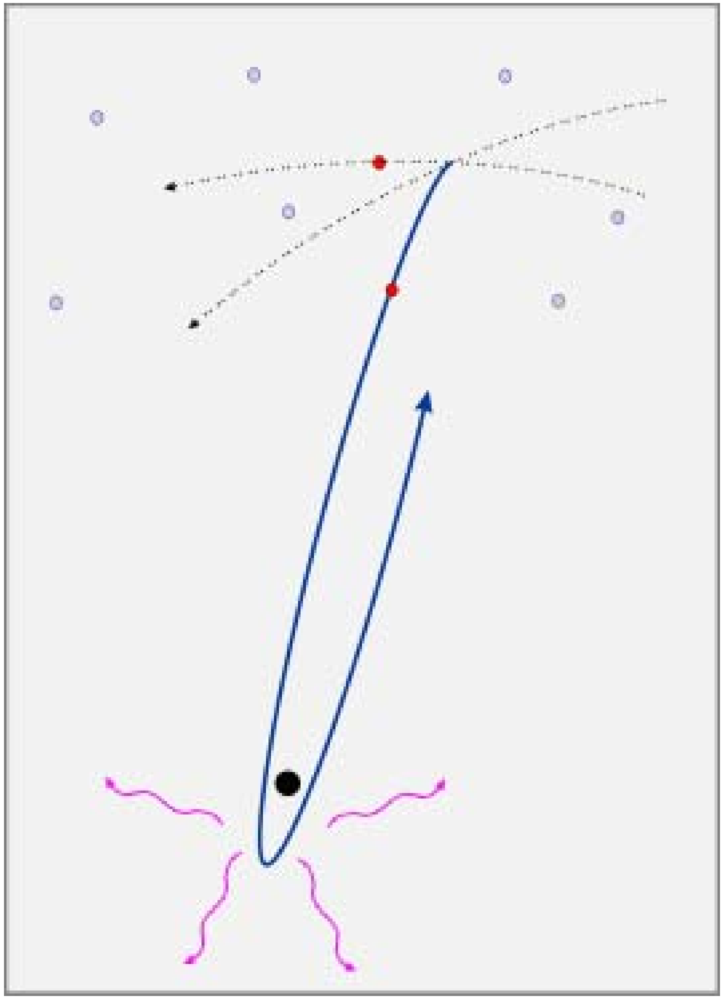
and discussions within

LIST's Working Group 1 (Sources and Data Analysis)

Outline

- Essential astrophysics of EMRI sources: frequencies, amplitude, time scales, etc.
- Data analysis of EMRIs: basic considerations in devising search strategy
- Use approximate waveforms to
  - Show how SNR builds up over time
  - Estimate LISA's parameter extraction accuracy
  - Estimate SNR threshold for detection, and (combined with astroph. rates) get detection rates ⇨ In J. Gair's talk
  - Confusion background from EMRIs

EMRI scenario: 3 Epochs



1st Epoch: high-eccentricity early inspiral

- Compact object in galaxy cusp kicked into "loss cone" through large-angle scattering.
- Rates (# per galaxy per yr) [Freitag 2003]:

$$4\times10^{-8} < R(WD) < 4\times10^{-6}$$

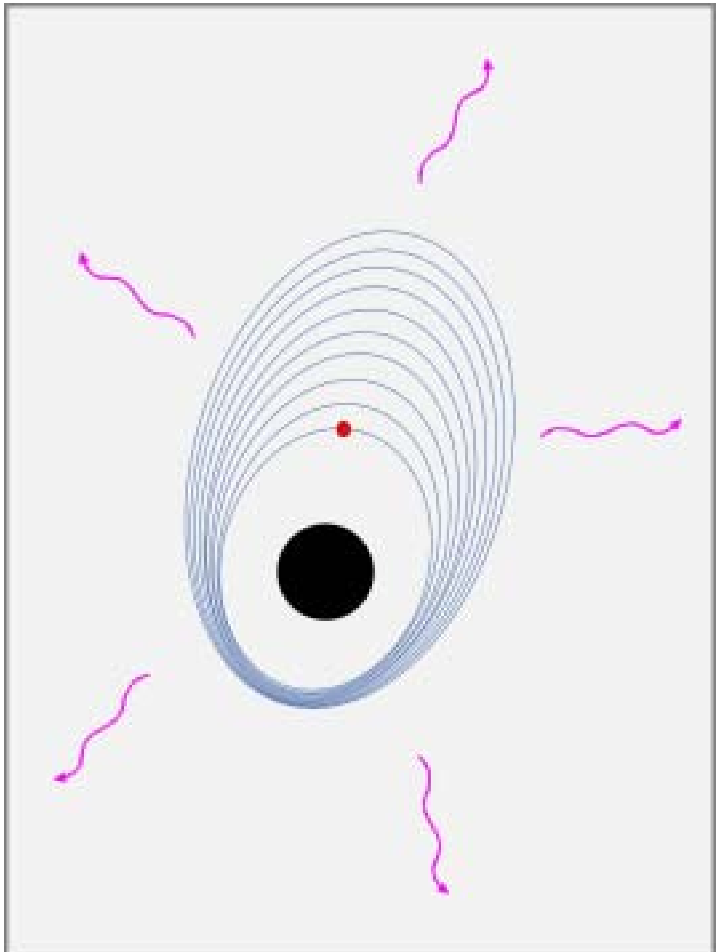
$$6\times10^{-8} < R(NS,BH) < 4\times10^{-7}$$

- For object to be "Captured":

$$1-10^{-3} < e_0 < 1-10^{-6}\ , \qquad 4r_s < r_{p,0} < 50r_s$$

- CO spends ~10^6 yrs inspiraling, losing energy & angular momentum through emission of GW during periastron passages. Orbit gradually shrinks & circularizes.
- GW signal still too weak (and low-f) for detection, but high-harmonics "leak" to LISA band, contribute to "confusion" background.

EMRI scenario



2nd Epoch: Strong-field inspiral

- A few yrs to plunge, ~10^5 cycles to go,

  f_{orb}” 2mHz ◇$(M/10^6 M_\odot)^{-1}$

- LISA sensitive to 5◇$10^5 M_\odot$<M<5◇$10^6 M_\odot$

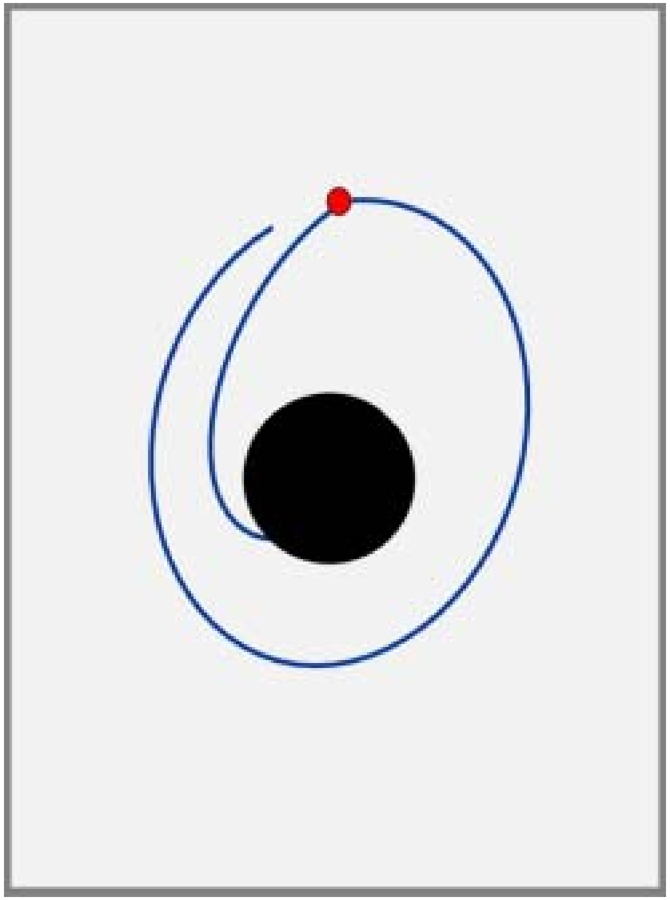
- GW intensity ($10M_\odot$ BH at 1Gpc): h”10^{-22}.
  ×10 smaller than LISA instrumental noise, but 1-yr long matched filtering can give SNR~100

- With SNR_{thresh}”35, LISA sees WDs to a few hundred Mpc, BHs to a few Gpc.

- Orbits: may stay moderately eccentric up till plunge; Show extreme GR features (e.g., “zoom-whirl”)

- Precise map of spacetime, test of GR

EMRI scenario



3^{rd} Epoch: Plunge

- Orbit transits from Inspiral to plunge
- Plunge is brief: Object falls through the event horizon within minutes.
- No imprint on detectable signal

DA of EMRI signals: basic considerations

- Waveforms (will be) well modeled (“problem of Radiation Reaction”)
  - Calls for search by Matched Filtering
- EMRI signal weak, complicated, parameter space huge (17d), need long integration
  - Optimal, coherent matched filtering is unrealistic;
  - Need to apply a (sub-optimal) hierarchical search $\rightarrow$ SNR_{thresh} goes up $\rightarrow$ Detection rate goes down ⇨ See J. Gair’s talk
  - Detection rate limited by computational resources!
- Time-Frequency techniques may detect brightest of EMRIs ⇨ See L. Wen’s poster
- “Confusion noise” from unresolvable galactic WD binaries obscures even strongest of EMRIs below ”2 mHz. Important to fit out WD binaries around 2-5 mHz.
- “Confusion noise” from unresolvable EMRIs may dominate noise at 2-5 mHz

Approximate EMRI waveforms

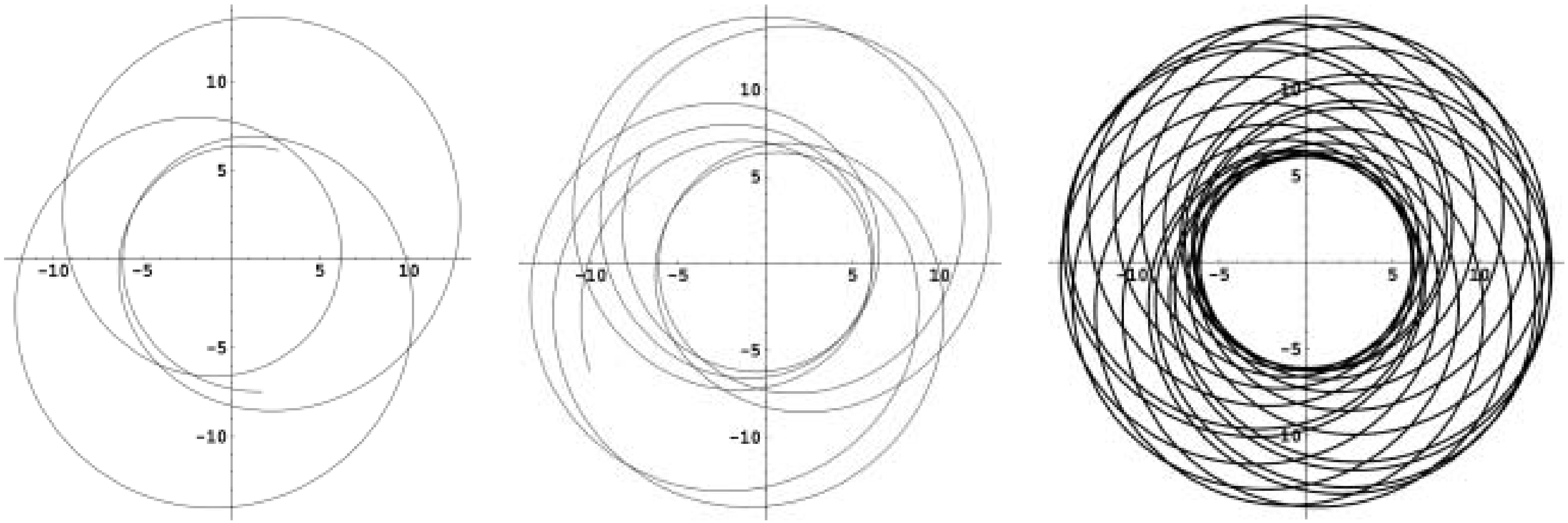
- To scope out data analysis issues we developed a family of approximate EMRI waveforms (Barack & Cutler, PRD 2004):
  - Orbit: “Instantaneously Keplerian”, evolved using post-Newtonian formulae
  - GW emission: Quadropolar, from eccentric orbits (Peters & Mathews 1963)
  - Incorporate the realistic LISA response function (inc. LISA motion & Doppler shift)
  - Waveforms have full dimensionality, feature most essential characteristics

- Gair *et al.* work with an alternative family of approximate waveforms:
  Same, except orbits are “instantaneously geodesic”

- Comparison provides an important sanity check.

Sample equatorial orbits

($m = 10\ M_{\odot}$, $M = 10^{6}\ M_{\odot}$, f= 1 mHz , Ecc=0.4, Spin=0)



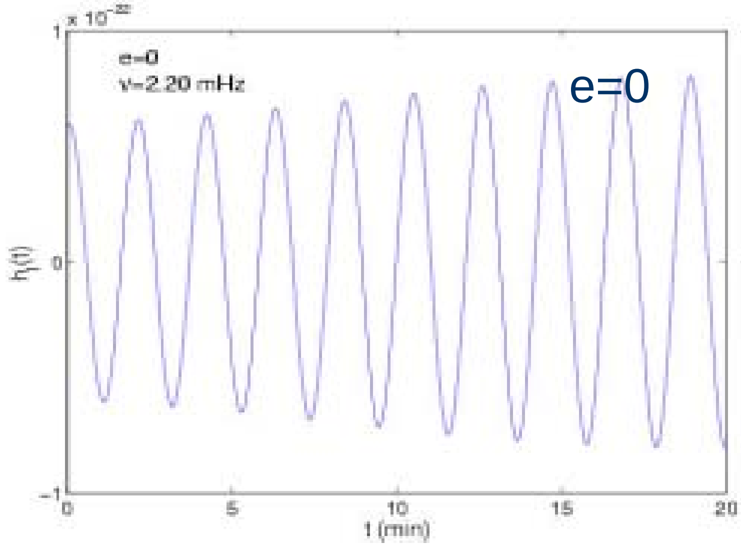
1 hour ⟹ 2 hours ⟹ 6 hours

Sample waveform stretches

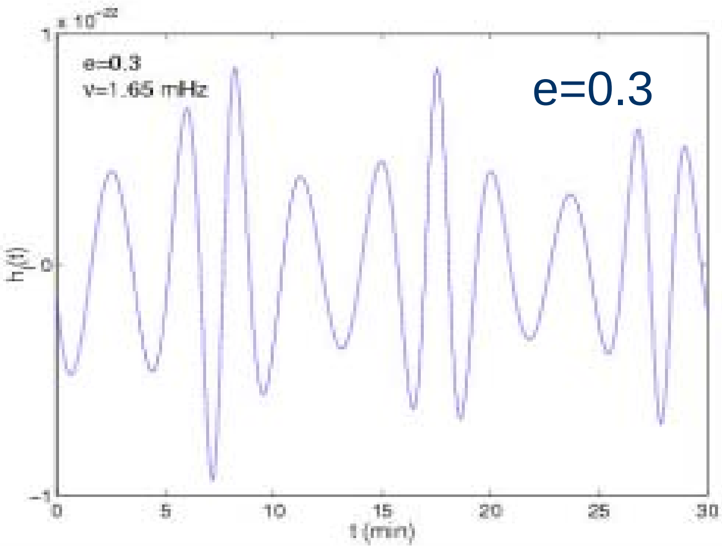
$m = 10\ M_\odot$

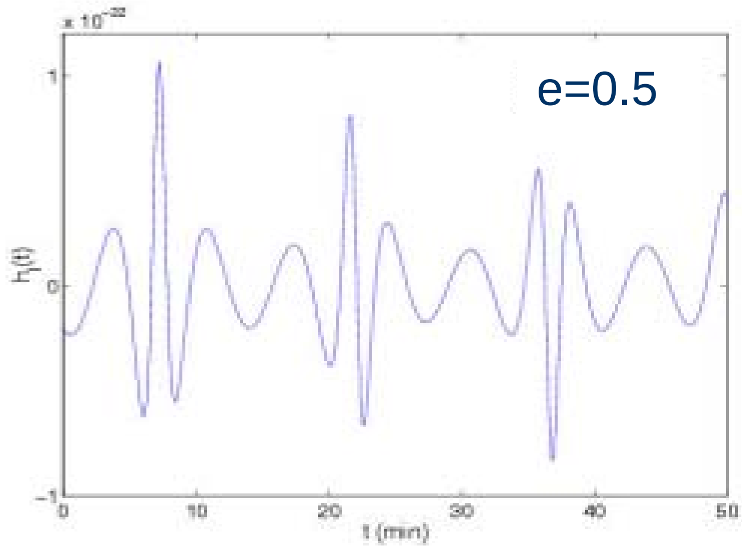
$M = 10^6\ M_\odot$

Spin=0

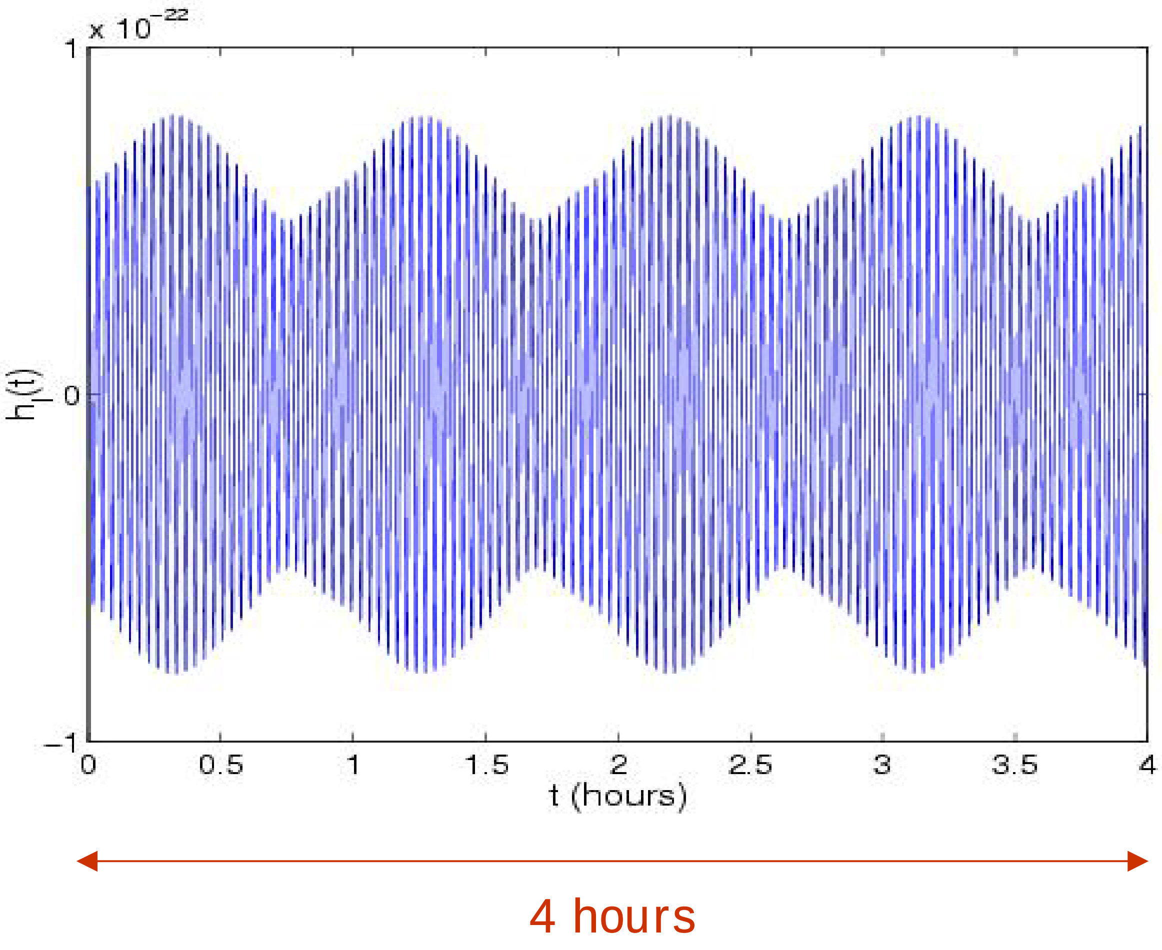


30 min



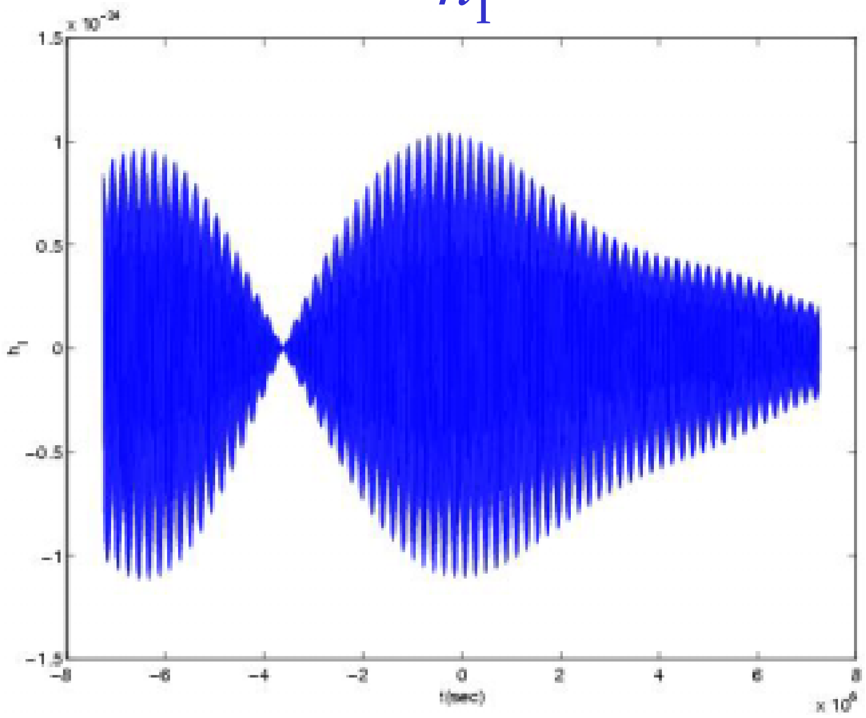


Sample waveform stretches



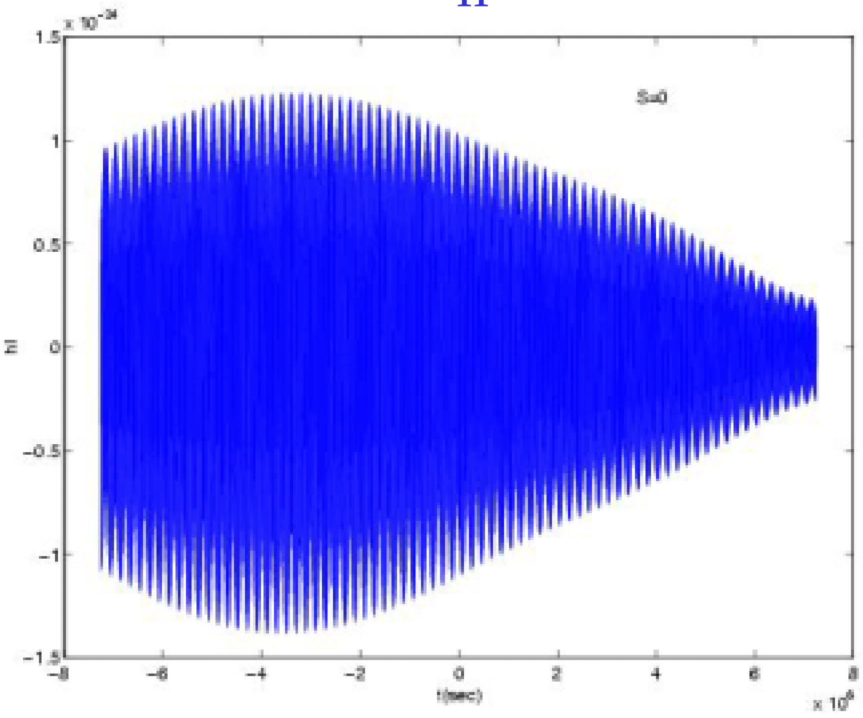
Sample waveform stretches

h_{I}



6 months

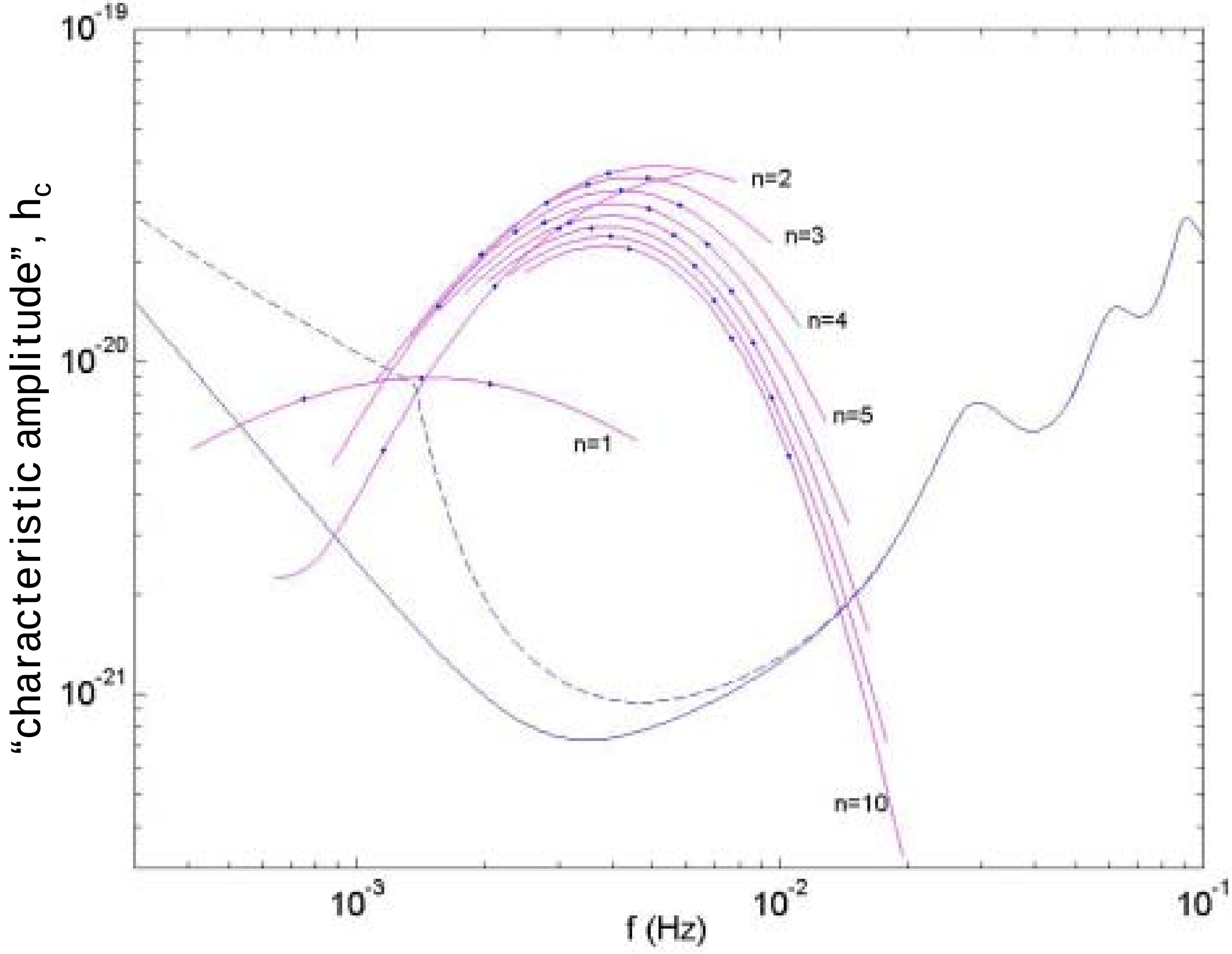
h_{II}



6 months

SNR output & distribution into harmonics

Capture of a BH



m = 10 $M_\odot$

M= 10^6 $M_\odot$

D= 1 Gpc

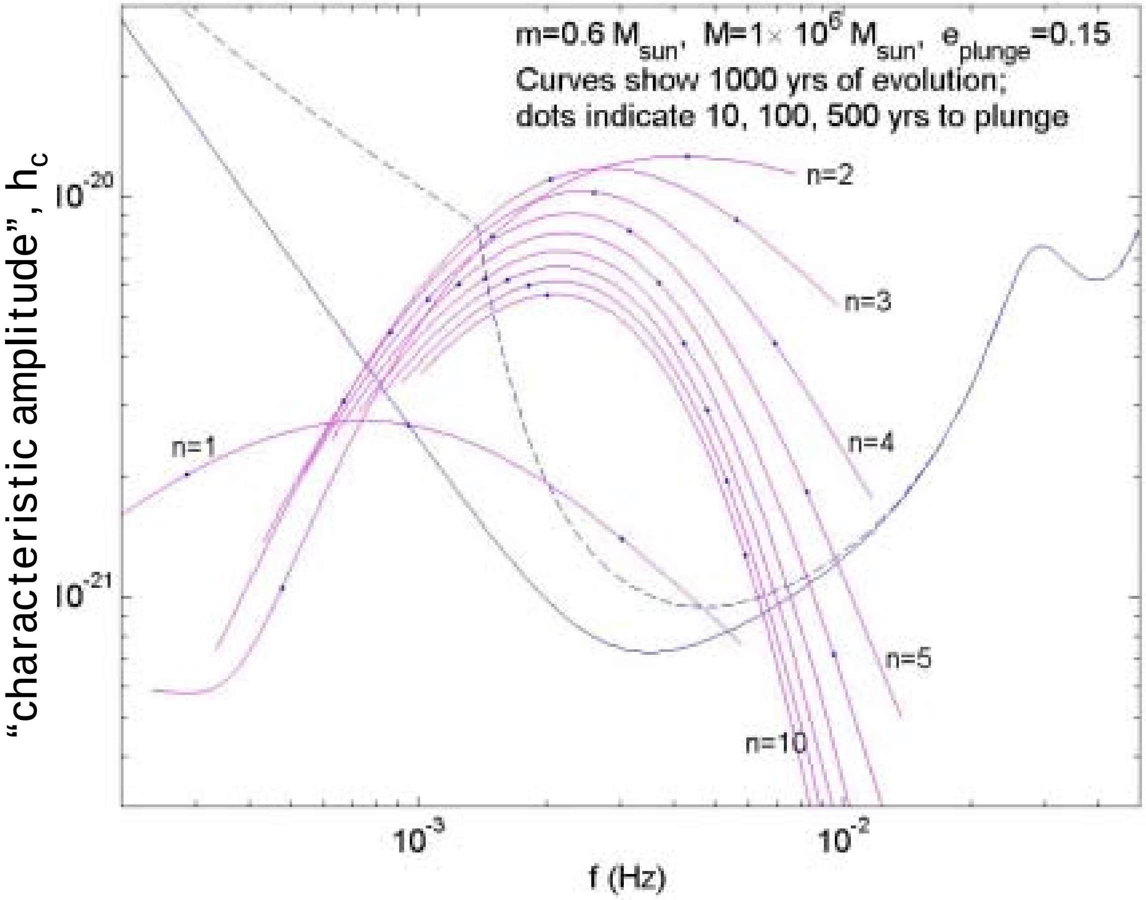
e(plunge)=0.3

e(plunge-10yr)=0.77

- Curves represent 10 yrs of source evolution
- Dots indicate (from left to right) state of system 5, 2, and 1 years before plunge.

SNR output & distribution into harmonics

Capture of a WD



$m = 0.6\ M_\odot$

$M = 10^6\ M_\odot$

$D = 1$ Gpc

e(plunge)=0.15

e(plunge-1000yr)=0.80

- Curves represent 1000 yrs of source evolution
- Dots indicate (from left to right) state of system 500, 100, and 10 years before plunge.

Parameter extraction accuracy

Formalism

- Construct Fisher information matrix: $\Gamma_{ij} \equiv \left\langle \frac{\partial h}{\partial \lambda^i} \middle| \frac{\partial h}{\partial \lambda^j} \right\rangle = \int_{-\infty}^{\infty} \frac{\partial \tilde{h}(f)}{\partial \lambda^i} \frac{\partial \tilde{h}(f)}{\partial \lambda^j} S^{-1}(f) df$
- Get accuracy for parameter λ^i: $\Delta\lambda^i = \sqrt{(\Gamma^{-1})^{ii}}$
- Get LISA's angular resolution: $\Delta\Omega = 2\pi \sqrt{(\Gamma^{-1})^{\cos\theta_S, \cos\theta_S} (\Gamma^{-1})^{\phi_S, \phi_S} - [(\Gamma^{-1})^{\cos\theta_S, \phi_S}]^2}$

Main Results

For inspiral of $10M_\odot$ BH into a $10^6M_\odot$ MBH (data from last year of inspiral, with SNR=30):

$\Delta(\ln m) \sim \Delta(\ln M) \sim \Delta|S|$ 🕓 10^{-4}

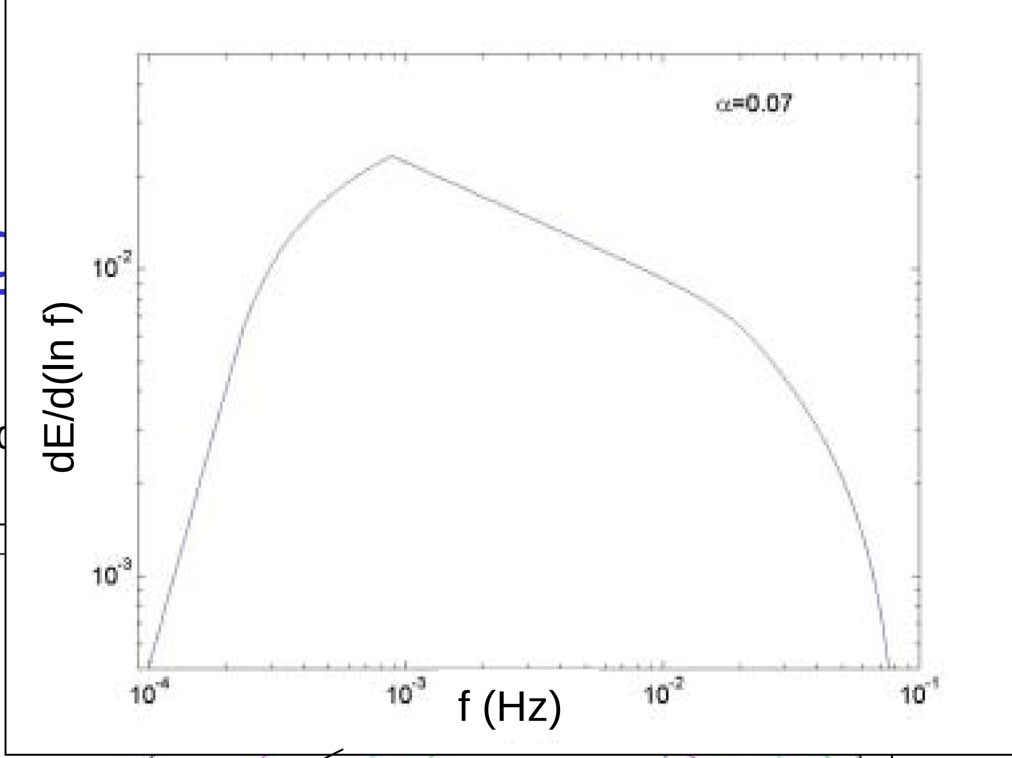
$\Delta\Omega$ 🕓 10^{-3} strd.

From inspiral of a low-mass MS star into the SBH at Sgr A*:

$\Delta|S|_{\text{Milky Way}}$ 🕓 5�10^{-3}

Confusion backg

- From EMRIs (mainly WDs) too far to
- From early history of EMRIs



From Approximate model, get Energy output and spectrum of a single EMRI

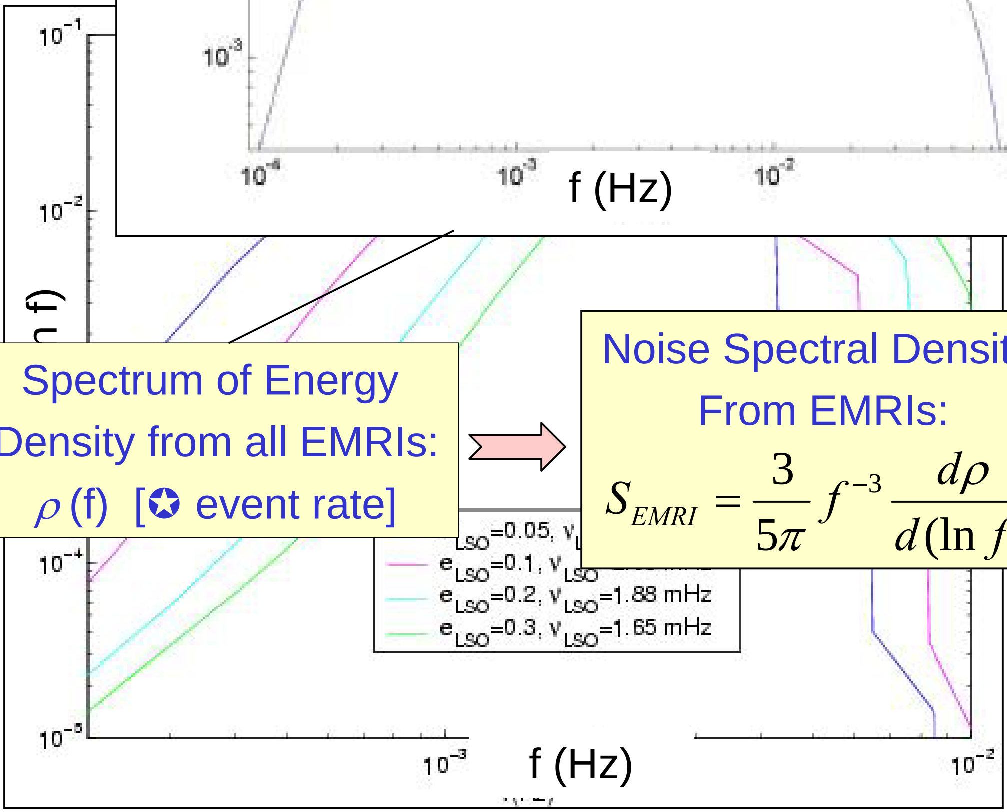
Astroph. Distribution of source parameters & event rates

Cosmology and MBH evolution (consider range of possible models)

Spectrum of Energy Density from all EMRIs: ρ(f) [✪ event rate]

Noise Spectral Density From EMRIs:

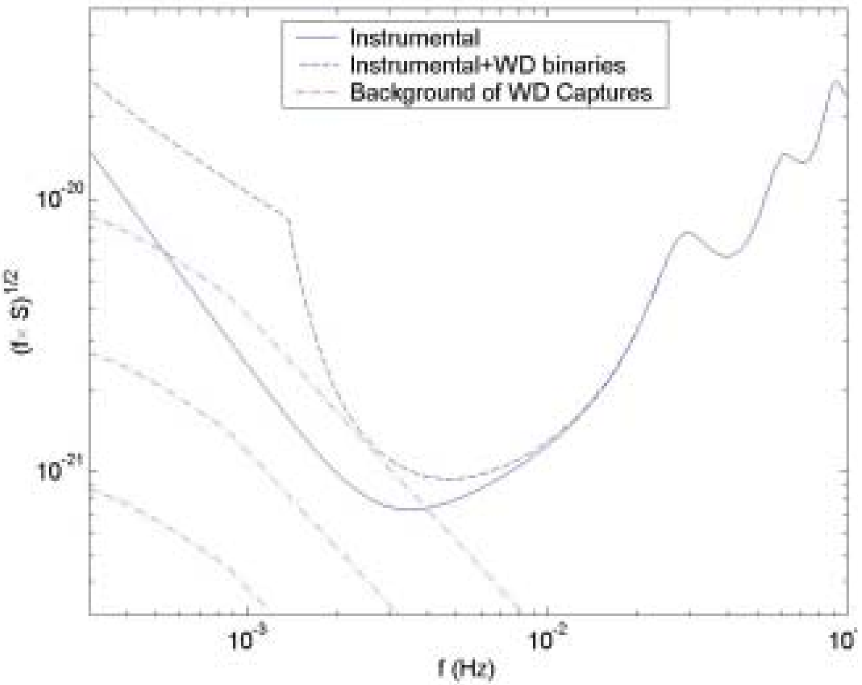
$$S_{EMRI} = \frac{3}{5\pi} f^{-3} \frac{d\rho}{d(\ln f)}$$



Confusion background from EMRIs

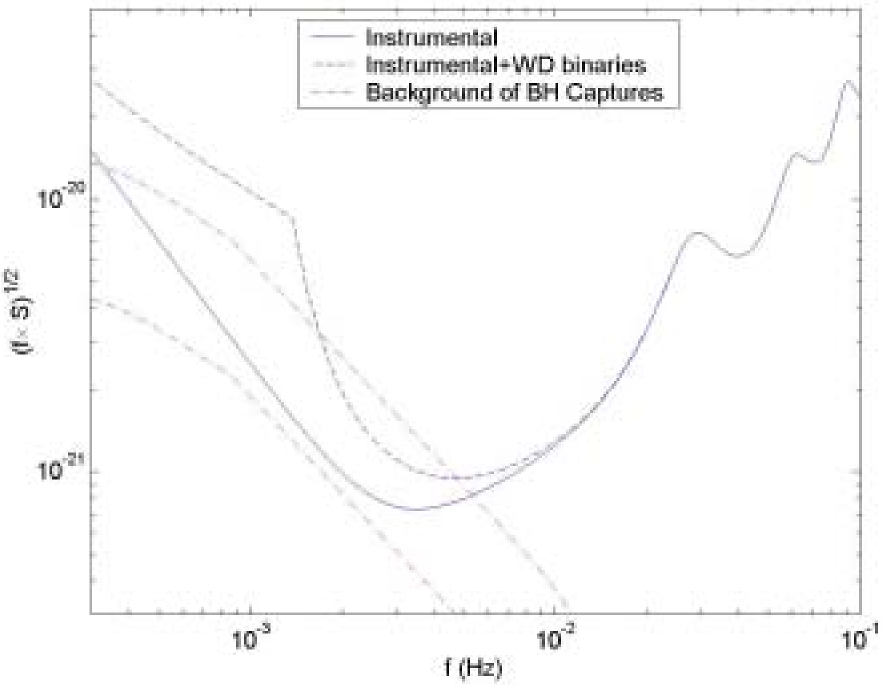
(before subtraction of resolvable sources)

From 0.6 $M_\odot$ White Dwarf EMRIs



Estimate: over 95% unsubtractable

From 10 $M_\odot$ Black holes EMRIs

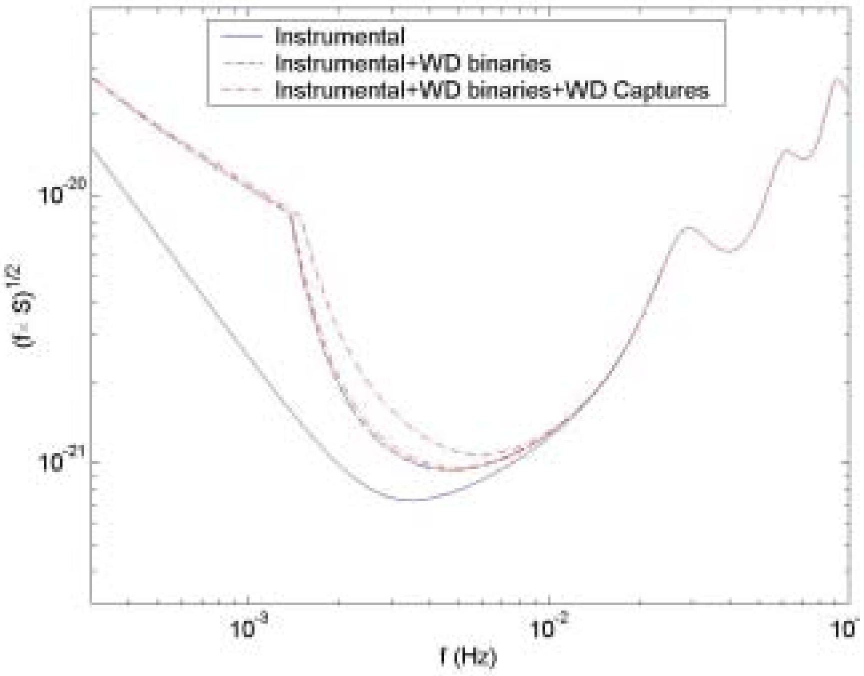


Estimate: 30-40% unsubtractable

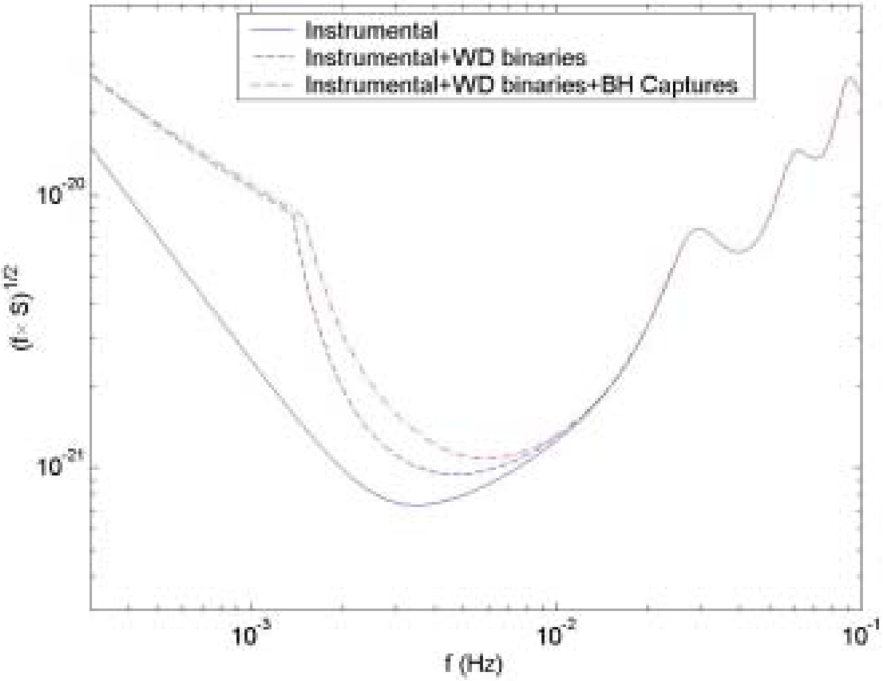
Total LISA Noise, inc. EMRI Confusion

(after subtraction of resolvable sources)

From 0.6 $M_\odot$ White Dwarf EMRIs



From 10 $M_\odot$ Black holes EMRIs



Still lots of work…

- Improve astrophysical event rates and distribution of source parameters
- Learn how to calculate accurate radiation-reaction evolution of orbits in Kerr, and construct waveform templates
- See if can improve extraction of WD-WD background near 2 mHz
- Complete design and simulation of hierarchical search scheme
- Study alternatives? E.g., T-F search may help in preliminary parameter extraction for brightest sources.