LISA Phase Measurement System: Simulation and Results

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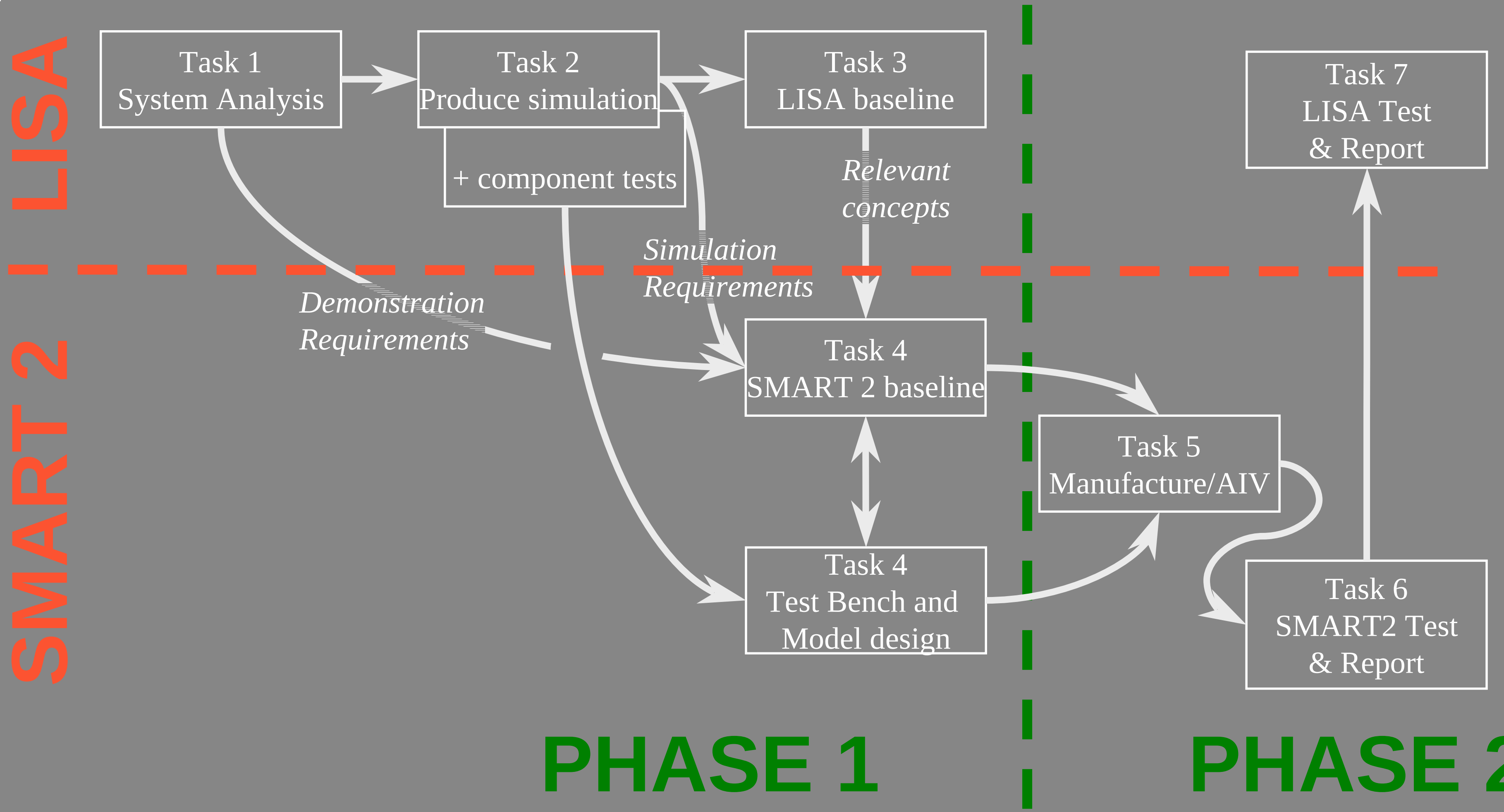
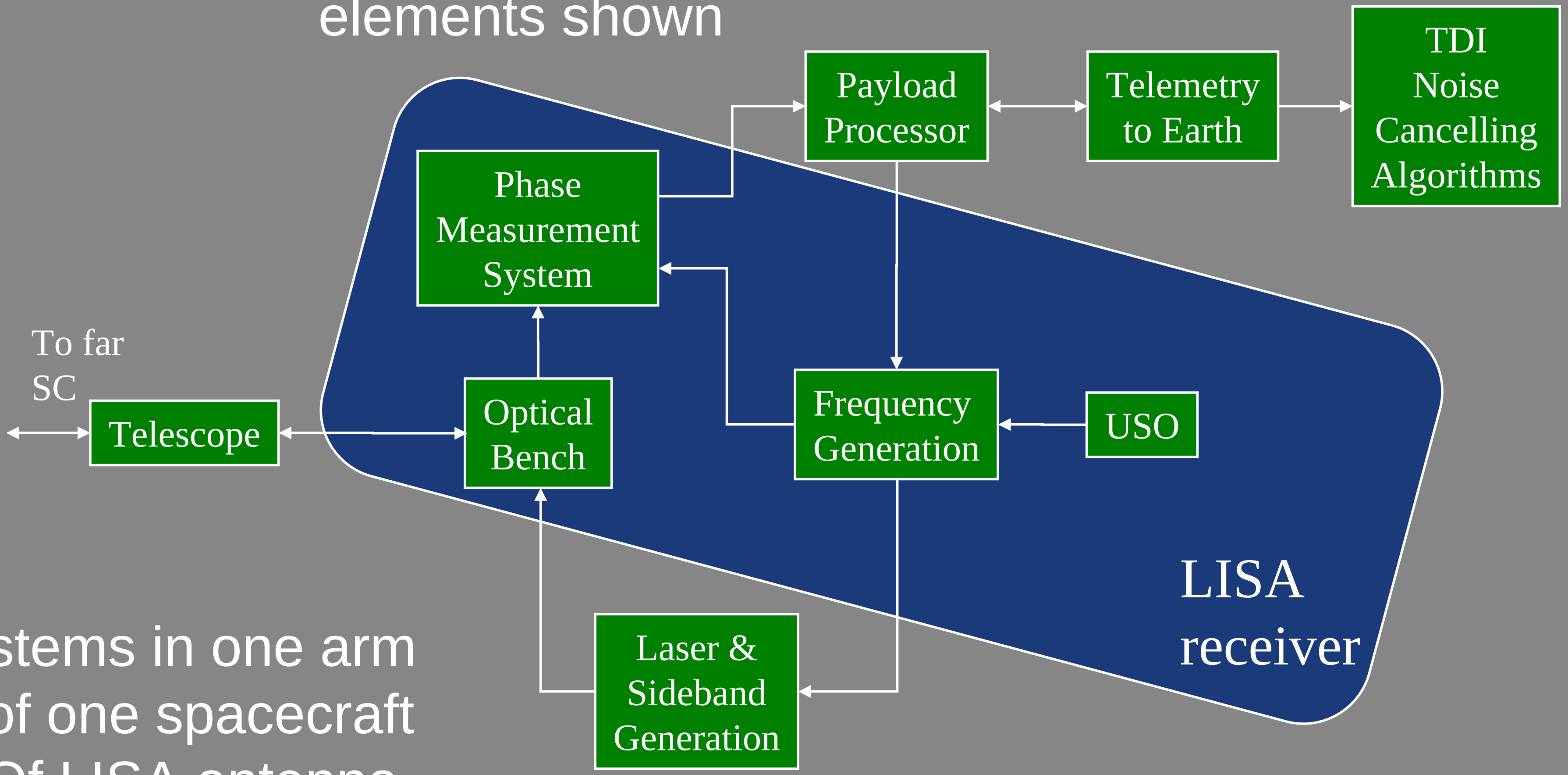
S·E·A

Structure of the LISA PMS Project



LISA receiver

In order to refine the PMS the Simulation covers all elements shown



Systems in one arm of one spacecraft Of LISA antenna

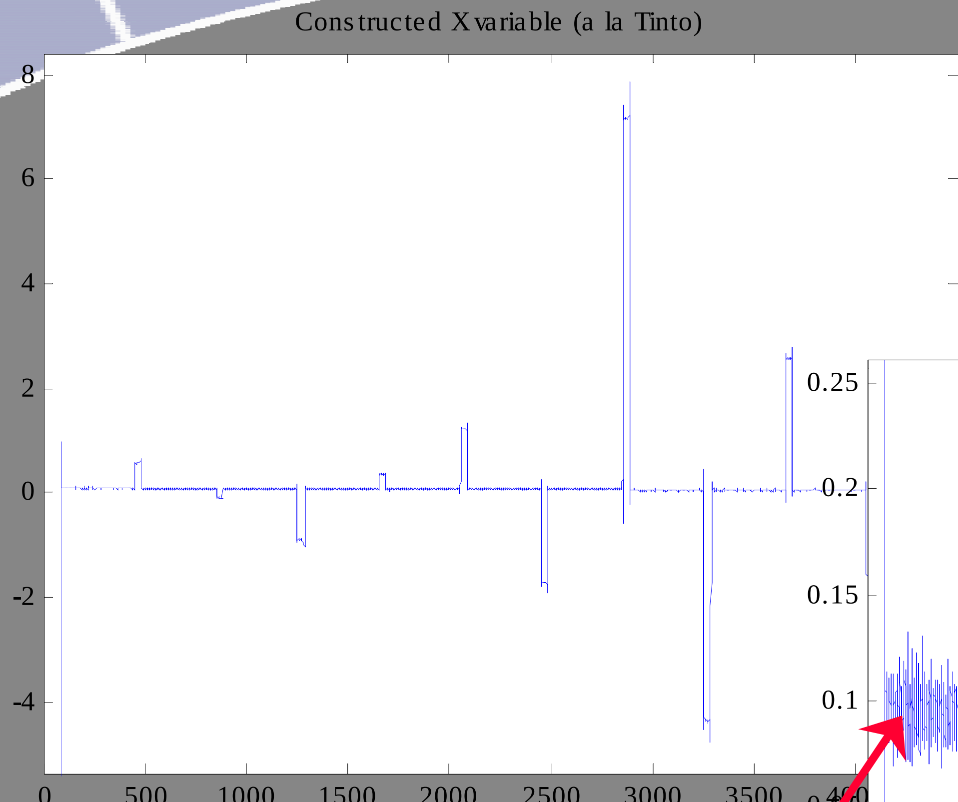
LISA PMS Simulation

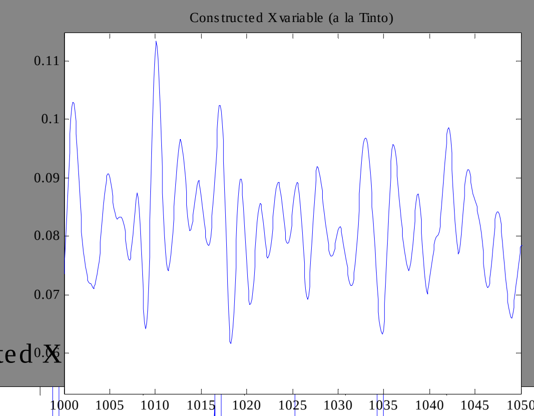
- End-to-End Simulation
- Covers:
  - 3 Spacecraft
  - Spacecraft AOCS and motion
  - Gravitational Waves
  - Lasers
  - USOs
  - Phase Measurement System
    - Analogue electronics and noise sources
    - Digital electronics and algorithms
  - Frequency Generation
  - Decimation
  - Telescope pointing error
  - Time Delay Interferometry version 1 X variable
- Used to refine the requirements for the Phase Measurement System
- 1.5 man years development time
- Developed under ECSS E40B

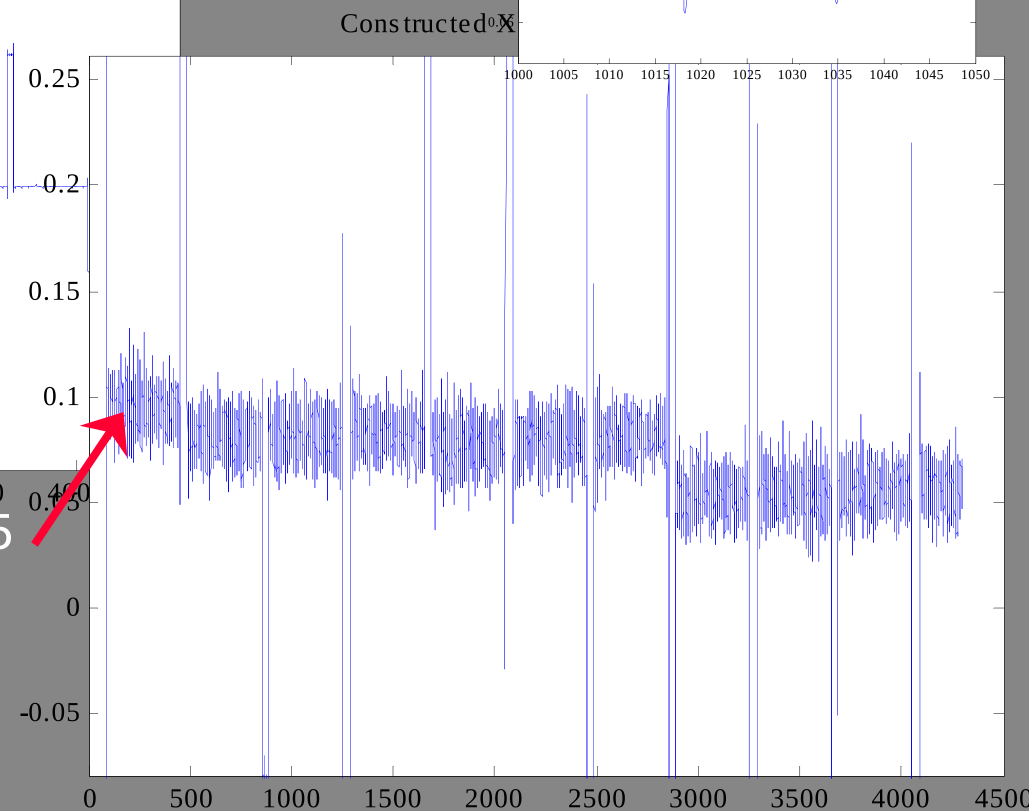
Primary Conclusions

1. Single Sideband vs. Dual Sideband:
   Recommendation: Single preferred.
2. Phase Measurement Technique:
   Recommendation: Digitisation + DFT/FFT
3. Synchronization Subsystem: (Discussed later)
   New Subsystem required by TDI
4. Multiple Digitization components to the PMS:
   Required by TDI
5. Broadband Noise Sources:
   Thermal Electronic noise comparable to shot noise
6. Phase Noise Cancellation: (Discussed later)
   Requirement: I and Q phase measurement

Noise Cancellation



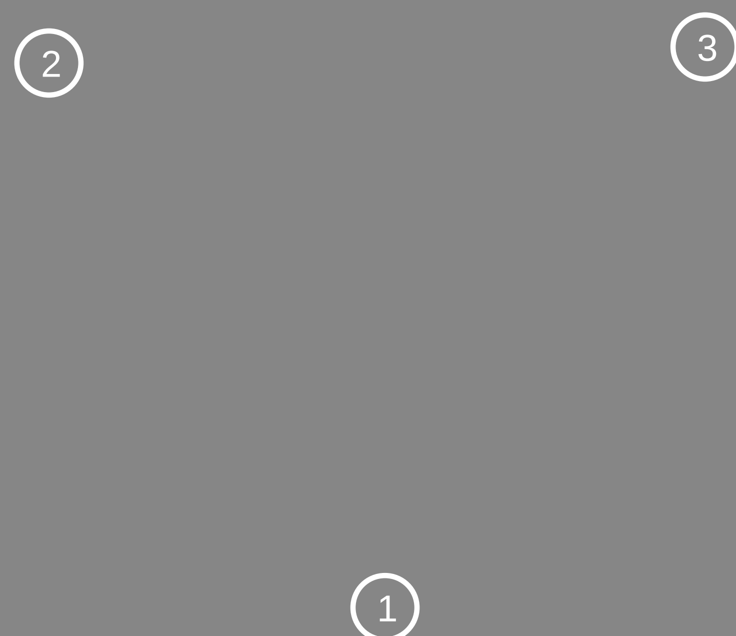




- Noise approximately 10^5 times to large!

TDI1: The X variable

$$X = s_{32,322} - s_{23,233} + s_{31,22} - s_{21,33} + s_{23,2} - s_{32,3} + s_{21} - s_{31}$$

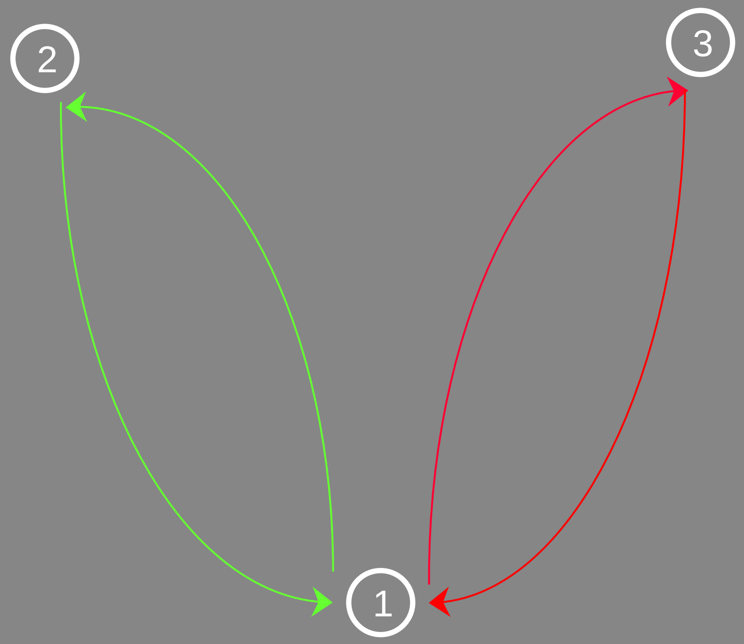


Main heterodyne
measurements
(s variables) only

Three spacecraft
unequal arm lengths

TDI1 X

$$X = s_{32,322} - s_{23,233} + s_{31,22} - s_{21,33} + s_{23,2} - s_{32,3} + s_{21} - s_{31}$$

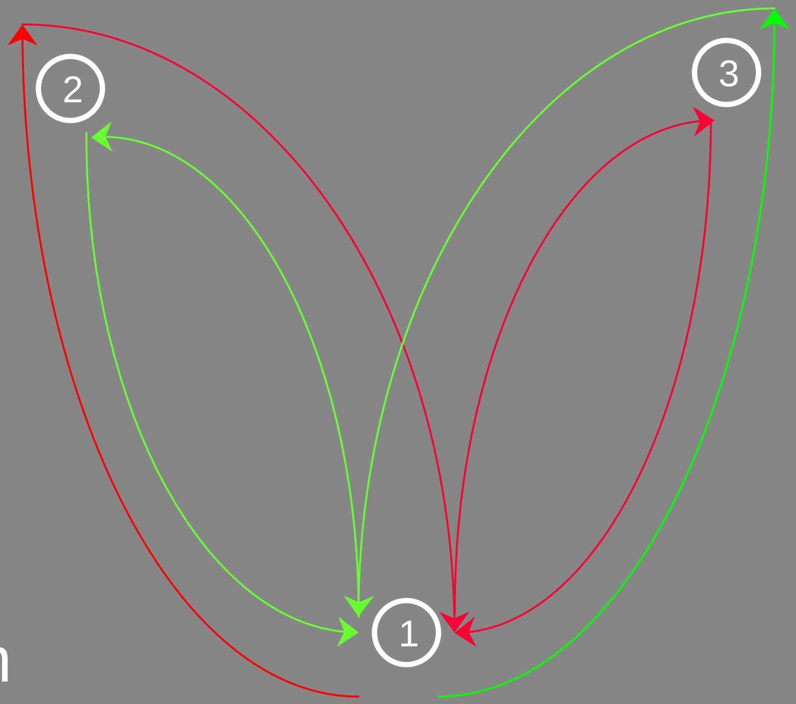


If SC1-2 = SC1-3 could form an equal arm interferometer at this stage

TDI1 X

$$X = S_{32,322} - S_{23,233} + S_{31,22} - S_{21,33} + S_{23,2} - S_{32,3} + S_{21} - S_{31}$$

Extended equal arm interferometer

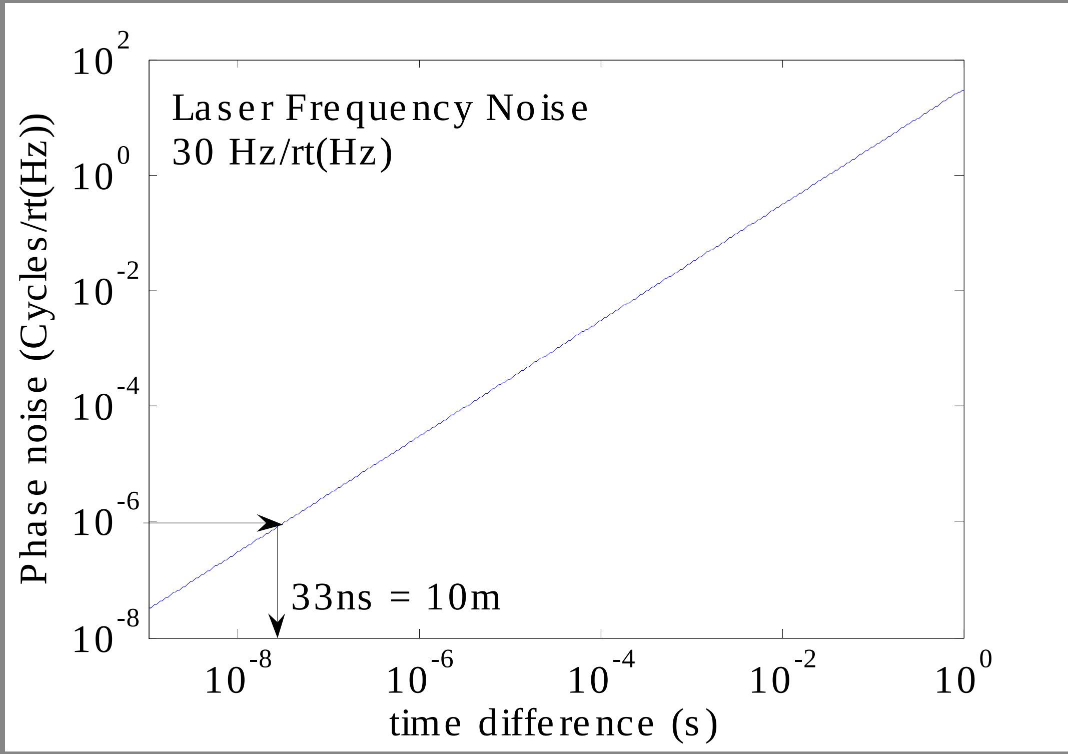


Rabbit Eared Diagram

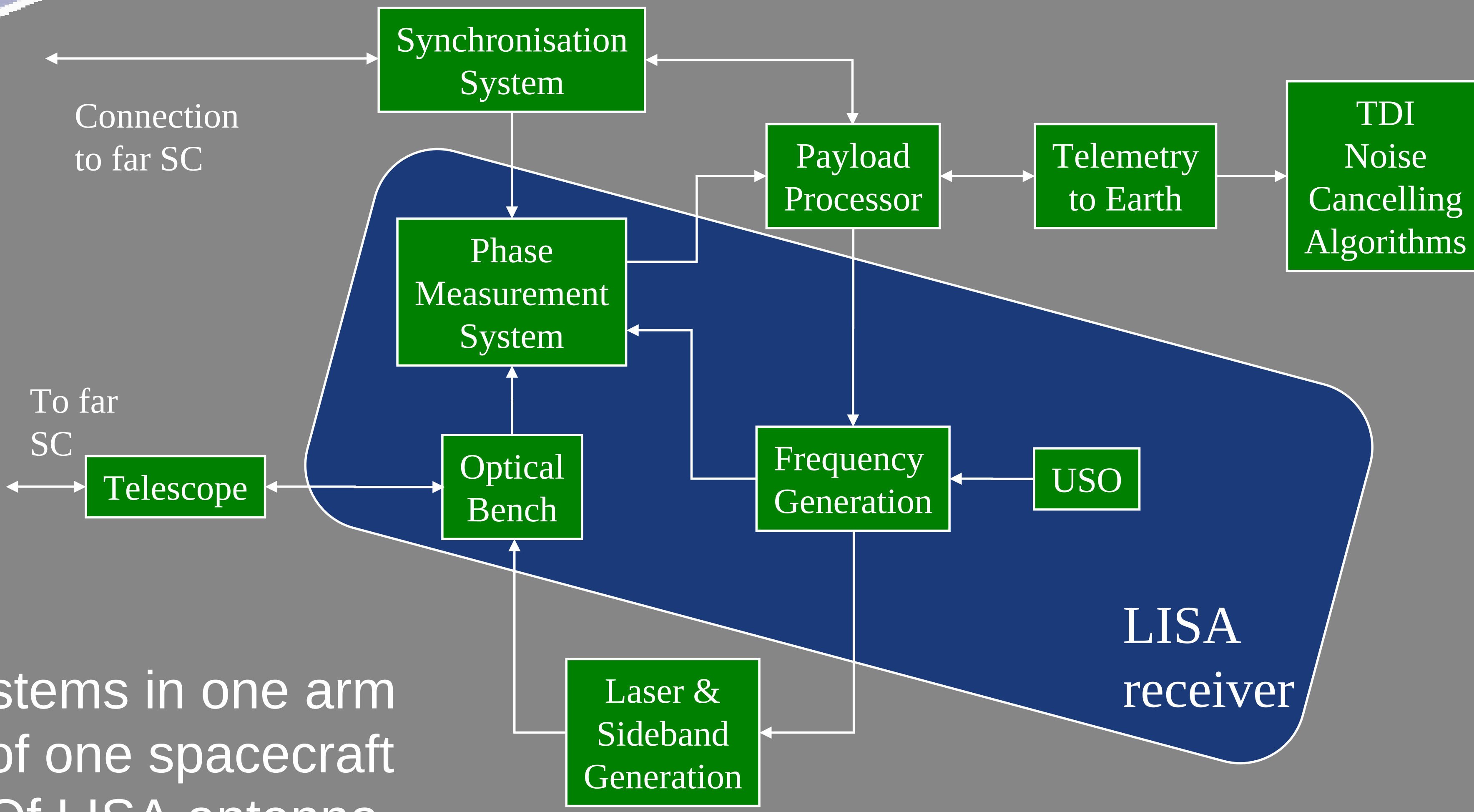
Green Path Length = Red Path Length ⇒ Phase Noise Cancels

Timing Precision

- How different can the path length/timing requirements be?
- Dictated by laser noise (coherence length)
- Flat laser frequency noise, gives flat phase noise in TDI when arm length/time differ by T (for f«1/T)
- Timing requirement also applies to measurement of phase at the spacecraft
- Requires a Synchronisation System to tie the SC together to time resolution



LISA receiver



Systems in one arm
of one spacecraft
Of LISA antenna

Synchronisation
System Requirements

Either

- Absolutely Synchronise spacecraft clocks to ~50ns
- Measure the arm lengths to ~10m

Or

- Synchronise phase measurements along null vectors to ~50ns

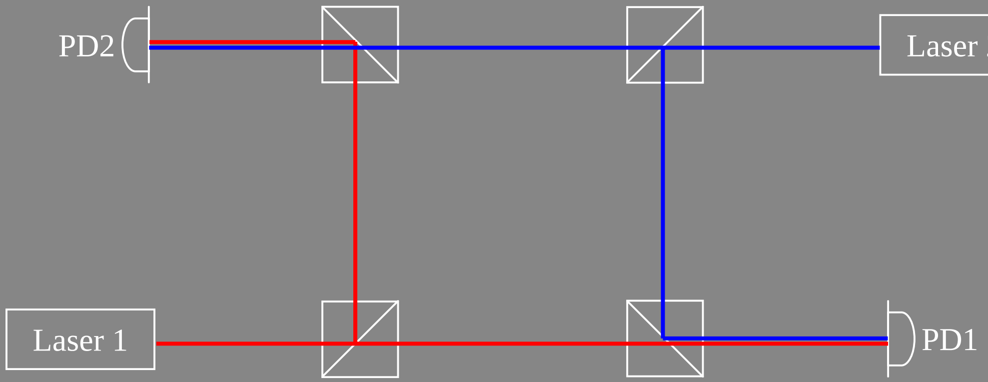
Plus

- Knowledge of which phase measurements are required
- Require multiple PMSs, one for each synchronised phase measurement

Phase Measurement and Phase Noise

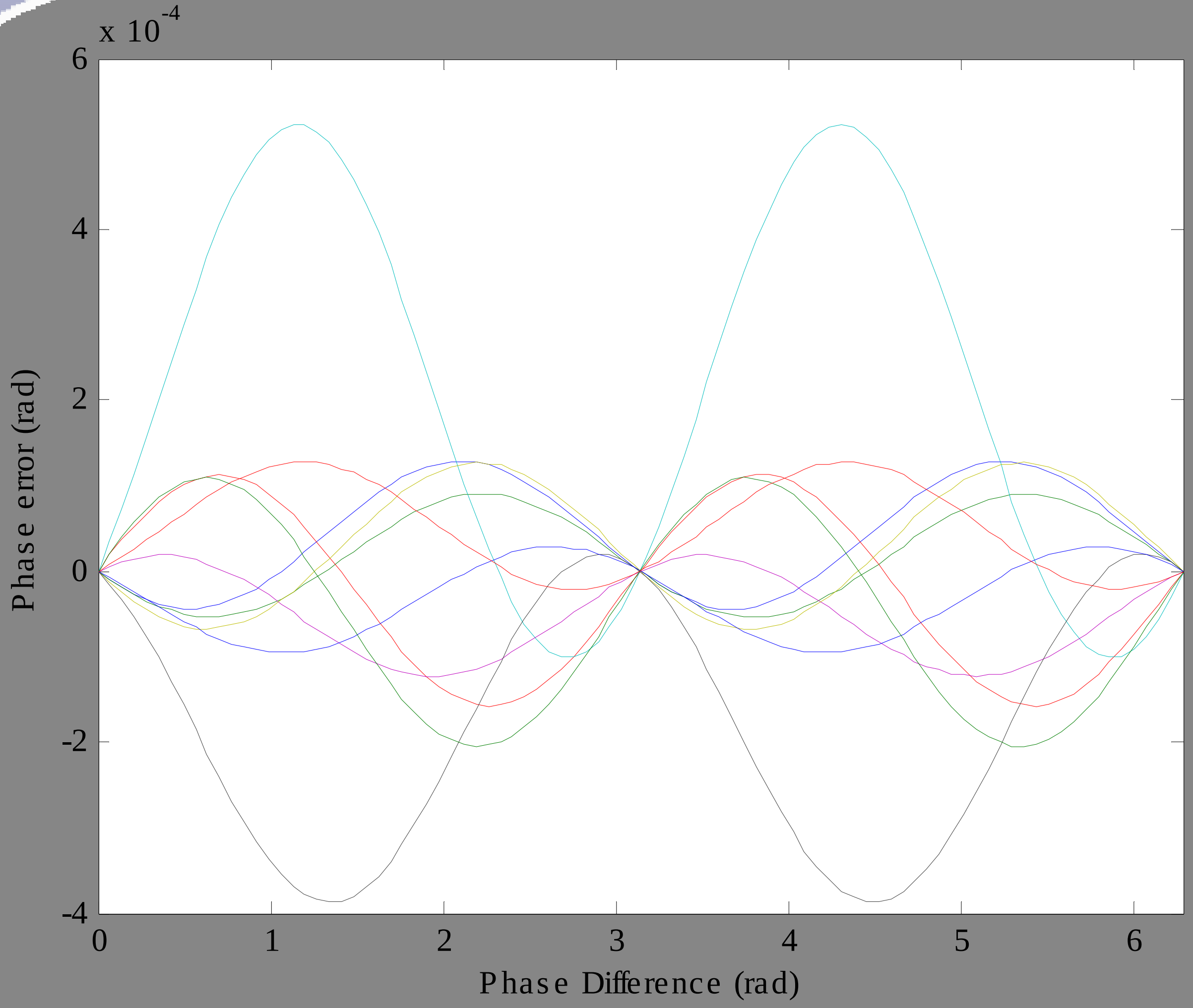
Or how do we measure a noisy heterodyne signal?

Gedankenexperiment:
Simpler than LISA cancellations, but require the same phase noise cancellations



- Stable Optical Paths
- Lasers have frequency noise
  - 1rad/s/√Hz for f<100Hz
  - 1/f for f>100Hz
- PD1 and PD2 heterodynes have noise due to laser noise
- PD1-PD2 heterodyne should be noise free as common mode noise cancels
- Phase measured by FFT. Heterodyne centred in bin
- 8192 point FFT phase measurement. Sample rate 81.92kHz
- Simulates the cancellation of laser noise needed by LISA in TDI where cancellations happen at ends of arms

Gedankenexperiment



- Each curve shows phase error in PD1-PD2 as a function of the phase difference between PD1 and PD2 for a particular set of laser phase noise
- Different curves are for different runs of the phase noise
- No phase error when $\Delta\varphi = 0$, π , 2π etc
- For all other $\Delta\varphi$ the phase errors are larger than is permissible for LISA

Phase Noise Cancellation Requirements

1. Pure heterodyne with phase :

$$\left\langle \hat{\phi} \right\rangle = \phi$$

2. Heterodyne with phase and phase noise:

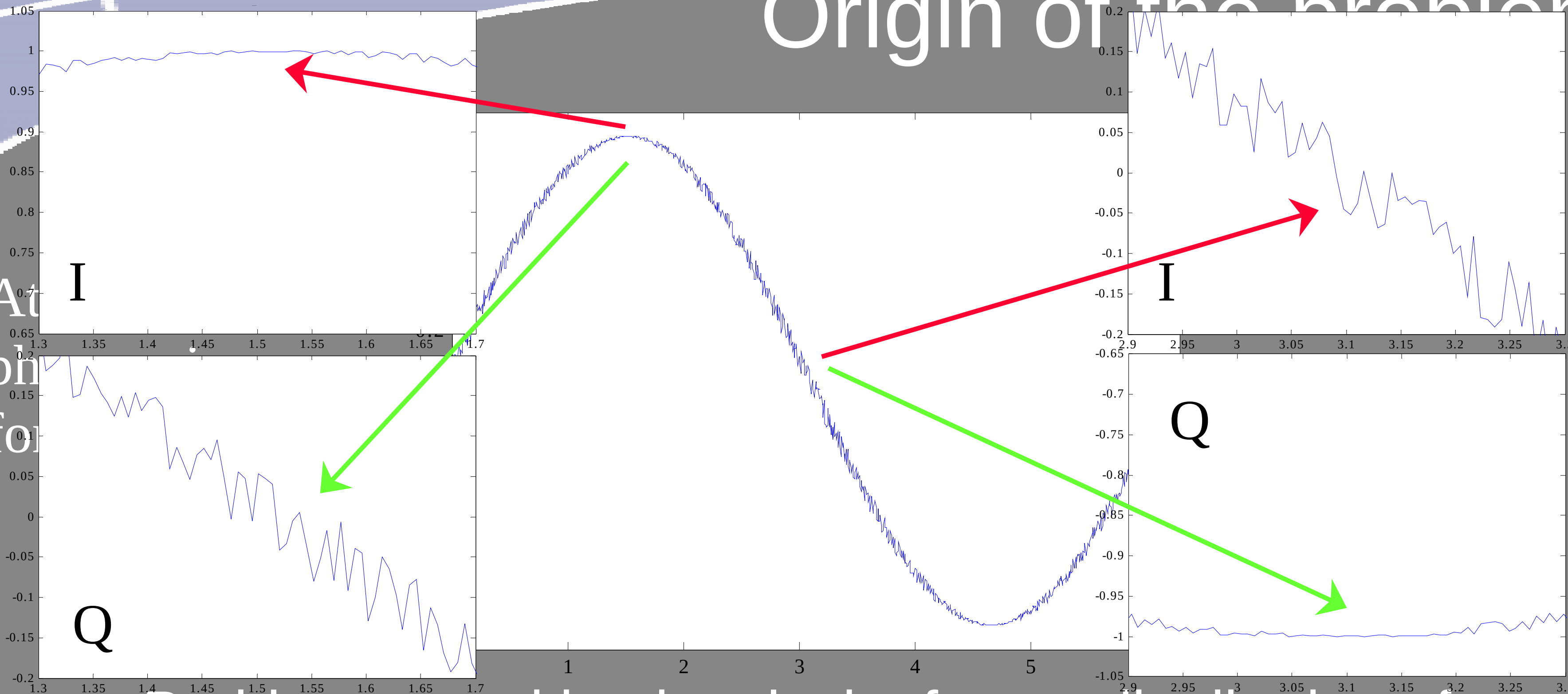
$$\left\langle \hat{\phi} \right\rangle = \phi + f\left(\text{phase noise}\right)$$

3. Heterodyne with phase and phase noise 1 ± phase noise 2:

$$\left\langle \hat{\phi} \right\rangle = \phi + f\left(\text{phase noise}\,1\right) \pm f\left(\text{phase noise}\,2\right)$$

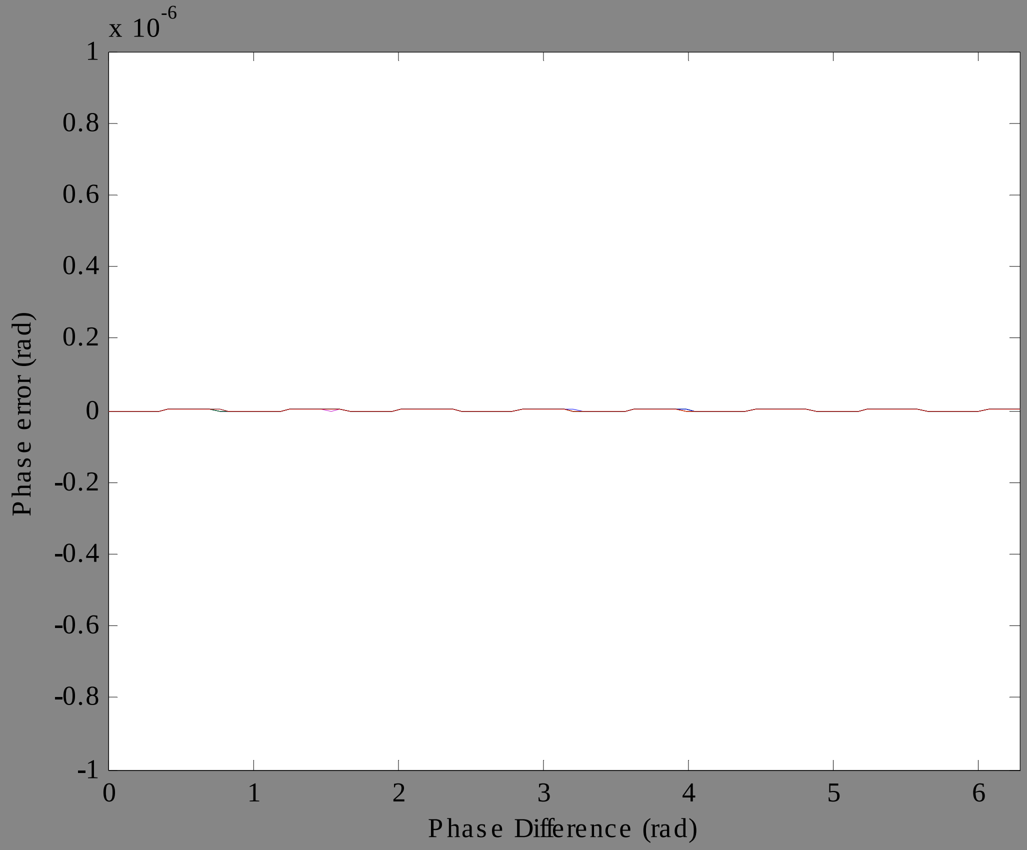
$\hat{\phi}$ is the measured phase, f has no dependence on ϕ

Origin of the problem



- Problem traced back to lack of even distribution of amplitude variations from phase noise on a sinusoid.
- Solution is to extract the I and Q components of the heterodyne signal
- Gives enhanced definition to phase

Gedankenexperiment I and Q results



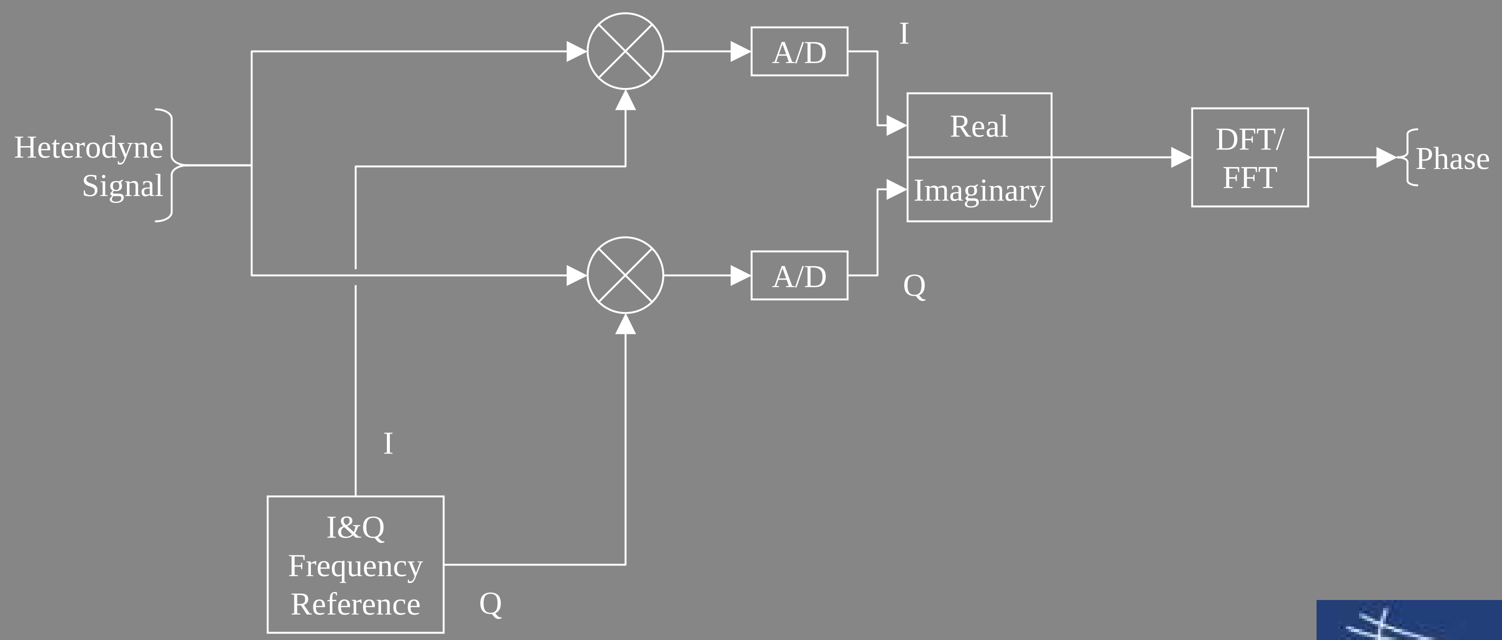
- Same gedanken-experiment as before
- Phase measured by I&Q DFT/FFT
- Real = I
  Imaginary = Q
- Phase Errors cancelled effectively

Math Results

- Analytical calculation of the effect of noise on a heterodyne (frequency *f)* shows:
  - In projection to the base band two frequencies of phase noise are present:
    - $f_N=0$
    - $f_N=2f$
  - The $f_N=0$ noise is cancelled in differences between phase measurements
  - The $f_N=2f$ noise is ***NOT*** cancelled in differences between phase measurements
  - With I and Q technique the f_N=2f noise is absent and hence differences between phase measurements cancel the phase noise

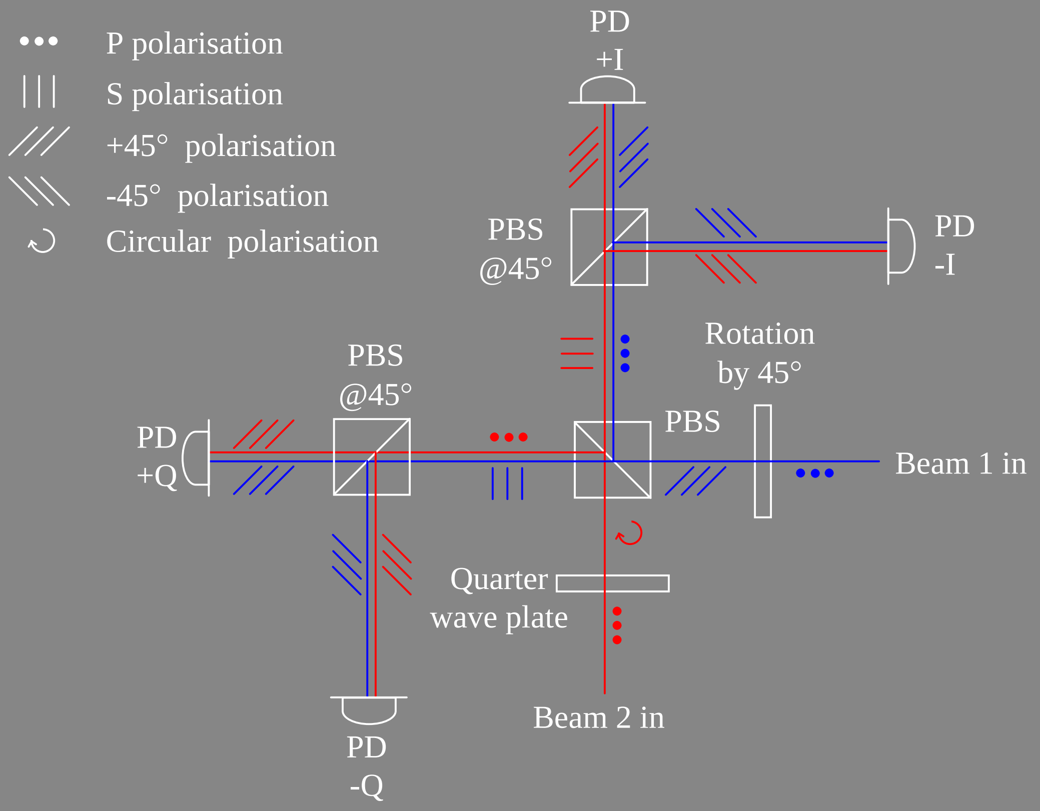
I and Q how: Electrical

- I and Q components can be generated in down conversion
- LISA PMS needs a down conversion
- Use this to generate I and Q



I and Q how: Optical

- Second down conversion in the interferometer from the optical (10^{14}Hz) to the heterodyne



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Secondary Conclusions

- Sign ambiguity in heterodyning methodology.
  - ⇒ When heterodyning can't tell if we are measuring
    - ϕ near laser - ϕ far laser
    - ϕ far laser - ϕ near laser
  - ⇒ Hence can't say if we need to sum, or difference phase measurements to cancel phase noise
  - ⇒ Either need to record this, or try all possible variations and see which minimises noise.
  - ⇒ What is the impact on TDI? Possible complications in USO noise cancellations?
- Relative Spacecraft Velocity constraint.
  - ⇒ Problem solved by JPL – more complex TDI2 variables
- Sideband-Carrier or Sideband-Sideband Heterodyne?
  - ⇒ Carrier-Sideband heterodyne as good as sideband-sideband heterodyne for cancelling USO noise
    - Higher frequency heterodyne – more challenging PMS
    - $10\times$ amplitude - lower shot noise contribution

Open Issues

- How to implement single sidebands?
  - Dual Lasers? AM +FM EOM?
- How is I and Q implemented?
  - Electrically? Optically?
- What are the full Synchronization System Requirements?
- How is the Synchronization System implemented?
  - RF between SC? Or modulation of the laser?