Detection of extreme mass ratio inspirals with LISA - search strategy and detection rate estimates

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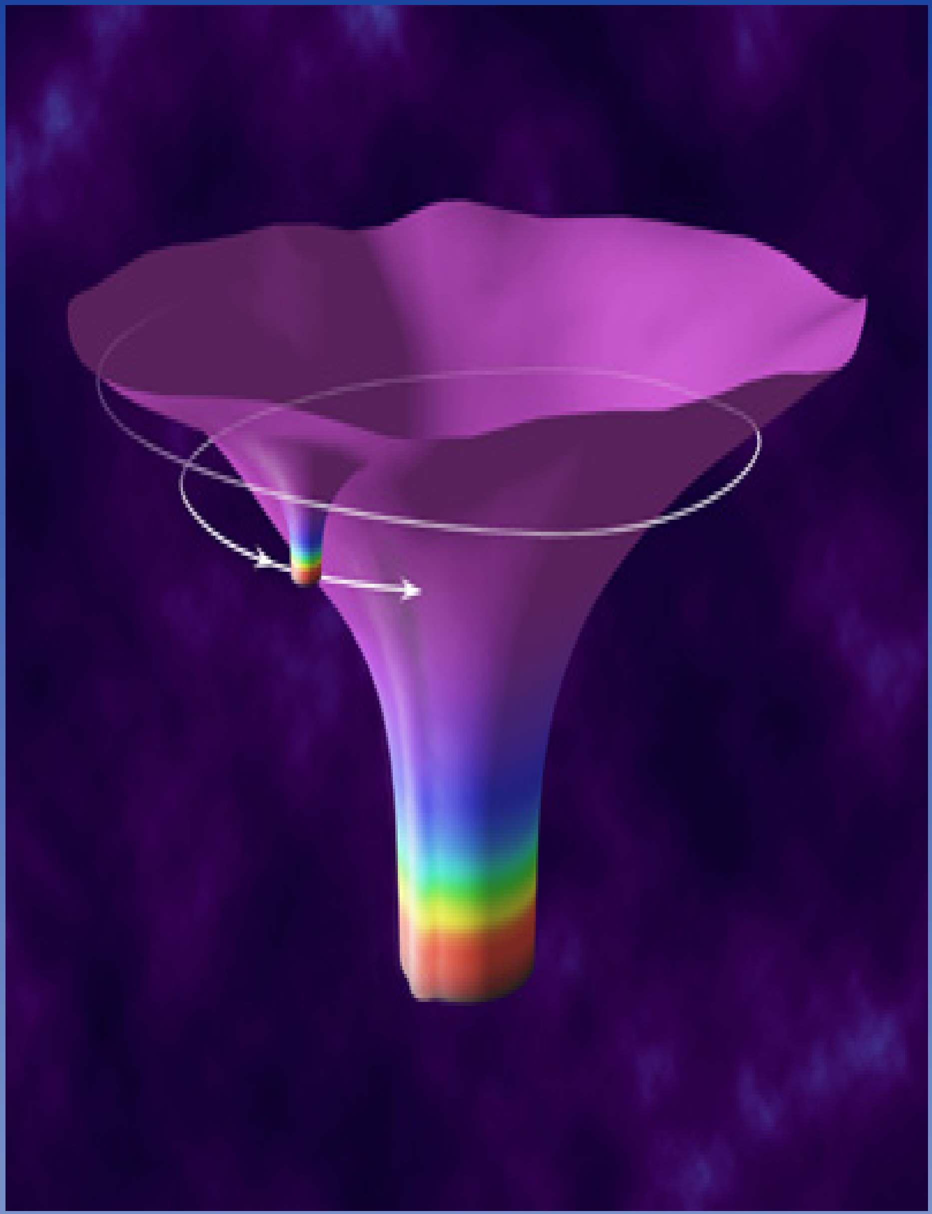

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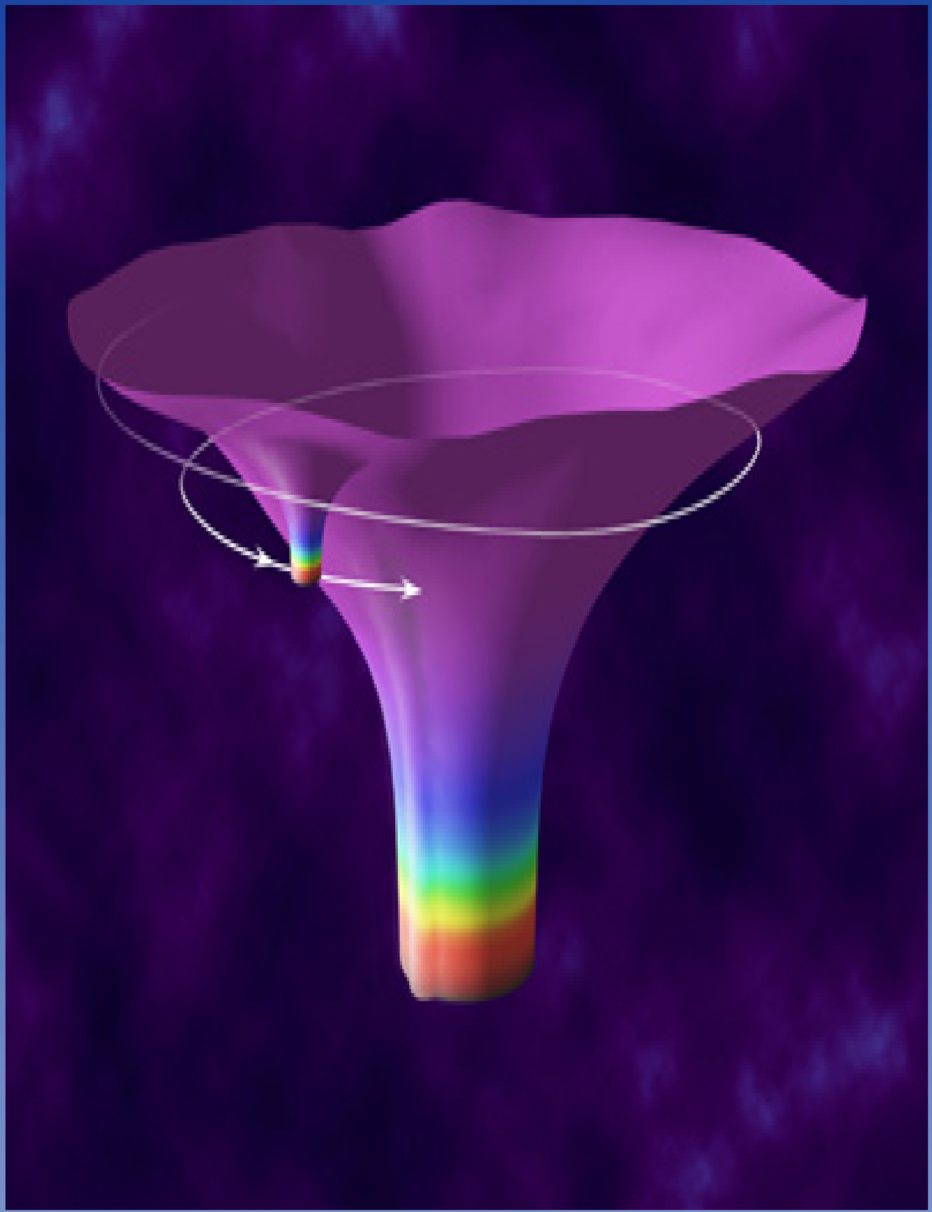

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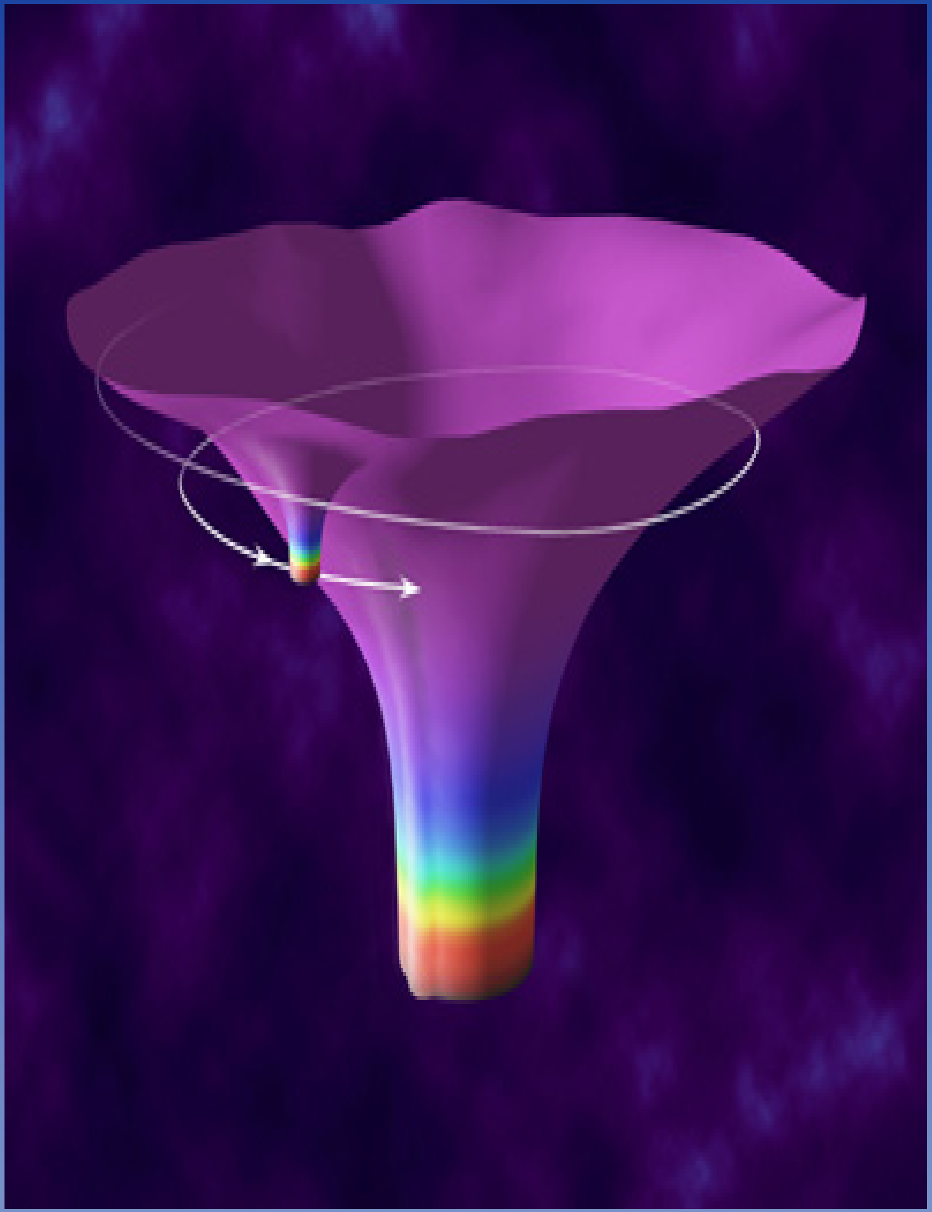

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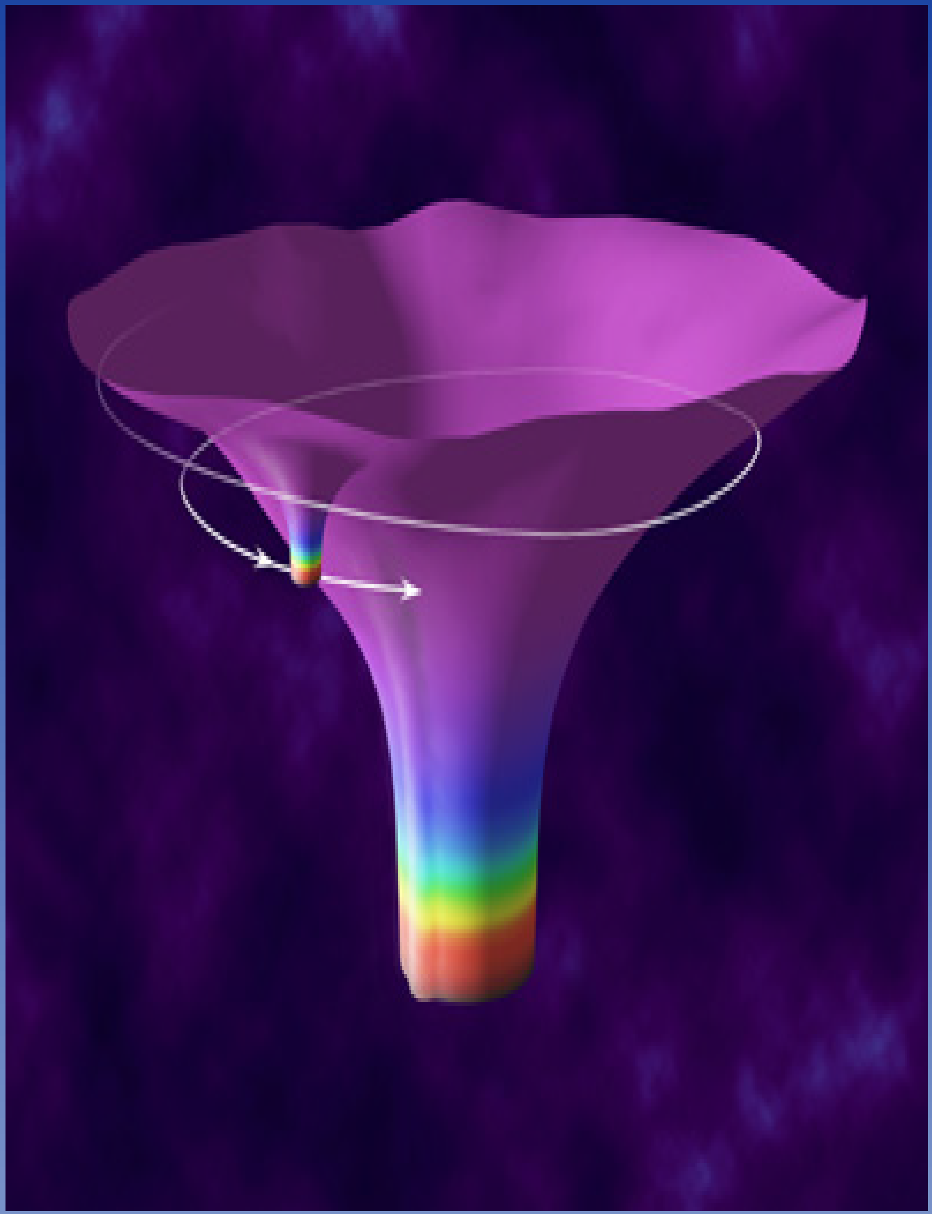

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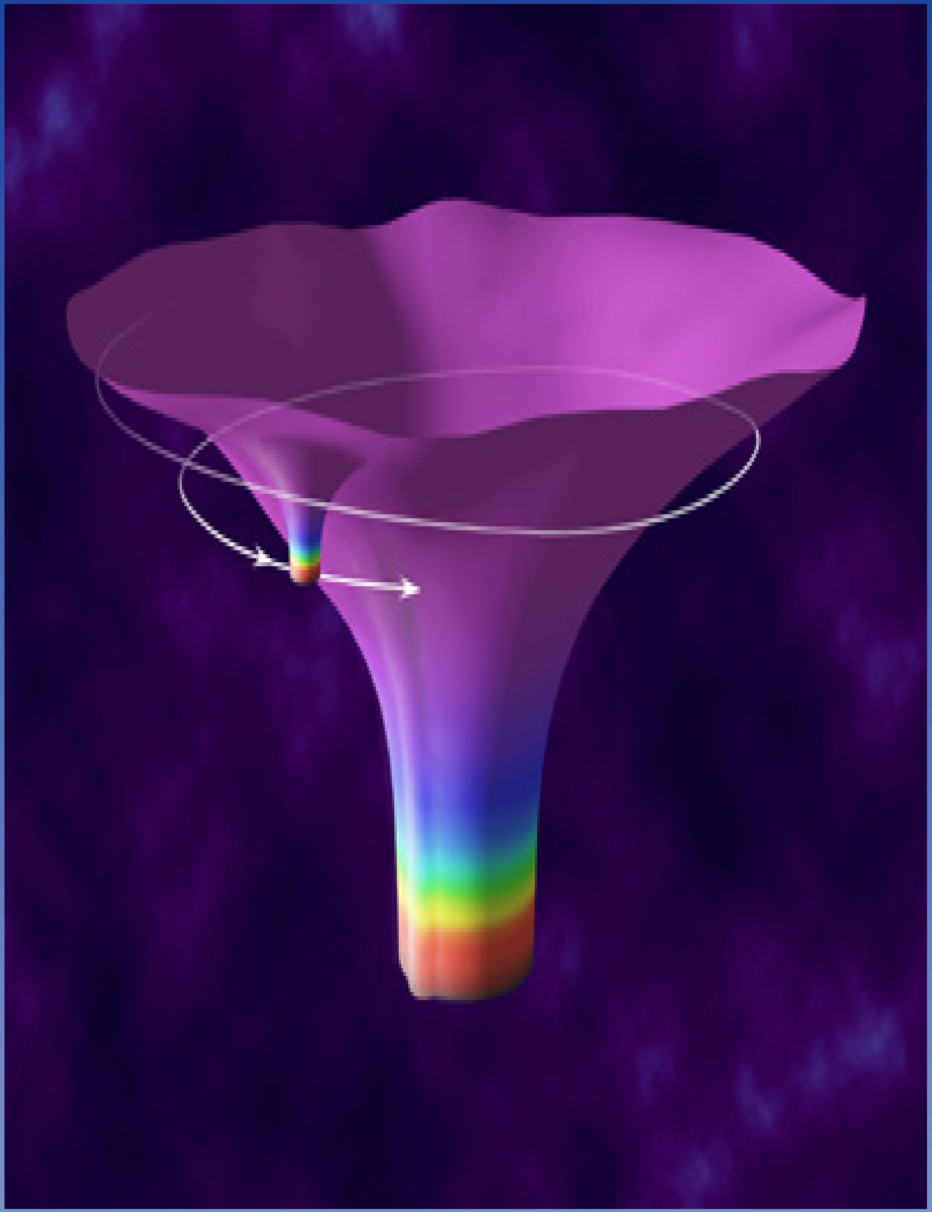
- Complicated gravitational waveforms provide a map of the spacetime geometry around spinning black holes.
- Desire to detect many EMRIs is driving the specification for the floor of the LISA noise curve.



Data analysis challenges

- The parameter space is very large, waveforms depend on 17 different parameters $- M$, S, m, e, r_p, ι, ψ_0, χ_0, ϕ_0, θ_K, ϕ_K, θ_s, ϕ_s, D plus 3 parameters describing the spin of the small body, but we ignore this for now.

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- Confusion from white dwarfs makes detection of EMRIs more difficult. Assume these can be removed to some level, although it is unlikely to be that simple in practice.

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- Analytic and numerical kludge waveforms drift out of phase over a few hours, but template counts agree to a few tens of percent. A useful sanity check!

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- Replace one parameter (e.g., r_p) with a time offset. Can search time offsets cheaply using inverse FFTs.

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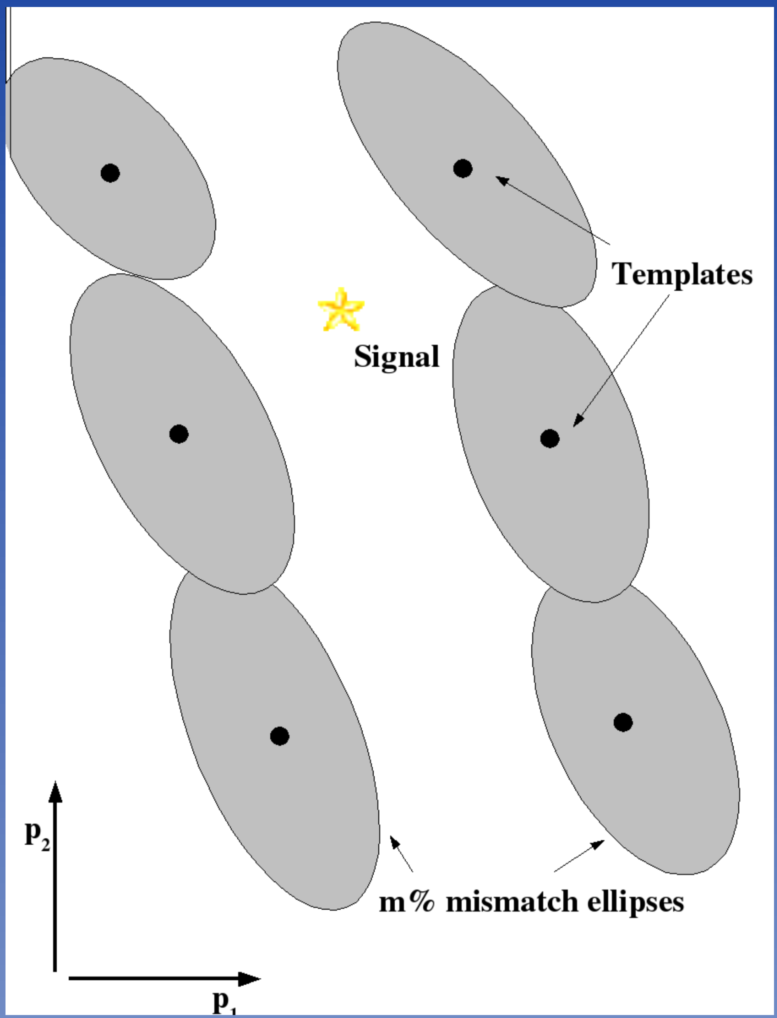

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- Define the usual metric on template space and use Monte Carlo simulations to estimate number of intrinsic templates required as a function of coherent integration time.
- Assuming 50 Teraflops computing power, and a 'match factor', $\mathcal{M} = 0.8$, find we are limited to coherent integrations of length ~ 3 weeks.

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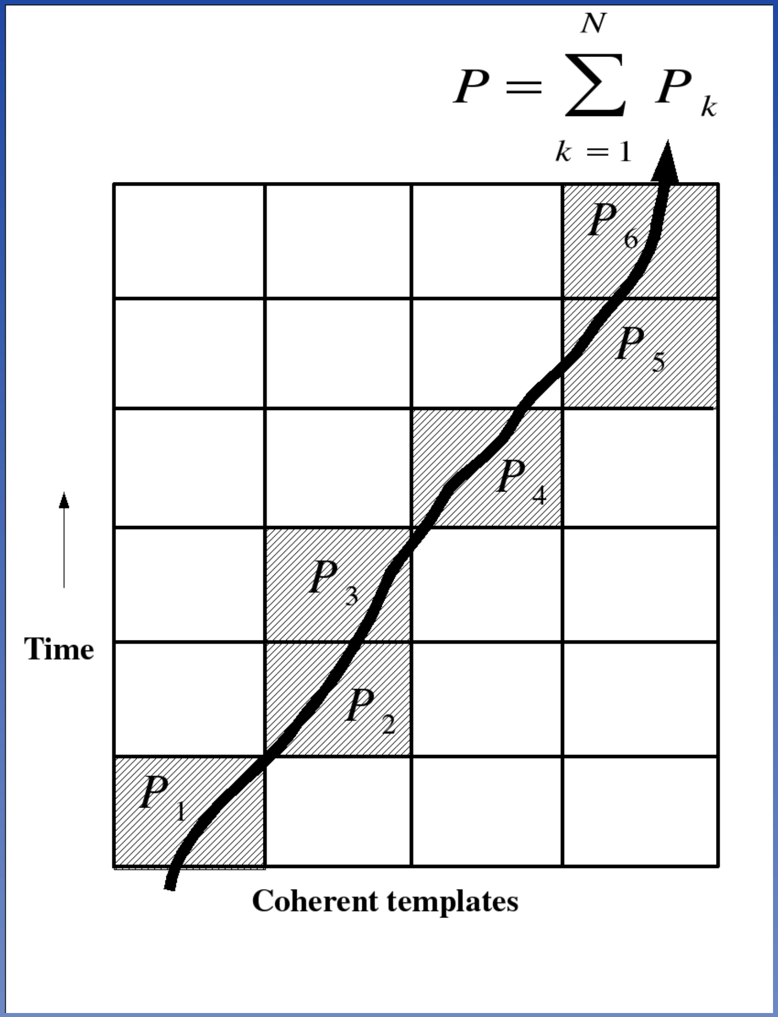

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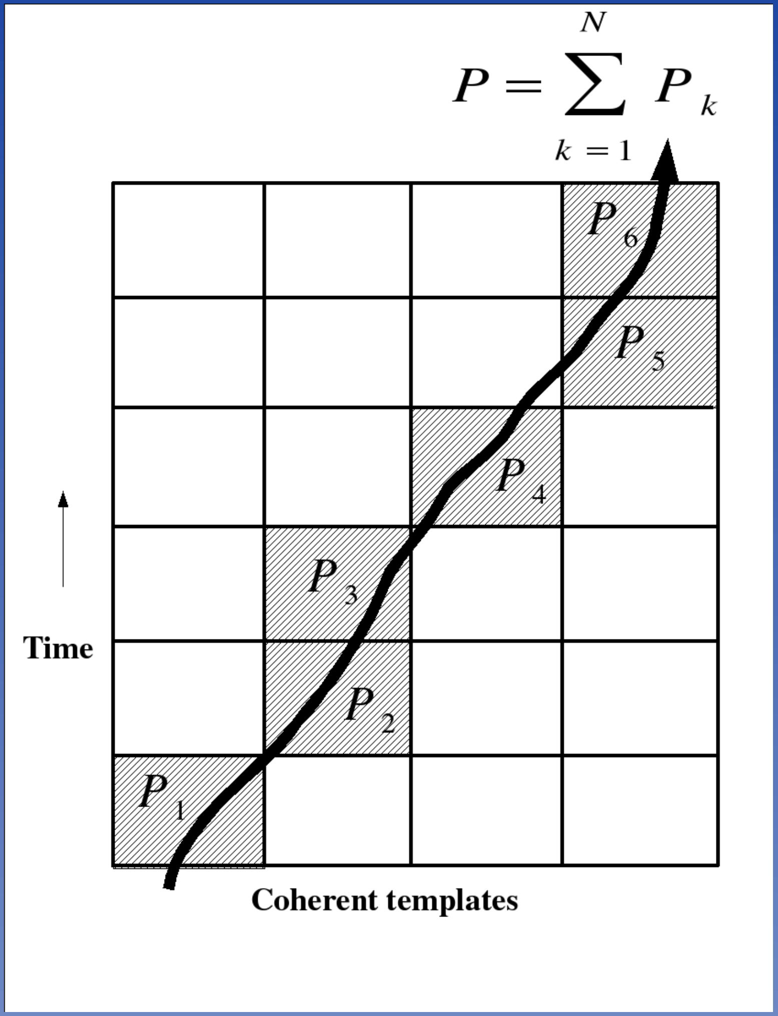
- Set threshold on P to give search an overall false alarm rate of 1%.



Astrophysical event rates

- Use galaxy luminosity function and $L - \sigma \;/\; M - \sigma$ relations to estimate space density of black holes $M_\bullet \frac{\mathrm{d}N}{\mathrm{d}M_\bullet} = 1.5 \times 10^{-3}\, h_{65}^2\, Mpc^{-3}$.

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		$0.6 M_\odot$ WD	$1.4 M_\odot$ MWD/NS	$10 M_\odot$ BH	$100 M_\odot$ PopIII
$10^{6.5\pm0.25}$	1.7	8.5	1.7	1.7	1.7×10^{-3}
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- Conservative rates could be a factor of ~ 100 smaller for WDs, or a factor of ~ 10 smaller for black holes.

Estimating the LISA event rate

- Using these astrophysical rates, we can estimate the number of LISA events, under two sets of assumptions –
  - ⋆ **Optimistic** – Assume a 5 year LISA lifetime; SNRs computed using optimal AET combination; optimistic white dwarf subtraction; three week coherent integrations (threshold SNR ~ 36).

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- Repeat the calculation for 'Short LISA' with 1.6×10^6km arms.

Estimated LISA event rates

- Final results are shown below. For $z > 1$, system evolution is uncertain and flat space extrapolation is no longer valid, so we quote $z < 1$ lower limits $(*)$.

$M_\bullet$	m	LISA		Short LISA	
		Optimistic	Pessimistic	Optimistic	Pessimistic
300 000	0.6	10	< 1	10	1
300 000	10	700*	90	900	120
300 000	100	1*	1*	1*	1*
1 000 000	0.6	90	10	80	10
1 000 000	10	1100*	660*	1100*	500
1 000 000	100	1*	1*	1*	1*
3 000 000	0.6	70	2	10	< 1
3 000 000	10	1700*	130	820	20
3 000 000	100	2*	1*	2*	1

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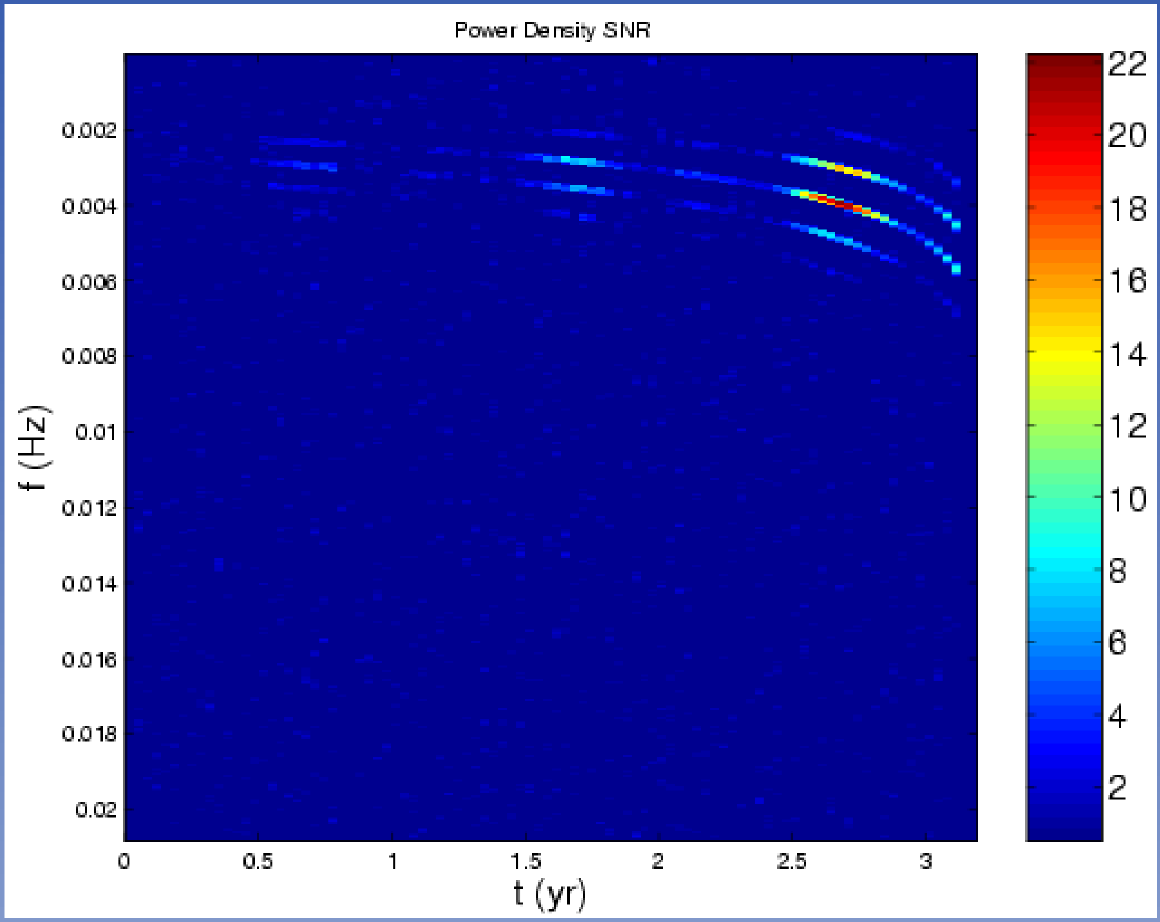

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- More details on poster by Linqing Wen et al.

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- Must integrate this search with other aspects of LISA data analysis.
- Search method can be improved in various ways – using higher waveform multipoles, more stages to the heirarchy etc.
- Pursue alternative methods to use in conjunction with this approach.