Parameter estimation

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5th LISA Symposium

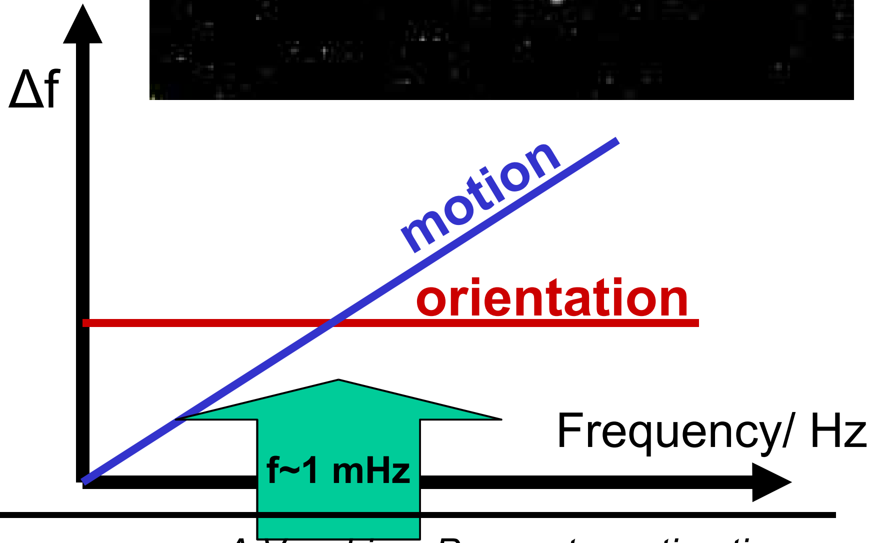
ESTEC, 12th – 15th July 2004

Outline

- Computing the expected minimum mean squared error:
  - Variance-covariance matrix
  - Assumptions
- Summary of results for:
  - (Quasi-) monochromatic signals
  - In-spiraling intermediate mass / massive / super-massive black hole binary systems
- What we have learned and future work

Astronomy with LISA

- Two key (and distinct) motions:
    1. LISA orbits the Sun: the signal frequency is Doppler shifted
    2. Spacecraft constellation rotates around the normal to the detector plane: the response of the detector is not fixed, that is the antenna pattern is time dependent
- The LISA motion is what provides the detector pointing capability
- Information on physical parameters are mainly (but not only) contained in the GW phase



Variance-covariance matrix

- Signal

$$s(t) = h(t; \boldsymbol{\lambda}) + n(t)$$

- Probability distribution of the errors

$$p(\Delta\boldsymbol{\lambda}) = \left(\frac{\det(\Gamma)}{2\pi}\right)^{1/2} e^{-\frac{1}{2}\Gamma_{jk}\Delta\lambda^j\Delta\lambda^k}$$

$$\Gamma_{jk}^{(i)} = \left(\frac{\partial h^{(i)}}{\partial\lambda^j}\middle|\frac{\partial h^{(i)}}{\partial\lambda^k}\right)$$

- Lower bound on the minimum mean squared errors (Cramer-Rao bound):

$$\langle(\Delta\lambda^i)^2\rangle = \Sigma^{jj}\,; \qquad c^{jk} = \frac{\Sigma^{jk}}{\sqrt{\Sigma^{jj}\,\Sigma^{kk}}} \qquad (-1 \le c^{jk} \le +1)$$

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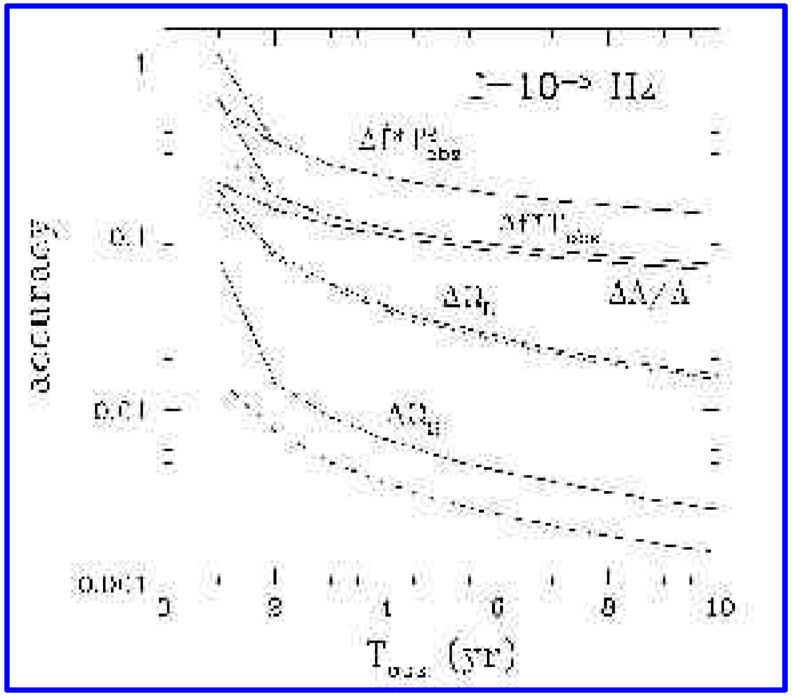
Assumptions

- Stationary and Gaussian noise
  - Instrumental noise non stationary/Gassian at some level
  - The "confusion noise" is intrinsically non stationary (Seto, 2004)
- High signal-to-noise ratio
  - Surely valid for several signals
  - … but not for all of them
- Only one source at the time in the data stream
  - De-facto correct for high SNR rare events, such as (super-) massive black hole binaries
  - Just "wrong" for all the other signals in LISA

(Quasi-)monochromatic signals

- Monochromatic signal:
  - 7 parameters: A, f, ϕ_0, $(\theta_N\ \phi_N)$, $(\theta_L\ ,\phi_L)$
  - If linearly chirping, one more parameter (df/dt)

f_{GW}	$\bar{\mu}_S$	$\bar{\phi}_S$	$\bar{\mu}_L$	$\bar{\phi}_L$	S/N	$\Delta\Omega_{S,L}$	$\Delta\Omega_S$
10^{-4}	0.3	5.0	-0.2	4.0	7.07	1.89 × 10 [illegible]	7.79×10^{-2}
10^{-4}	0.3	5.0	0.2	0.0	7.19	1.87 × 10 [illegible]	7.11×10^{-2}
10^{-4}	0.3	1.0	0.2	4.0	6.89	1.17 × 10 [illegible]	7.10×10^{-1}
10^{-4}	-0.3	1.0	0.8	0.0	6.80	1.26 × 10 [illegible]	7.10×10^{-2}
3×10^{-4}	0.3	5.0	-0.2	4.0	7.07	1.47 × 10 [illegible]	6.41×10^{-3}
3×10^{-4}	0.3	5.0	0.2	0.0	7.19	1.41 × 10 [illegible]	6.15×10^{-2}
3×10^{-4}	0.3	1.0	0.2	4.0	6.89	1.04 × 10 [illegible]	6.20×10^{-2}
3×10^{-4}	-0.3	1.0	0.8	0.0	6.80	1.17 × 10 [illegible]	6.28×10^{-2}
10^{-3}	0.3	5.0	-0.2	4.0	7.07	6.15×10^{-2}	2.91×10^{-3}
10^{-3}	0.3	5.0	0.2	0.0	7.19	6.04×10^{-2}	2.87×10^{-4}
10^{-3}	-0.3	1.0	-0.2	4.0	6.89	6.02×10^{-2}	3.17×10^{-1}
10^{-3}	-0.3	1.0	0.8	0.0	6.80	6.85×10^{-2}	3.10×10^{-2}
3×10^{-3}	0.3	5.0	-0.2	4.0	7.07	1.50×10^{-2}	7.23×10^{-3}
3×10^{-3}	0.3	5.0	0.2	0.0	7.19	1.58×10^{-2}	7.41×10^{-4}
3×10^{-3}	-0.3	1.0	-0.2	4.0	6.89	1.71×10^{-2}	7.60×10^{-5}
3×10^{-3}	-0.3	1.0	0.8	0.0	6.80	1.75×10^{-2}	7.04×10^{-5}
10^{-2}	0.3	5.0	-0.2	4.0	7.07	1.93×10^{-3}	9.05×10^{-1}
10^{-2}	0.3	5.0	0.2	0.0	7.19	2.16×10^{-3}	9.55×10^{-1}
10^{-2}	-0.3	1.0	-0.2	4.0	6.89	1.97×10^{-3}	7.98×10^{-1}
10^{-2}	-0.3	1.0	0.8	0.0	6.80	1.95×10^{-3}	7.60×10^{-1}



(Cutler, 1998; Takahashi and Seto, 2002)

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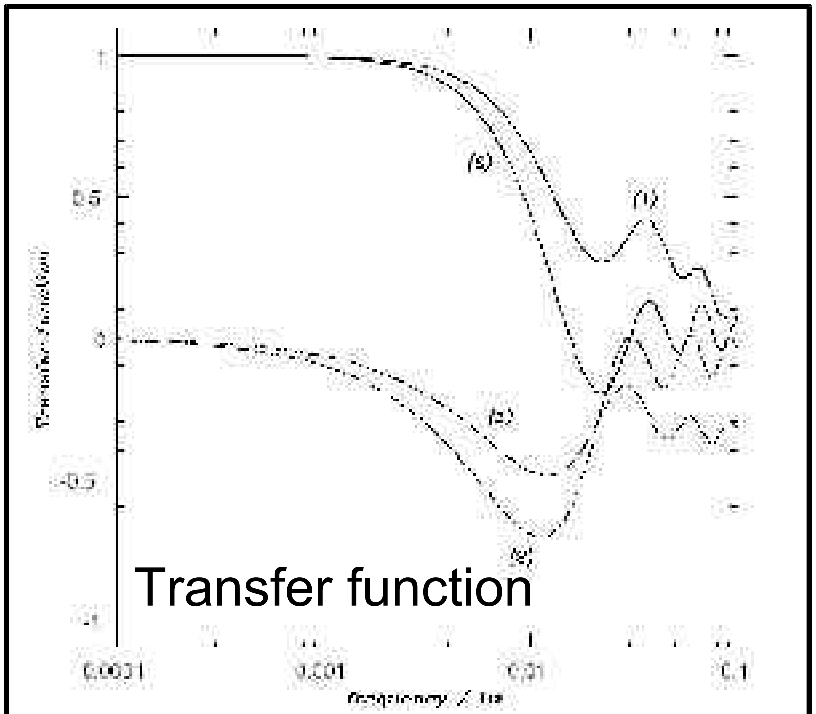
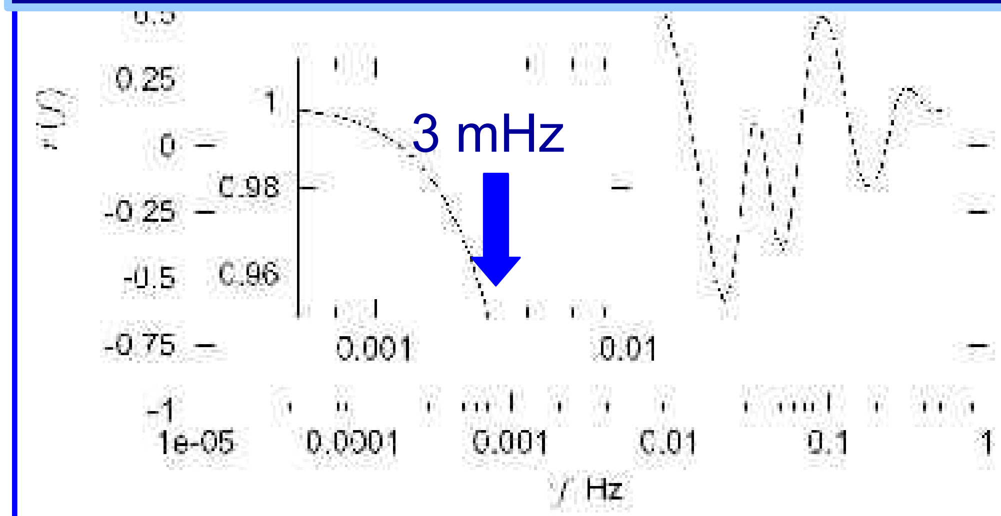
Finite arm length

Finite arm length (L ~ 16 sec) induce amplitude and phase modulations that depend on source frequency and position



However for **detection** the long wavelength approximation is fine up to 10 mHz: **FF > 0.97**



(Rubbo et al, 2004)

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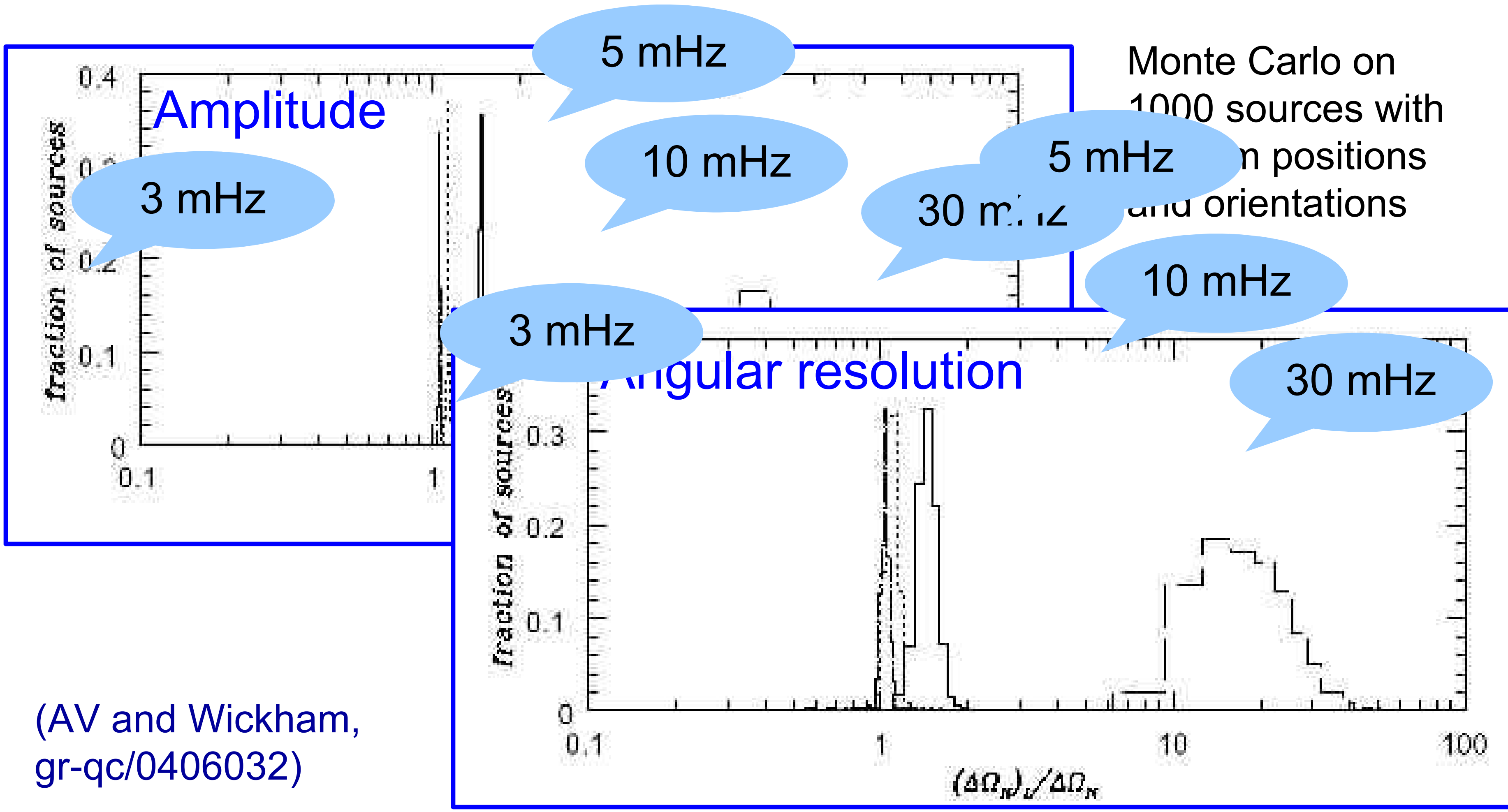
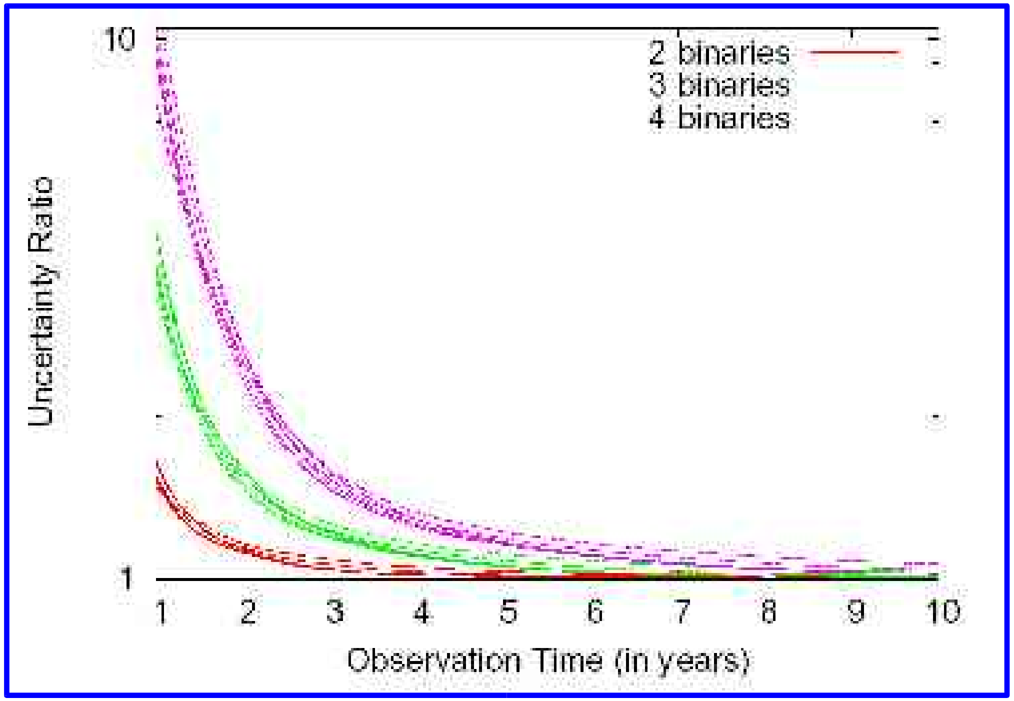
Impact of finite arm length



Monte Carlo on 1000 sources with [illegible] positions and orientations

(AV and Wickham, gr-qc/0406032)

Multiple sources



(Crowder and Cornish, gr-qc/0404129)

- If just 4 binaries are present in the data set, for 1 year of observation the errors increase by a factor~ 10 (with respect to the 1 source case)
- However long(er) integration time helps considerably:
  - **T > 3 years**

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Massive black hole binaries

- In-spiral signal (last year of coalescence):
  - 11 parameters (2PN): m_1, m_2 β σ, D, (θ_N ϕ_N), (θ_L ,ϕ_L), t_0, ϕ_0

z = 1

$m_1(=m_2)$	ρ_{insp}	T_{insp}	ρ_{ring}	$\delta D/D$	$\delta z_1/z$	$\delta z_2/z$	$\delta\mathcal{M}_z/\mathcal{M}_z$	$\delta\mu_z/\mu_z$
$10^3 M_\odot$	20	575 days	10^{-3}	0.08	0.15	0.05	5×10^{-5}	0.05
$10^4 M_\odot$	150	550 days	0.25	0.05	0.15	0.04	5×10^{-5}	0.03
$10^5 M_\odot$	1000	430 days	60	0.02	0.15	0.02	1×10^{-4}	0.04
$10^6 M_\odot$	200	15 days	3500	0.2	0.2	0.2	5×10^{-3}	0.5
$10^7 M_\odot$	40	100 minutes	612	70	70	70	150	300

- Restricted post-Newtonian approximation
- Pha
- No sp nduced p

10% error in cosmology

Cosmology known exactly

(Hughes, 2002)

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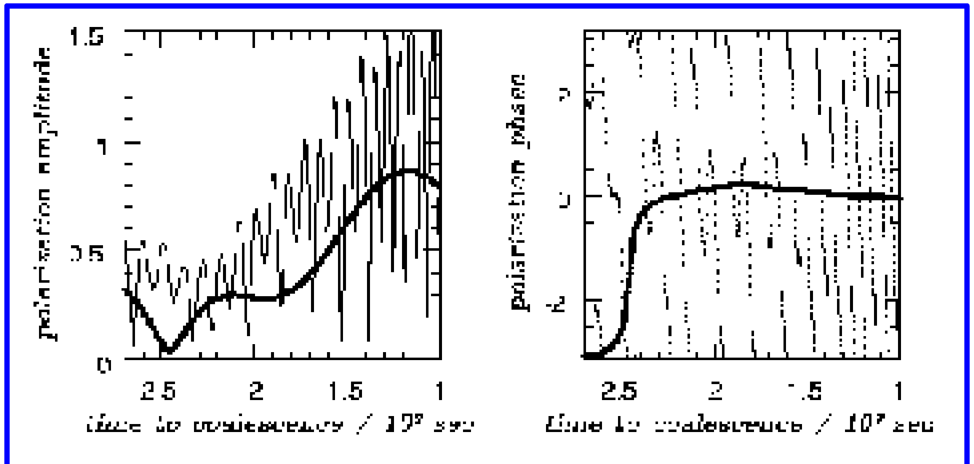
(Main) effect of spins: precession of the orbital plane

$m_1 = 10^7\ M_{sun}$

$m_2 = 10^5\ M_{sun}$

SdotL = 0.5

$S/m^2 = 0.95$



$m_1 = 10^6$ M

Number of precession cycles in the LISA band

$$\mathcal{N}_p \approx \begin{cases} 10\left(1+\frac{3\,m_2}{4\,m_1}\right)\left(\frac{m}{10^6\,M_\odot}\right)^{-1}\left(\frac{f_a}{10^{-4}\,\mathrm{Hz}}\right)^{-1} & (L \gg S) \\ 2\left(1+\frac{3\,m_2}{4\,m_1}\right)\left(\frac{m_1}{m_2}\right)\left(\frac{S}{m_1^2}\right)\left(\frac{m}{10^6\,M_\odot}\right)^{-2/3}\left(\frac{f_a}{10^{-4}\,\mathrm{Hz}}\right)^{-2/3} & (L \ll S) \end{cases}$$

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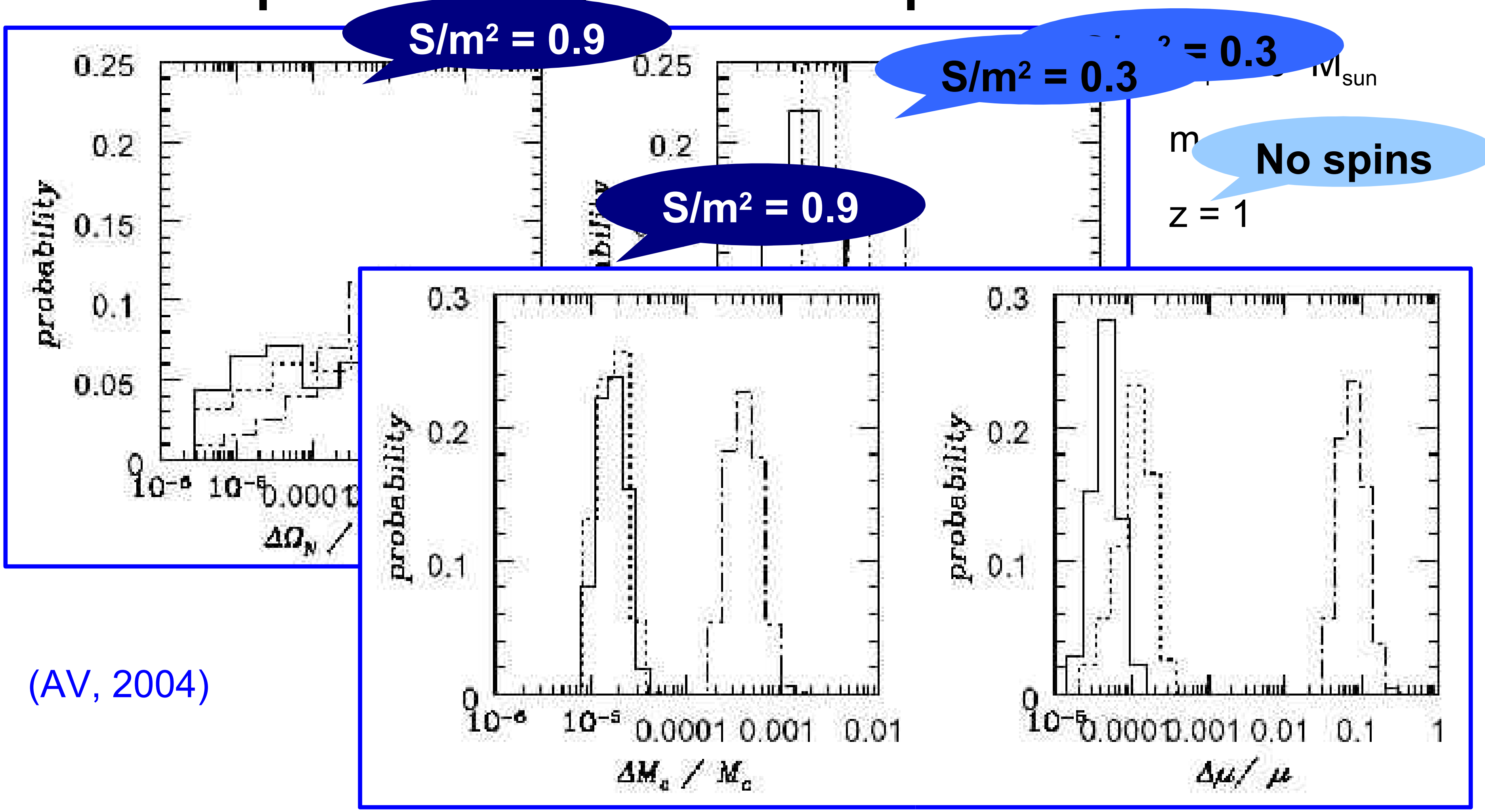
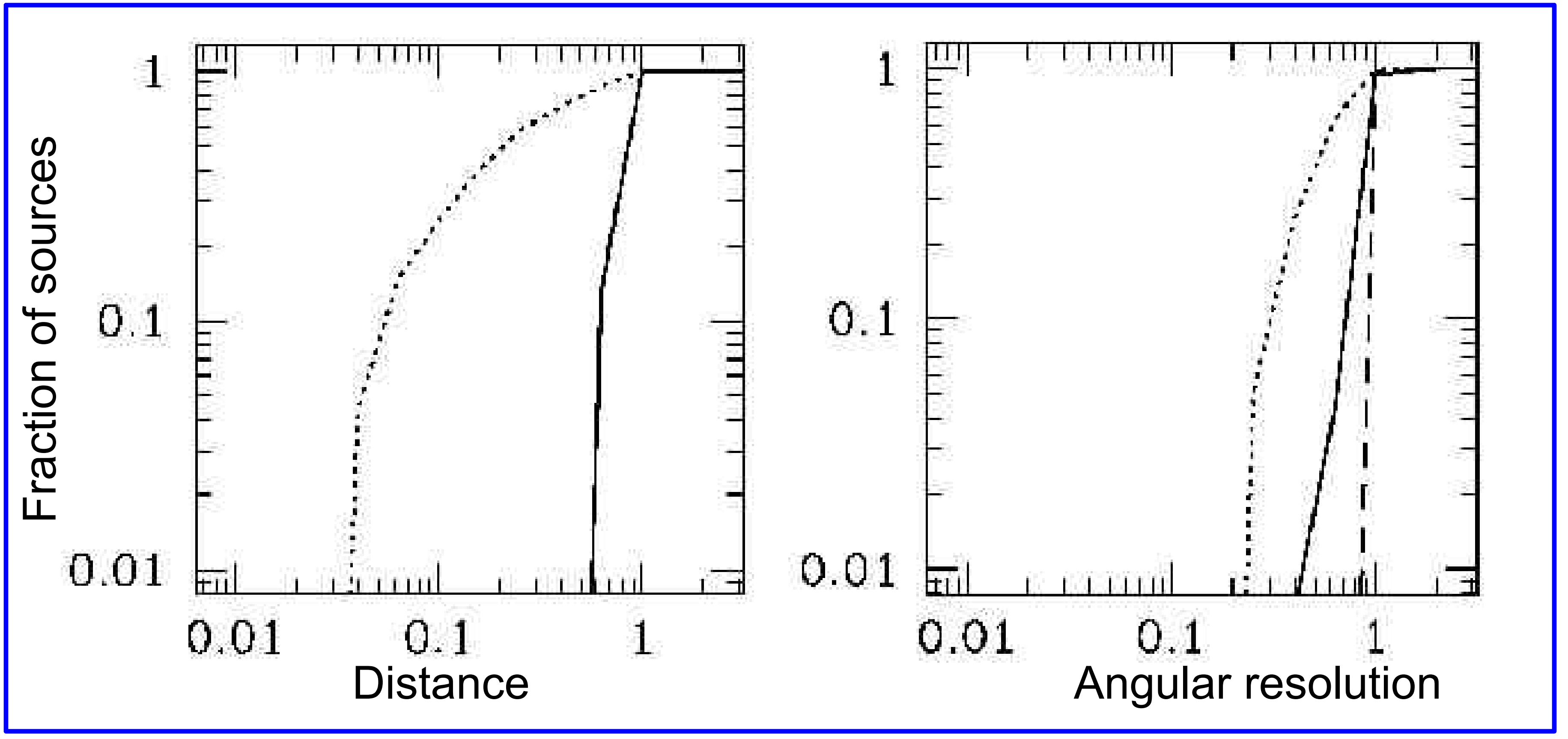
Impact of spin-orbit precession



(AV, 2004)

Impact of finite arm-length



(Seto, 2002)

Other effects

- **Use <u>full</u> post-Newtonian approximation (include the other harmonics):**
  - Additional information
  - Errors are reduced by a factor ~ 3 (Moore and Hellings, 2002; Sintes and AV, 2000)

$$h_{+,\times}(t) = \Re\left\{\sum_k H_{+,\times}^{(k)}(t)\, e^{i k \phi_{orb}(t)}\right\}$$

- **Weak lensing:**
  - Measure of the luminosity distance is affected by the magnification μ
  - Introduce systematic error (Holz and Hughes, astro-ph/ 0212218)

$$\Delta D_L / D_L = 1 - 1/\sqrt{\mu}$$

- **Strong lensing:**
  - Multiple images help in reconstructing the parameters
  - Improve significantly parameter extraction, but likely very rare (Seto, 2004)

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Impact of low frequency for high mass/redshift binaries

z = 1

$m_1(=m_2)$	ρ_{insp}	T_{insp}	ρ_{ring}	$\delta D/D$	$\delta z_1/z$	$\delta z_2/z$	$\delta\mathcal{M}_z/\mathcal{M}_z$	$\delta\mu_z/\mu_z$
$10^3\,M_\odot$	20	575 days	10^{-3}	0.08	0.15	0.05	5×10^{-5}	0.05
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$10^5\,M_\odot$	1000	430 days	60	0.02	0.15	0.02	1×10^{-4}	0.04
$10^6\,M_\odot$	200	15 days	3500	0.2	0.2	0.2	5×10^{-3}	0.5
$10^7\,M_\odot$	40	100 minutes	612	70	70	70	150	300

(see also talks by Centrella and Phinney)

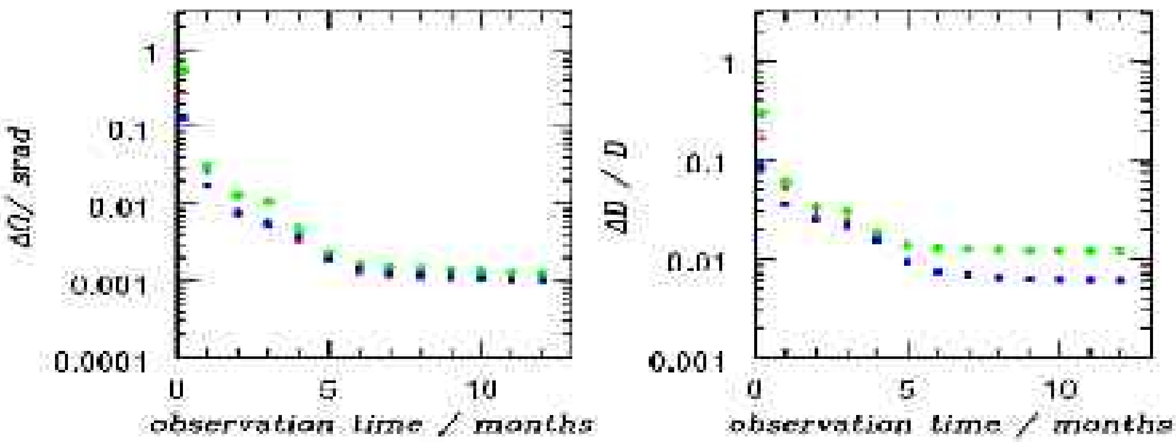
Low frequency cut-off at 0.1 mHz

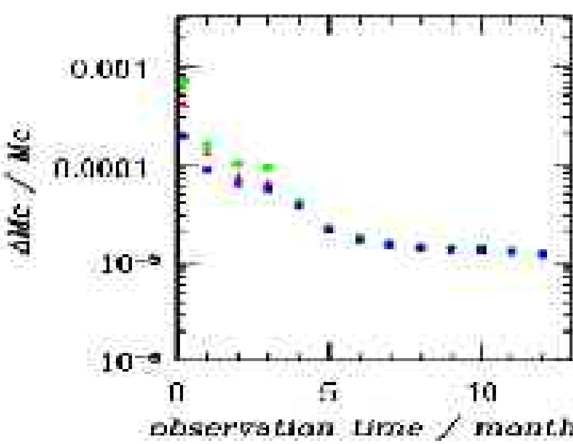
Information extraction vs observation time

$m_1 = 10^6\ M_{sun}$

$m_2 = 10^5\ M_{sun}$

$z = 1$





The signal needs to be visible for 6 months before coalescence in order to preserve information extraction

Conclusions

- Typical results:
  - WD binaries: angular resolution ~ 1-100 square degrees [not great but good enough for follow on observations in the mHz range]
  - Massive black hole binaries:
    - Physical parameters (masses and spins) measured with high accuracy (< 1%)
    - Distance: probably hard to do better than a few %
    - Angular resolution: in general several square degrees (but we might be lucky and detect an event with error box 5 arcsec x arcsec)
- We have a fairly good understanding of what are the key factors that affect information extraction in the case of one source
- We need to work on
  - Multiple sources
  - Mock data analysis to see what we can actually do in “battle conditions”
- LISA astronomy would definitely benefit from a <u>long</u> mission (T > 5 years)