Detection of extreme mass ratio inspirals with LISA - search strategy and detection rate estimates

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- Desire to detect many EMRI is driving the specification for the floor of the LISA noise curve.
Data analysis challenges

- The parameter space is very large, waveforms depend on 17 different parameters — $M, S, m, e, r_p, \iota, \psi_0, \chi_0, \phi_0, \theta_K, \phi_K, \theta_s, \phi_s, D$ plus 3 parameters describing the spin of the small body, but we ignore this for now.
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- Confusion from white dwarfs makes detection of EMRIs more difficult. Assume these can be removed to some level, although it is unlikely to be that simple in practice.
Scoping out data analysis – ‘numerical kludge’ waveforms

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- Analytic and numerical kludge waveforms drift out of phase over a few hours, but template counts agree to a few tens of percent. A useful sanity check!
Coherent search – computational simplifications

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- On these timescales, five parameters — \(\theta_S, \phi_S, \theta_K, \phi_K\) and \(\phi_0\) are extrinsic. Assuming a pure quadrupole gravitational waveform, the \(\rho^2\) statistic maximizes over these parameters automatically.
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\rho^2 = \sum_{\alpha=I}^{II} \sum_{i=1}^{5} \langle h_i(\lambda_I), s_{\alpha} \rangle^2 , \quad \text{where} \quad \langle a, b \rangle = 4 \Re \left[ \int_{0}^{\infty} \frac{\tilde{a}^*(f) \tilde{b}(f)}{S_b(f)} df \right] \tag{1}
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\]

- Replace one parameter (e.g., \( r_p \)) with a time offset. Can search time offsets cheaply using inverse FFTs.
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- Assuming 50 Teraflops computing power, and a ‘match factor’, $M = 0.8$, find we are limited to coherent integrations of length $\sim 3$ weeks.
Second stage - incoherent summation

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- Final search statistic is the sum $P = \sum P_k$ along trajectories through the coherent segments.
- Set threshold on $P$ to give search an overall false alarm rate of 1%.
Astrophysical event rates

- Use galaxy luminosity function and $L - \sigma / M - \sigma$ relations to estimate space density of black holes $M_\bullet \frac{dN}{dM_\bullet} = 1.5 \times 10^{-3} h_{65}^2 \ Mpc^{-3}$. 
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<thead>
<tr>
<th>$M_\bullet \ M_\odot$</th>
<th>space density $10^{-3} h_{65}^2 \text{Mpc}^{-3}$</th>
<th>Merger rate $R \ Gpc^{-3} \text{yr}^{-1}$</th>
</tr>
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<tbody>
<tr>
<td>$10^{6.5 \pm 0.25}$</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>$10^{6.0 \pm 0.25}$</td>
<td>1.7</td>
<td>6</td>
</tr>
<tr>
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<td>3.5</td>
</tr>
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Table I: Estimated capture rates
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<tbody>
<tr>
<td></td>
<td>0.6 (M_\odot) WD</td>
<td>1.4 (M_\odot) MWD/NS</td>
</tr>
<tr>
<td>(10^6.5 \pm 0.25)</td>
<td>1.7</td>
<td>8.5</td>
</tr>
<tr>
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<td>1.7</td>
<td>6</td>
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- Conservative rates could be a factor of \(\sim 100\) smaller for WDs, or a factor of \(\sim 10\) smaller for black holes.
Estimating the LISA event rate

- Using these astrophysical rates, we can estimate the number of LISA events, under two sets of assumptions —

  - Optimistic — Assume a 5 year LISA lifetime; SNRs computed using optimal AET combination; optimistic white dwarf subtraction; three week coherent integrations (threshold SNR $\sim 36$).
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- Repeat the calculation for ‘Short LISA’ with $1.6 \times 10^6$ km arms.
Estimated LISA event rates

- Final results are shown below. For $z > 1$, system evolution is uncertain and flat space extrapolation is no longer valid, so we quote $z < 1$ lower limits (*)..

<table>
<thead>
<tr>
<th>$M_\bullet$</th>
<th>$m$</th>
<th>LISA</th>
<th>Short LISA</th>
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<tr>
<td></td>
<td></td>
<td>Optimistic</td>
<td>Pessimistic</td>
</tr>
<tr>
<td>300 000</td>
<td>0.6</td>
<td>10</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>300 000</td>
<td>10</td>
<td>700*</td>
<td>90</td>
</tr>
<tr>
<td>300 000</td>
<td>100</td>
<td>1*</td>
<td>1*</td>
</tr>
<tr>
<td>1 000 000</td>
<td>0.6</td>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>1 000 000</td>
<td>10</td>
<td>1 100*</td>
<td>660*</td>
</tr>
<tr>
<td>1 000 000</td>
<td>100</td>
<td>1*</td>
<td>1*</td>
</tr>
<tr>
<td>3 000 000</td>
<td>0.6</td>
<td>70</td>
<td>2</td>
</tr>
<tr>
<td>3 000 000</td>
<td>10</td>
<td>1 700*</td>
<td>130</td>
</tr>
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- More details on poster by Linqing Wen et al.
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- Pursue alternative methods to use in conjunction with this approach.