



*Beyond Einstein: From the Big Bang to Black Holes*

# *Self-gravity modeling for LISA*

*5<sup>th</sup> International LISA Symposium*

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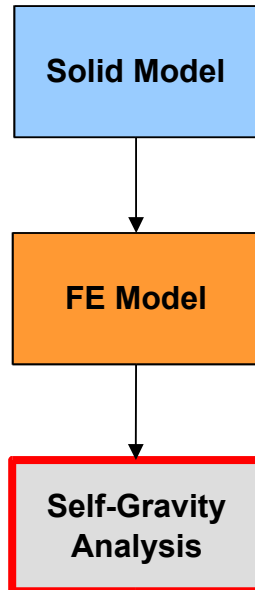
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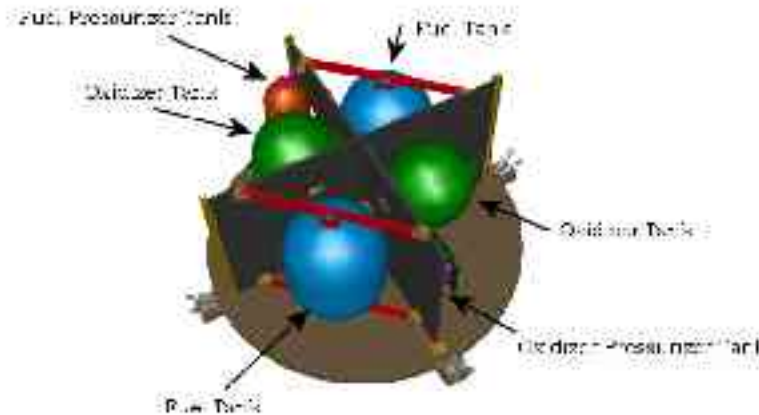
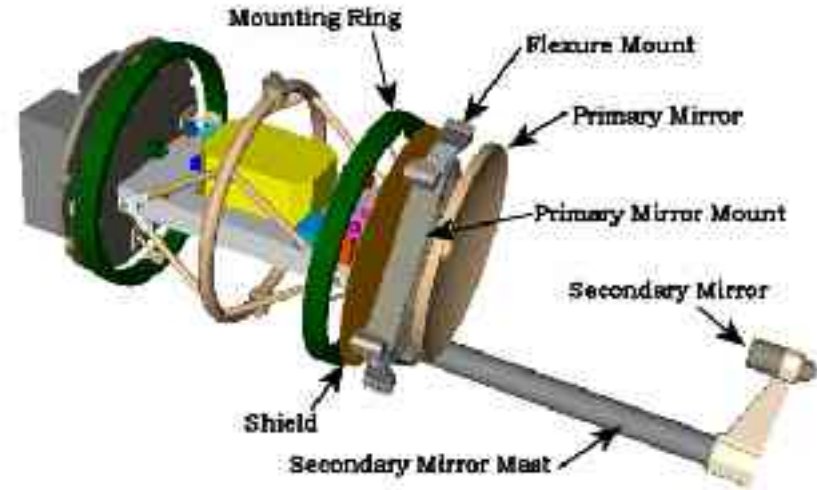


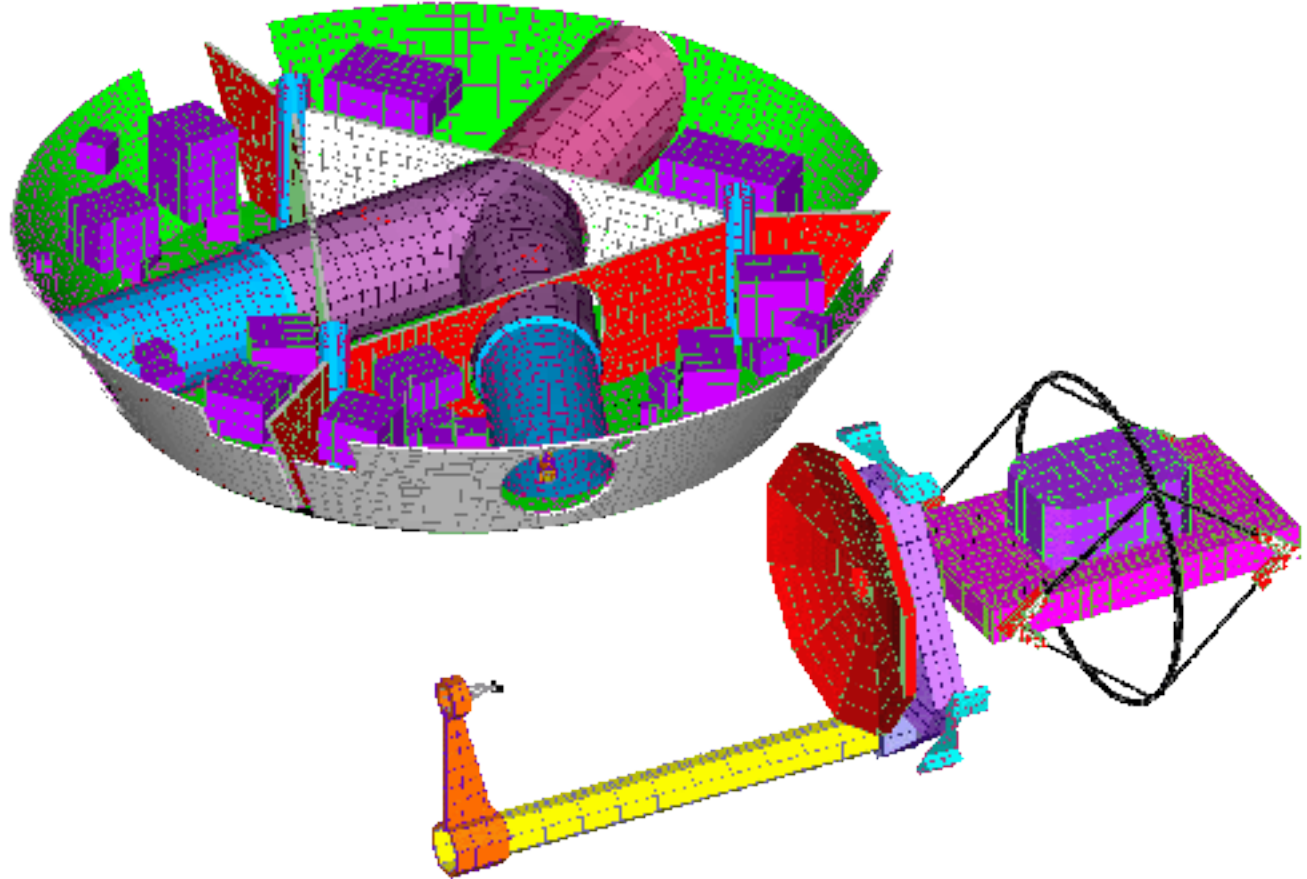
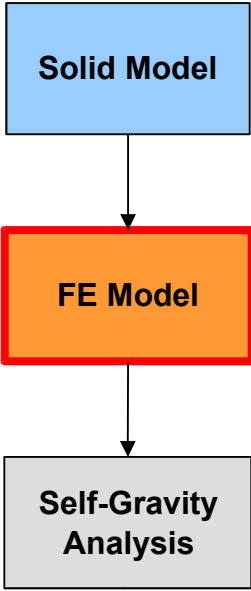
- Mission level requirements:
  - Total Static Self-Gravity  $\leq 5 \times 10^{-10} \text{ m/s}^2$ .
    - Equivalent to acceleration from 1 kg mass 36 cm away.
    - Acceleration of Mars on you is  $\sim 100 \times 10^{-10} \text{ m/s}^2$  (20 x LISA budget).
  - Total Static Self-Gravity Gradient  $\leq 3 \times 10^{-8} \text{ s}^{-2}$ .
    - Equivalent to gradient from 1 kg mass 16 cm away.
- Self-gravity analysis is a critical part of LISA Integrated Modeling.
- STOPG analysis used to analytically determine spacecraft distortions due to structural, thermal, and release of 1g loads.
  - Based on current practices used in integrated Structural, Thermal, OPTical (STOP) analysis
  - Addition of self-Gravity analysis leads to STOPG
- Self-Gravity tools accept distorted and undistorted cases.
- Outputs forces, moments, and their gradients on the proof masses.

Solid Model

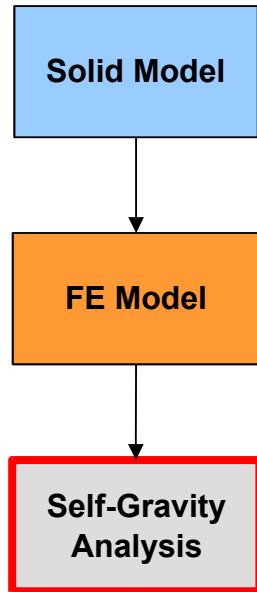
FE Model

Self-Gravity Analysis



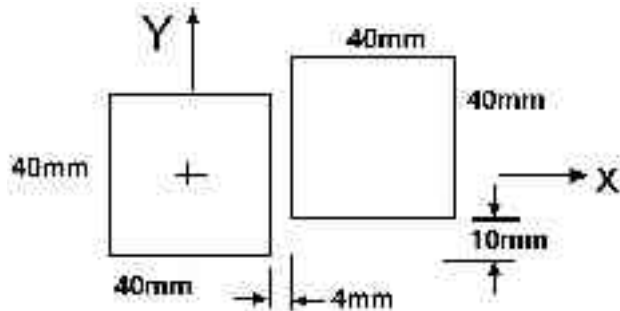


- Meshing performed in FEMAP.
- A single mesh is used for all portions of the STOPG analysis.
- Current model has ~43,000 nodes

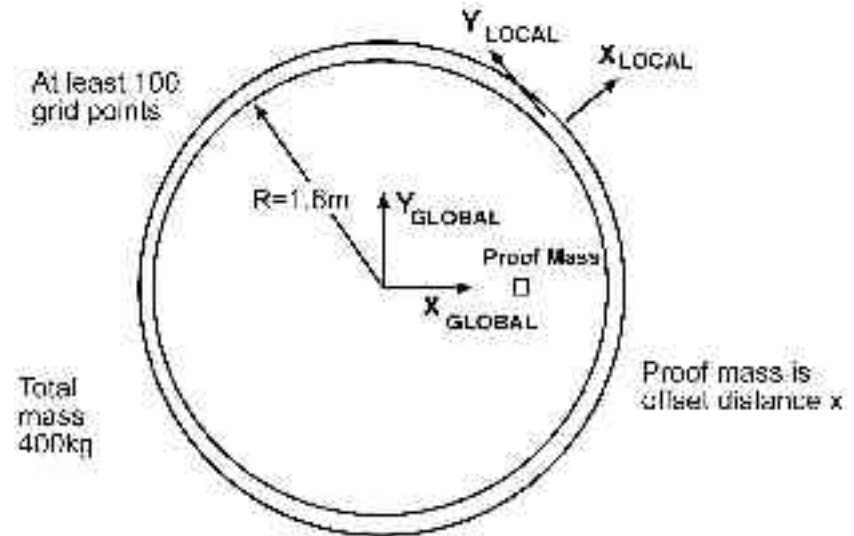


- Two independent self-gravity tools developed
  - Fortran 77 version (Swales developed)
  - Matlab version
- Point mass approximation used
- Inputs
  - FEM mass matrix and rigid body vectors
  - Spacecraft deformed node locations
- Outputs
  - Forces and moments on a proof mass (6 terms each)
  - Gradients on forces and moments (36 terms each)
  - Numerous checks on the accuracy of the inputs and calculations.
- Tools accept distorted and undistorted cases
  - Thermal distortions
  - One-g release distortions


- Two independent tools developed and cross-checked
- Verified independently using analytically solvable problems
- Two examples:



Two cubes in proximity



Circular ring with proof mass

- 
**Conflicting mesh requirements:**
  - Coarse mesh size for thermal (except in regions of high gradient)
  - Fine mesh size for structural analysis of deformed shapes
  - Very fine mesh size for self-gravity near the proof masses

- 
**Self gravity guidelines for the FEM mesh size were developed:**



	Maximum D/R for Grid Size	Maximum L/C for point mass approximation
Forces	0.9	0.3
Moments	0.5	0.1
Force gradients	0.4	0.2
Moment gradients	0.25	0.15

D = FEM grid spacing, the mesh size

R = Distance from the grid point to the surface of the proof mass

L = Largest dimension of the solid body

C = Distance from the center of the body to the surface of the proof mass

- 
**Tool estimates errors caused by D/R ratios.**
- 
**A separate tool is used to remesh structure close to proof mass**



# Self-Gravity Division



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Subdivision	# Grid Points	Mass (kg)
+Y housings with proof mass	2601	5.546
+Y optical block with mount and proof mass vacuum cover	4851	7.415
+Y telescope with primary mirror, mounts and sec. mirror assy.	1093	4.915
+Y OB cylindrical thermal shields, rings and flexures	1577	8.154
-Y housing with proof mass	2601	5.546
-Y optical block with mount and vacuum cover	4851	7.416
-Y telescope with primary mirror, mounts and sec. mirror assy.	1093	4.915
-Y OB cylindrical thermal shields rings and flexures	1577	8.157
Y tube with sun shade	1486	24.163
S/C structure top plate, bottom plate, bulkheads and support tubes	11842	93.037
S/C thermal shield, solar array and radiator panels	7832	36.490
E-boxes with their isolators	1922	68.243
Two HGAs	2	3.000
<b>Total</b>	<b>43336</b>	<b>276.998</b>





# Self-Gravity Results



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	For the +Y Test Mass	For the -Y Test Mass
$A_x \text{ (m/s}^2 \times 10^{-10}) =$	-26.8925	-20.6873
$A_y \text{ (m/s}^2 \times 10^{-10}) =$	-58.7772	58.1310
$A_z \text{ (m/s}^2 \times 10^{-10}) =$	-40.7371	-40.8112
$\alpha_\xi \text{ (}\rho\alpha\delta\sigma^2\xi 10^{-10}) =$	1.2278	1.2652
$\alpha_\psi \text{ (}\rho\alpha\delta\sigma^2\xi 10^{-10}) =$	0.0610	0.0486
$\alpha_\zeta \text{ (}\rho\delta/\sigma^2\xi 10^{-10}) =$	0.1161	0.0205

Requirement = 5

	$/\delta\xi (\mu)$	$/\delta\psi (\mu)$	$/\delta\zeta (\mu)$	$/\delta\xi (\mu)$	$/\delta\psi (\mu)$	$/\delta\zeta (\mu)$
$\delta A_\xi \text{ (}1/\sigma^2\xi 10^{-10}) =$	-1264.32	85.39	-25.74	-1258.04	-104.25	-24.69
$\delta A_\psi \text{ (}1/\sigma^2\xi 10^{-10}) =$	85.39	1669.91	7.20	-104.25	1666.05	-26.13
$\delta A_\zeta \text{ (}1/\sigma^2\xi 10^{-10}) =$	-25.74	7.19	-405.60	-24.69	-26.13	-408.01
$\delta\alpha_\xi \text{ (}\rho\sigma^2/\mu\xi 10^{-10}) =$	1.81	1137.84	49.92	1.42	1138.13	50.69
$\delta\alpha_\psi \text{ (}\rho\sigma^2/\mu\xi 10^{-10}) =$	-300.77	-0.70	-9.01	-300.88	-0.45	-6.62
$\delta\alpha_\zeta \text{ (}\rho\sigma^2/\mu\xi 10^{-10}) =$	-1.73	-8.19	-1.11	-0.16	-4.63	-0.97

Requirement = 300

	$/\delta\theta_\xi (\rho)$	$/\delta\theta_\psi (\rho)$	$/\delta\theta_\zeta (\rho)$	$/\delta\theta_\xi (\rho)$	$/\delta\theta_\psi (\rho)$	$/\delta\theta_\zeta (\rho)$
$\delta A_\xi \text{ (}\mu/\sigma^2/\rho\xi 10^{-10}) =$	0.00	40.66	-58.78	0.00	40.73	58.13
$\delta A_\psi \text{ (}\mu/\sigma^2/\rho\xi 10^{-10}) =$	-40.43	0.00	26.89	-40.51	0.00	20.69
$\delta A_\zeta \text{ (}\mu/\sigma^2/\rho\xi 10^{-10}) =$	58.79	-26.89	0.00	-58.12	-20.69	0.00
$\delta\alpha_\xi \text{ (}1/\sigma^2\xi 10^{-10}) =$	1812.3	-0.171	-0.118	1812.23	-0.011	-0.019
$\delta\alpha_\psi \text{ (}1/\sigma^2\xi 10^{-10}) =$	-0.055	335.90	-1.260	0.009	335.90	-1.264
$\delta\alpha_\zeta \text{ (}1/\sigma^2\xi 10^{-10}) =$	-0.178	-0.032	297.85	-0.068	0.001	297.84



# Self-Gravity Breakdown



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Results on +Y proof mass in local proof mass coordinates

Source	Ax (m/s <sup>2</sup> x 10 <sup>-10</sup> )	Ay (m/s <sup>2</sup> x 10 <sup>-10</sup> )	Az (m/s <sup>2</sup> x 10 <sup>-10</sup> )	dAx/dx (1/s <sup>2</sup> x 10 <sup>-10</sup> )	dAy/dy (1/s <sup>2</sup> x 10 <sup>-10</sup> )	dAz/dz (1/s <sup>2</sup> x 10 <sup>-10</sup> )
+ GRS housing	-0.43	-0.03	-0.39	-2274.9	1130.3	1144.7
+ Optical block	0.18	0.53	-43.24	600.8	676.8	-1277.5
+ Telescope	33.32	-0.25	-2.72	196.1	-105.1	-90.9
+ Thermal shield	-35.44	-0.30	-0.31	225.8	-145.7	-80.1
∅Proof mass	-1.26	-2.19	0.00	-1.1	5.4	-4.3
∅GRS housing	-4.24	-7.25	0.01	-3.2	17.7	-14.5
∅Optical block	-7.68	-12.53	-0.26	-3.3	29.7	-26.3
∅Telescope	-0.78	-5.19	-0.17	-5.5	12.6	-7.1
∅Thermal shield	-12.94	-13.01	-0.44	19.8	14.9	-34.8
Y-tube	4.65	-7.67	-3.45	14.9	-0.4	-14.5
S/C structure	-7.39	-4.77	-3.52	-37.0	-7.2	44.2
S/C thermal	-1.02	-0.88	14.82	-4.8	-4.6	9.3
Electronic boxes	6.17	-5.74	-1.49	9.0	44.3	-53.3
High Gain Antennas	0.23	0.40	0.44	-0.4	1.1	-0.7

X-axis is telescope line-of-sight



# Telescope Articulation



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QuickTime® and a  
Video decompressor  
are needed to see this picture.



# Telescope Articulation



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- Analysis completed for change in self-gravity due to telescope articulation.
- Nine models produced with the telescope articulation angle incremented
  - Expected articulation is  $\pm 0.5^\circ$
  - Linear change :  $-1.03 \times 10^{-10}$  m/s<sup>2</sup>/degree for sensitive axis (x)

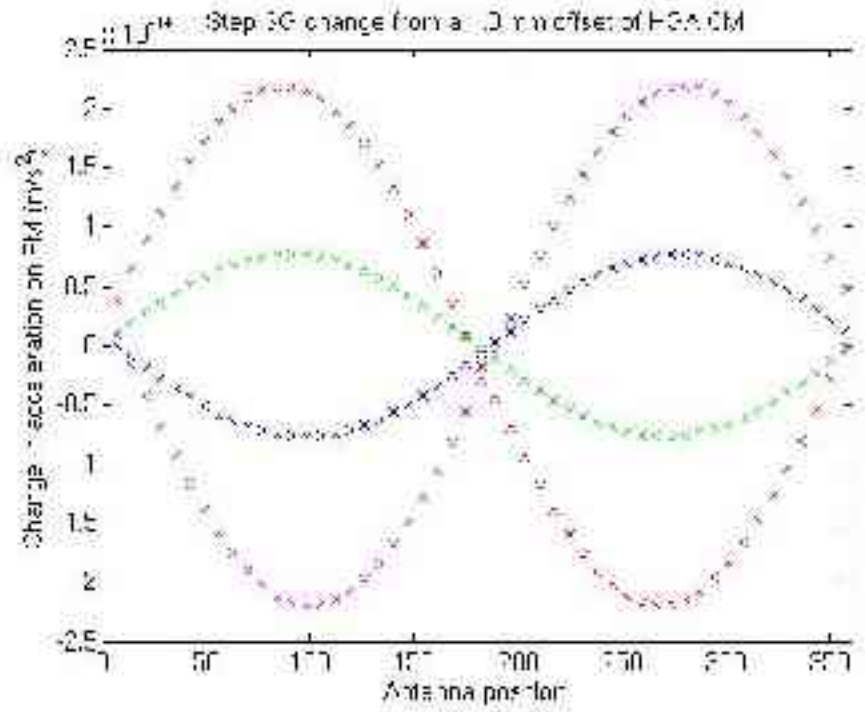
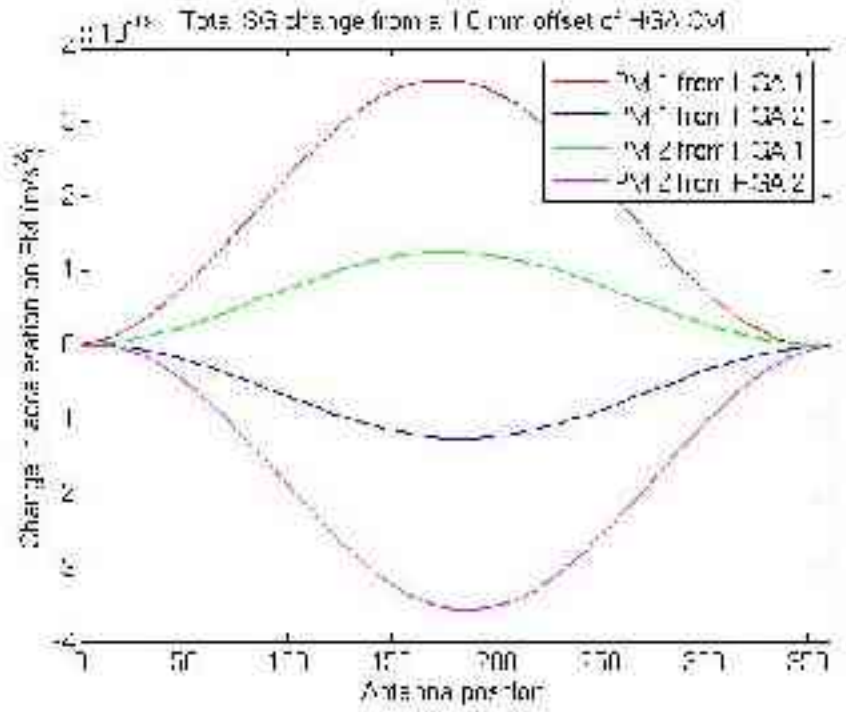
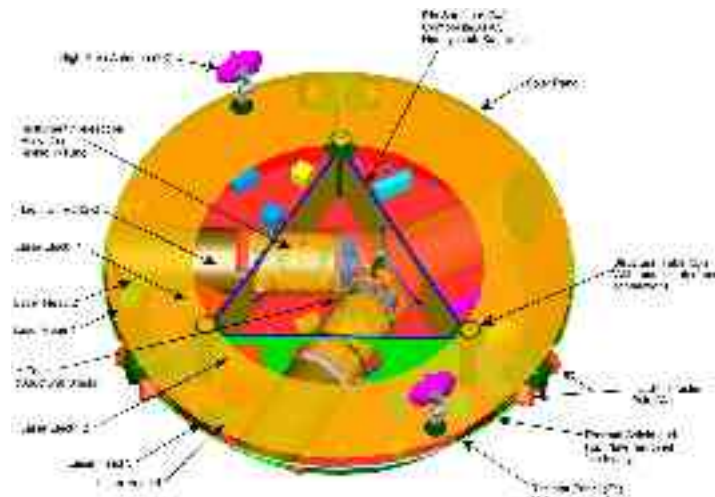
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Change In Translational Accelerations and Gradients on the +Y Proof Mass

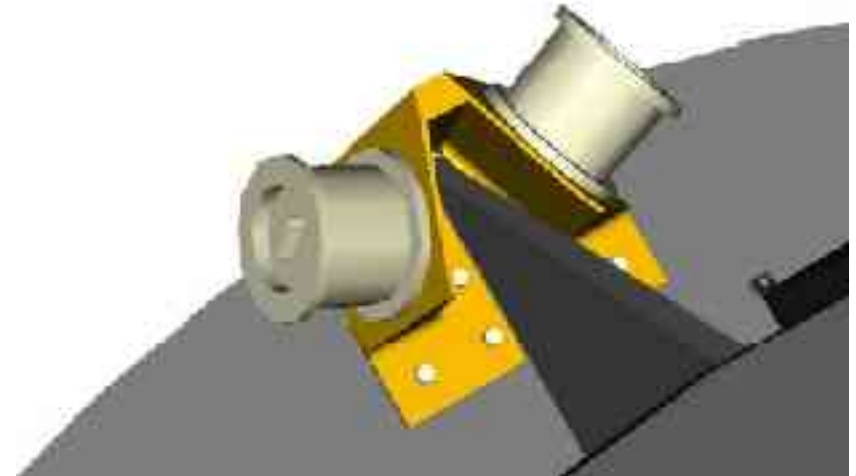
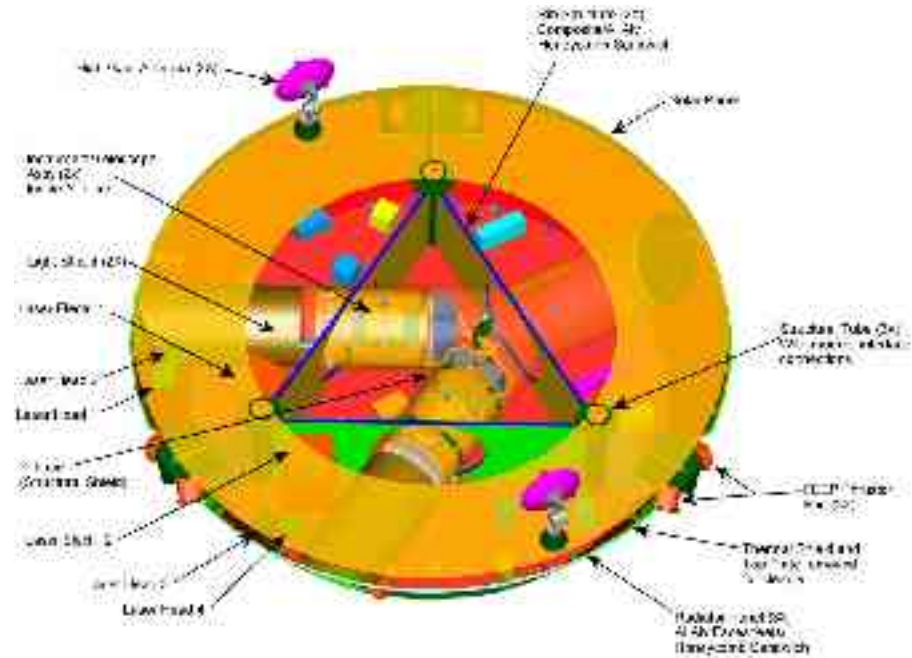
Articulation Angle $\theta_z$	$\Delta A_\xi$ ( $\mu / \sigma^2$ ) $\xi 10^{-10}$	$\Delta A_\psi$ ( $\mu / \sigma^2$ ) $\xi 10^{-10}$	$\Delta A_\zeta$ ( $\mu / \sigma^2$ ) $\xi 10^{-10}$	$\Delta \delta A_\xi / \delta \xi$ ( $1 / \sigma^2$ ) $\xi 10^{-10}$	$\Delta \delta A_\psi / \delta \psi$ ( $1 / \sigma^2$ ) $\xi 10^{-10}$	$\Delta \delta A_\zeta / \delta \zeta$ ( $1 / \sigma^2$ ) $\xi 10^{-10}$
+0.8 <sup>0</sup>	-0.824	0.351	2ε-6	2.32	-2.32	2ε-4
+0.6 <sup>0</sup>	-0.619	0.262	2ε-6	1.74	-1.74	1ε-4
+0.4 <sup>0</sup>	-0.413	0.174	1ε-6	1.15	-1.15	7ε-5
+0.2 <sup>0</sup>	-0.207	0.087	4ε-7	0.58	-0.58	4ε-5
0	0	0	0	0	0	0
-0.2 <sup>0</sup>	0.207	-0.086	-5ε-7	-0.58	0.57	-4ε-5
-0.4 <sup>0</sup>	0.414	-0.171	-1ε-6	-1.14	1.14	-8ε-5
-0.6 <sup>0</sup>	0.622	-0.256	-2ε-6	-1.71	1.71	-1ε-4
-0.8 <sup>0</sup>	0.829	-0.339	-2ε-6	-2.27	2.27	-2ε-4

~20 % of requirement

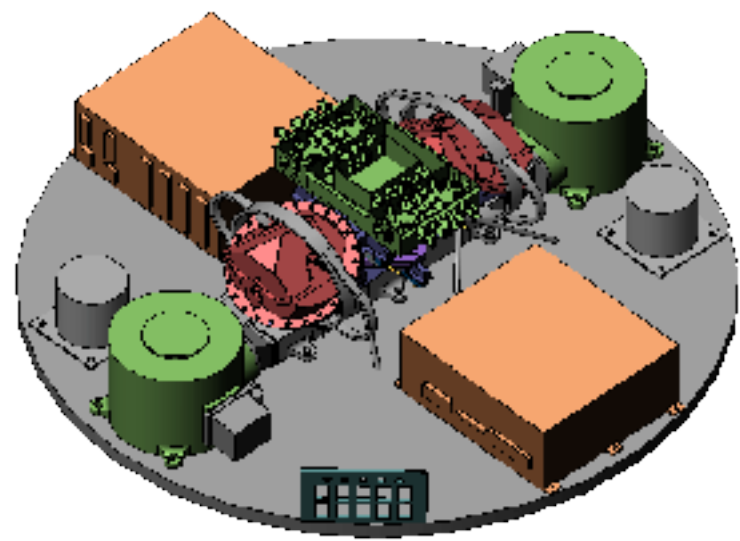
- Two high gain antennas rotate  $\sim 7^\circ$  once per week.
- For a 1 mm CM offset of 1 antenna:
  - $3.6 \times 10^{-13} \text{ m/s}^2 \text{ pp}$  over 1 year.
  - $2.2 \times 10^{-14} \text{ m/s}^2$  max step



- ☛ Change in self-gravity from  $\mu$ N-thrusters propellant loss is out of band.
- ☛ Over life of mission, worst case total change is:
  - $0.7 \times 10^{-10} \text{ m/s}^2$  (14% of requirement) for colloid thruster with  $I_{sp}=1000\text{s}$ , total mass = 8 kg.
  - $12 \times 10^{-10} \text{ m/s}^2$  (240% of requirement) for cold gas with  $I_{sp}=60\text{s}$ , total mass = 20 kg.

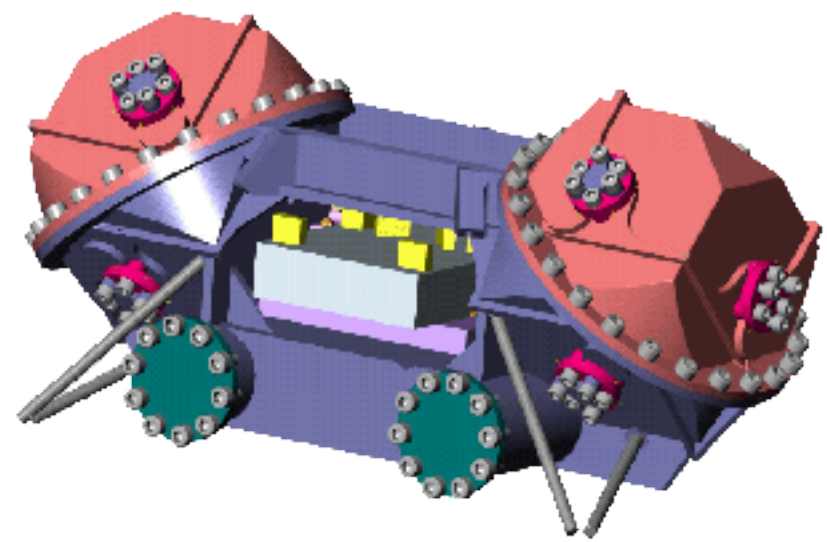


- 🌀 ST-7 DRS flies on the ESA LISA Pathfinder spacecraft
- 🌀 L1 orbit
- 🌀 Launch date: late 2007
- 🌀 Operational life: 3 months
- 🌀 The key new technologies are gravitational reference sensors and microthrusters.
  - DRS validate spacecraft position control to an accuracy of  $<10\text{nm}/\sqrt{\text{Hz}}$  over frequency range 1 mHz to 10 mHz
  - DRS validates that a test mass follows trajectory determined by gravitational forces only within  $3 \times 10^{-14} \text{ m/s}^2/\sqrt{\text{Hz}}$  over frequency range 1 mHz to 10 mHz

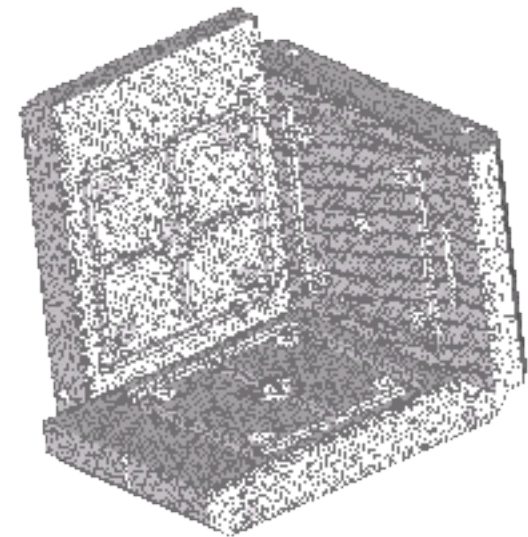
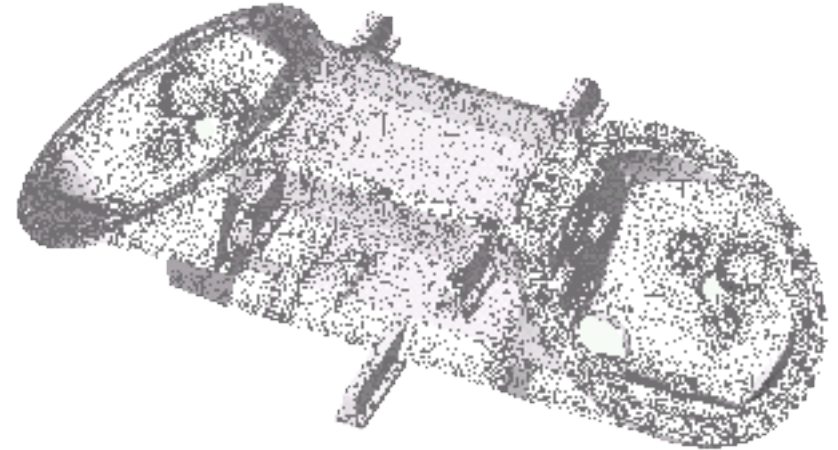


🌀 Self-Gravity Requirements:

Type	Allowed Max. Acceleration
DC acceleration	$1.0 \times 10^{-9} \text{ m/s}^2$
DC acceleration gradient	$80.0 \times 10^{-9} \text{ 1/s}^2$
AC fluctuations	$5.0 \times 10^{-15} \text{ m/s}^2/\sqrt{\text{Hz}}$



- The FEM consisted of 39 sub-assembly models
- Overall – 1,266,463 grid points (7,598,778 DOF)
- Mathematical errors estimated to be less than 0.1%
- Results in the sensing direction:

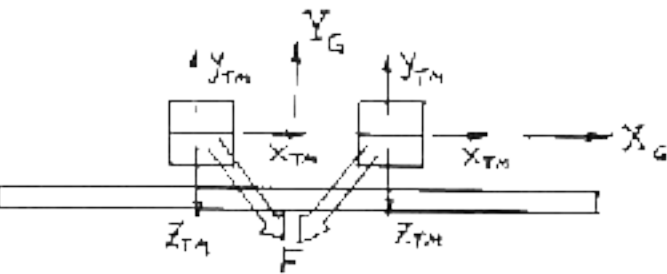
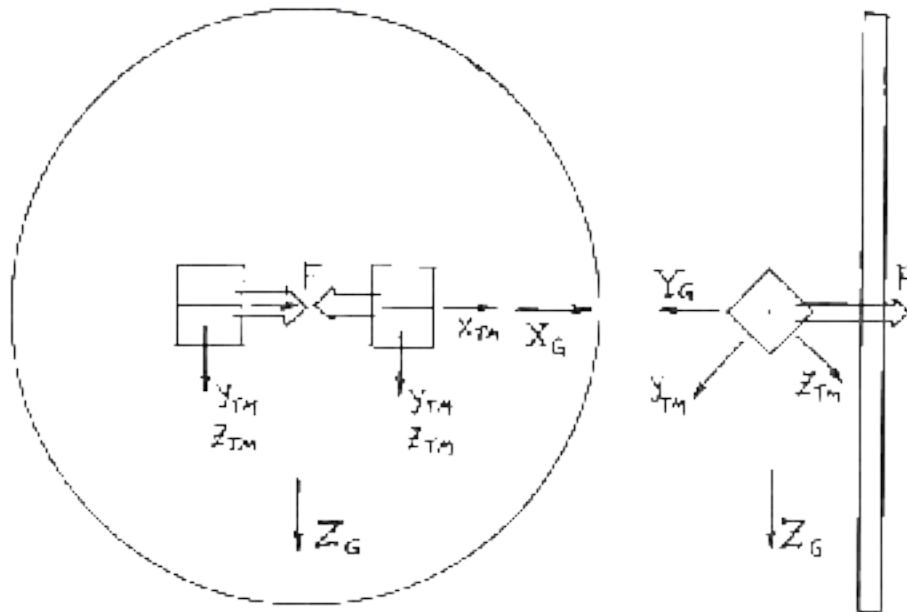


	$A_x$ ( $m/s^2 \times 10^{-9}$ )	$dA_x/dx$ ( $1/s^2 \times 10^{-9}$ )
From the other proof mass	-1.3	10.0
From housing walls alone	-0.4	8.7
From chassis alone	-9.1	50.7
From GRS assembly	-15.6	174.8
Complete assembly	-21.7	124.8
Budget	1.0	80.0

∴ **ST-7 flight provides a self-gravity anchor point.**

Ref: SAI-RPT-570, "Preliminary Self-Gravity of the ST7 DRS", by William Haile, 10Feb'04





	+X PM	-X PM	Budget
$A_x$ ( $m/s^2 \times 10^{-9}$ )	-21.7	22.0	1.0
$A_y$ ( $m/s^2 \times 10^{-9}$ )	-21.6	-21.5	
$A_z$ ( $m/s^2 \times 10^{-9}$ )	21.2	21.2	
$\alpha_x$ ( $rad/s^2 \times 10^{-9}$ )	-0.01	-0.03	
$\alpha_y$ ( $rad/s^2 \times 10^{-9}$ )	0.14	-0.15	
$\alpha_z$ ( $rad/s^2 \times 10^{-9}$ )	0.04	-0.05	
$dA_x/dx$ ( $1/s^2 \times 10^{-9}$ )	124.8	124.8	80.0
$dA_y/dy$ ( $1/s^2 \times 10^{-9}$ )	-60.7	-62.0	
$dA_z/dz$ ( $1/s^2 \times 10^{-9}$ )	-64.2	-62.8	

- Complete thermal distortion study (in progress).
- Repeat analysis on evolving LISA design as necessary.
- Update ST-7 analysis after CDR.
- Study effects of non-uniform proof mass density.
- Develop self-gravity balancing techniques and tools.