Untangling LISA Source Confusion

Neil Cornish
Louis Rubbo, Seth Timpano, Jeff Crowder, Ron Hellings

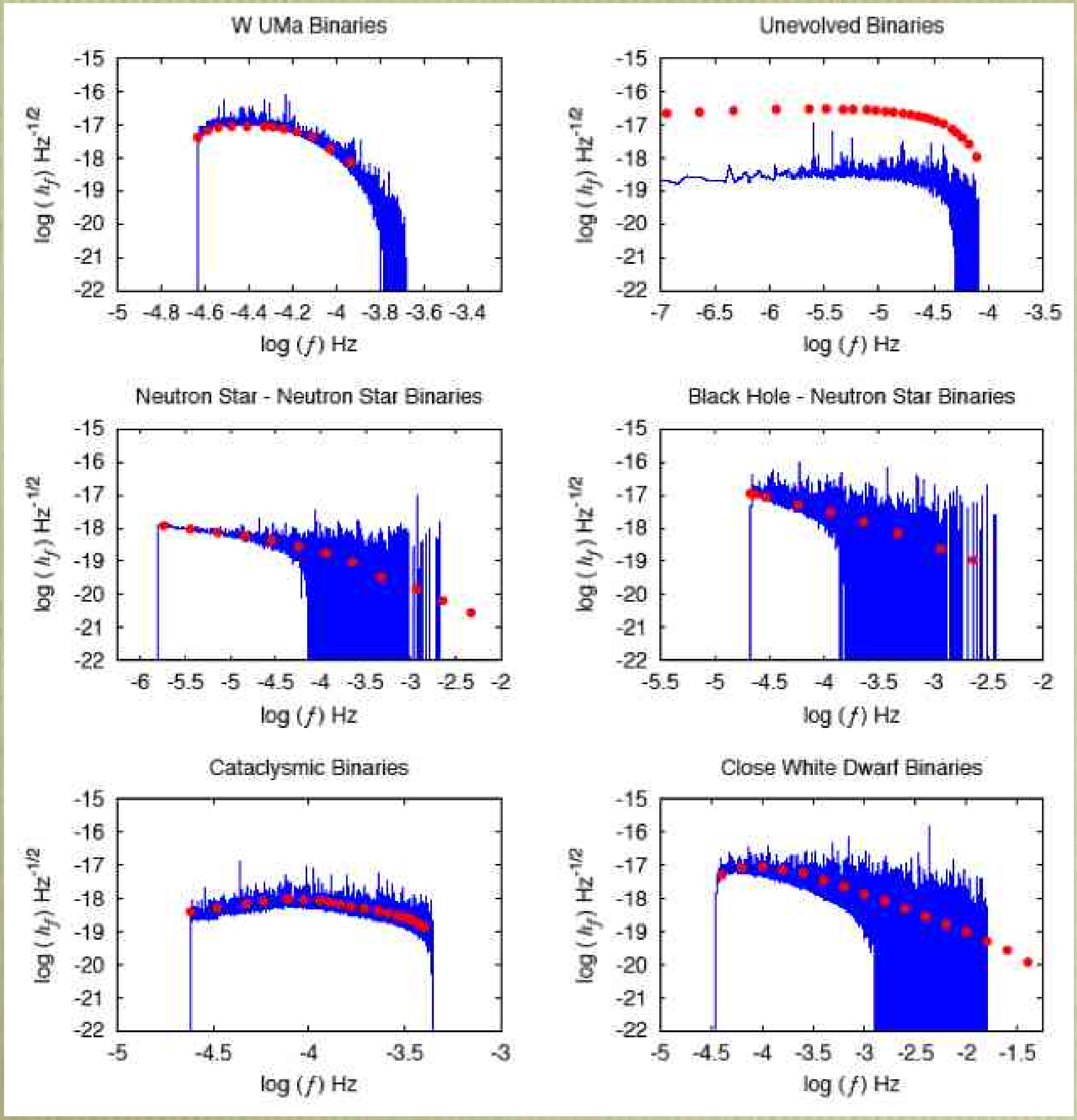
Embarrassment of Riches

- Galactic & Extra-Galactic Stellar Binaries
- Super Massive Black Hole Binaries
- Extreme Mass Ratio Inspirals
- Cosmic String Cusps
- Bursts
- etc.

"All Sky, All the Time"

Non-Orthogonal Signals → Overlap and Confusion

Galactic Background

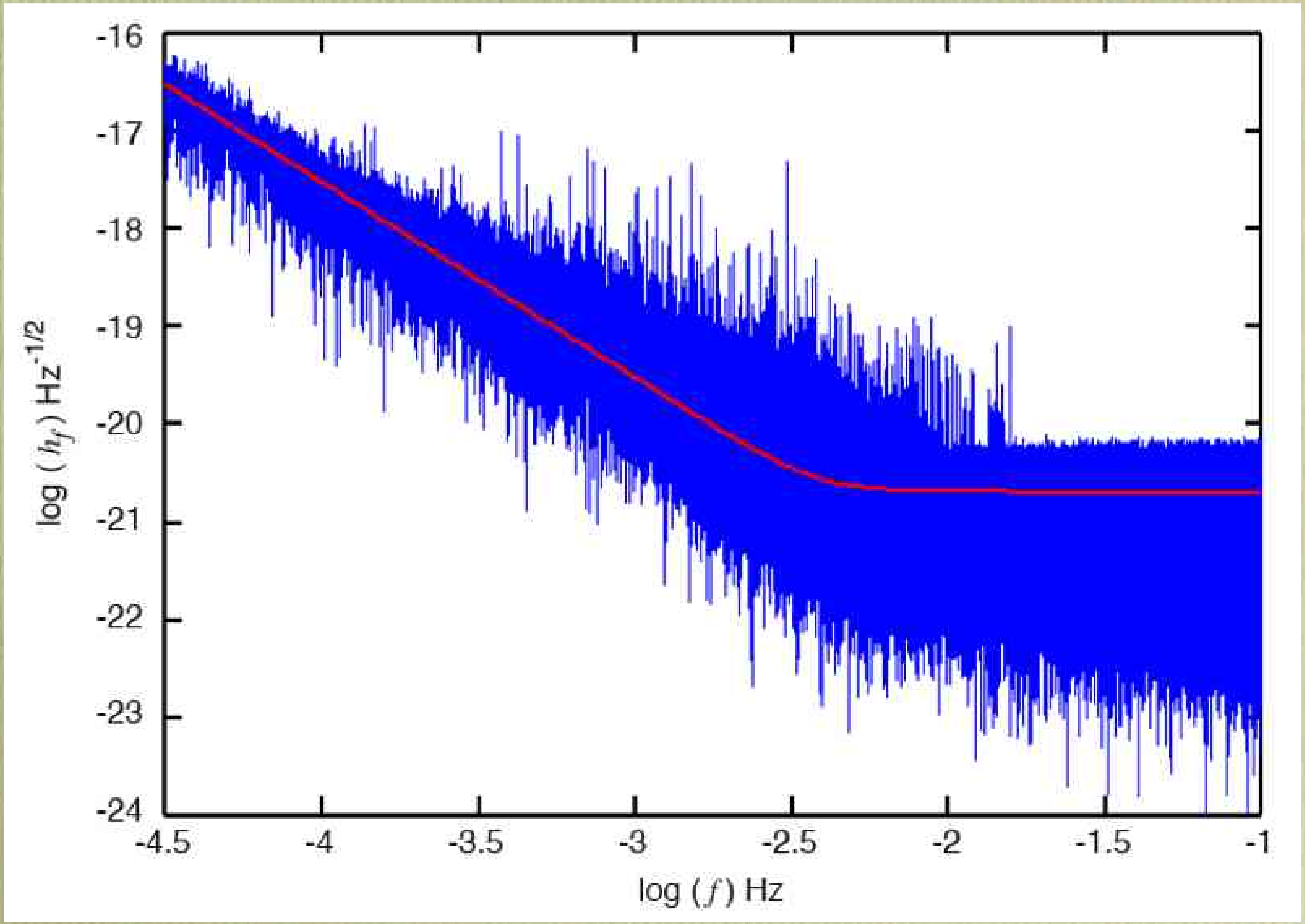


Hils, Bender & Webbink, ApJ 75, 360 (1990)

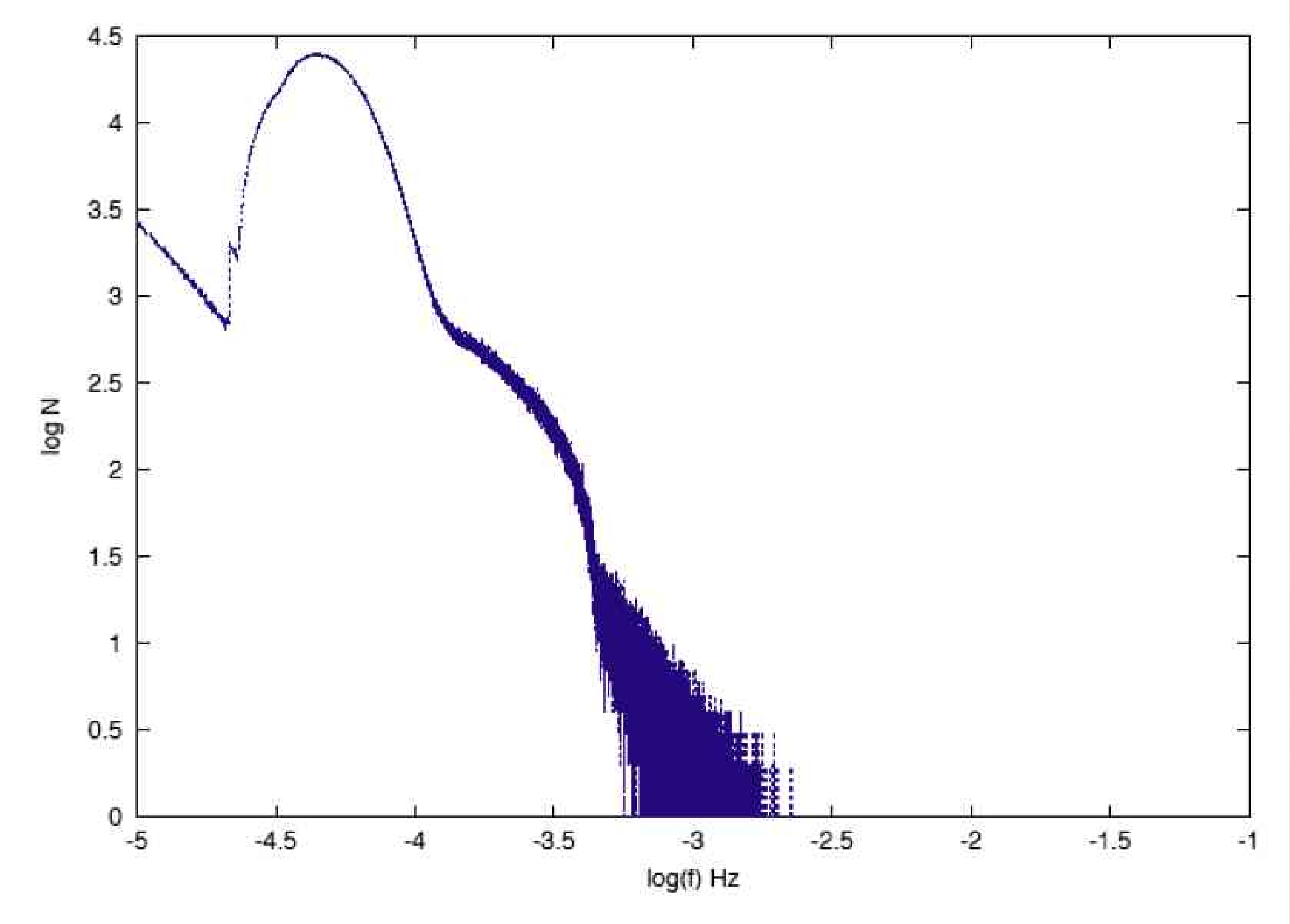
Rubbo, Timpano & Cornish, *in preparation*

3×10^7	W UMa
7×10^6	Unevolved
1×10^6	NS – NS
5×10^5	BH – NS
1×10^6	Cataclysimic
3×10^6	WD – WD

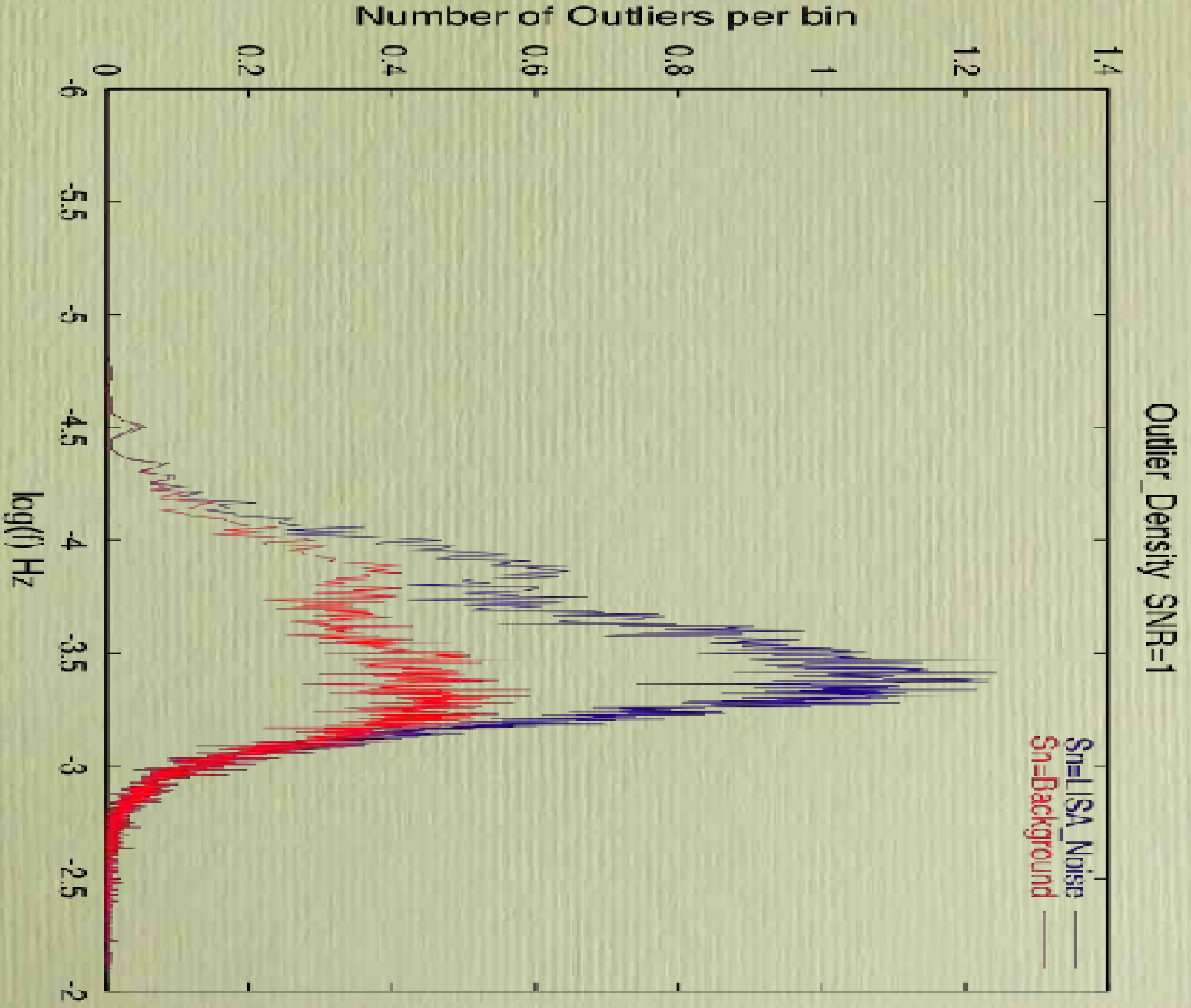
Galactic Background as seen by LISA



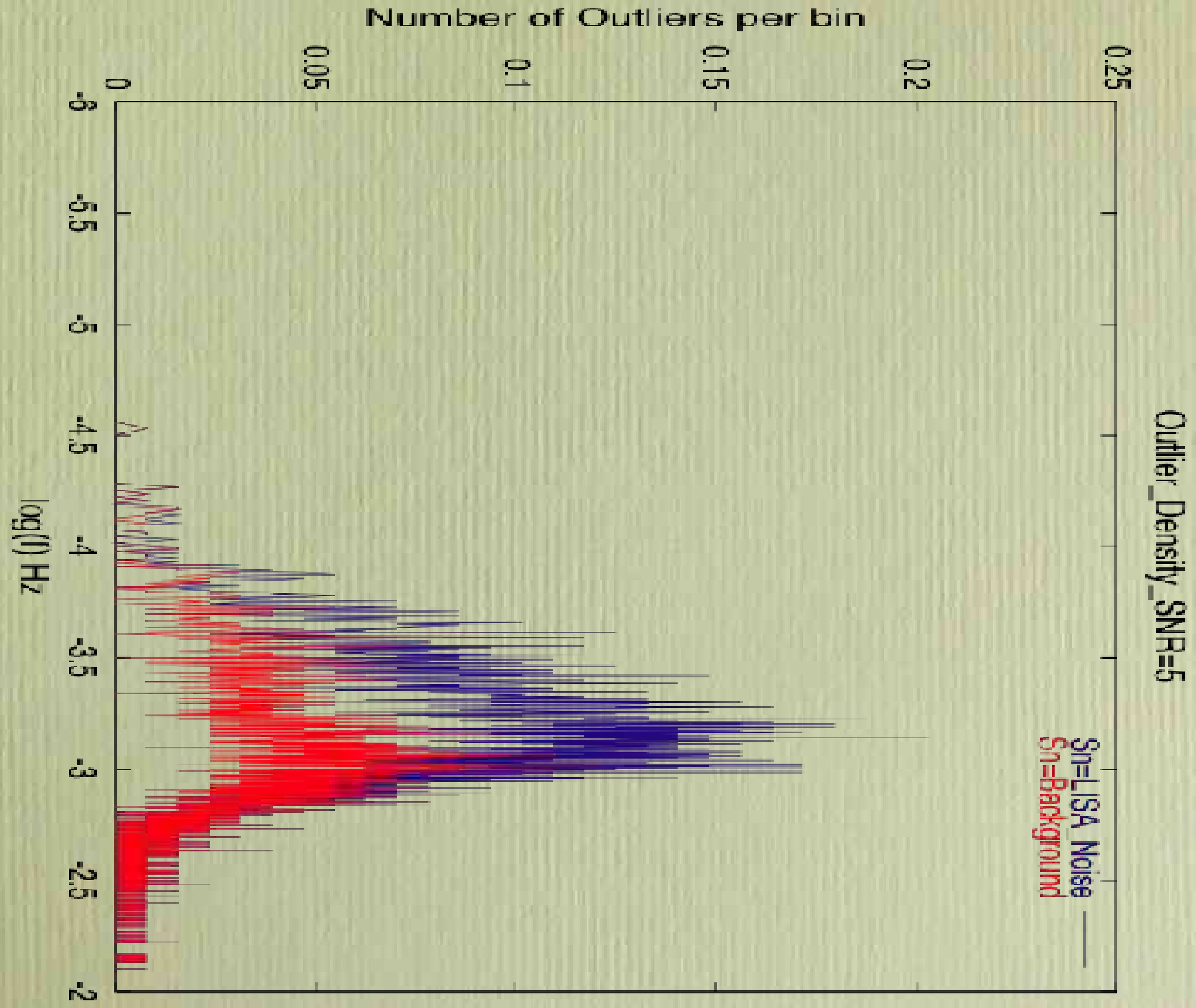
Source Density I



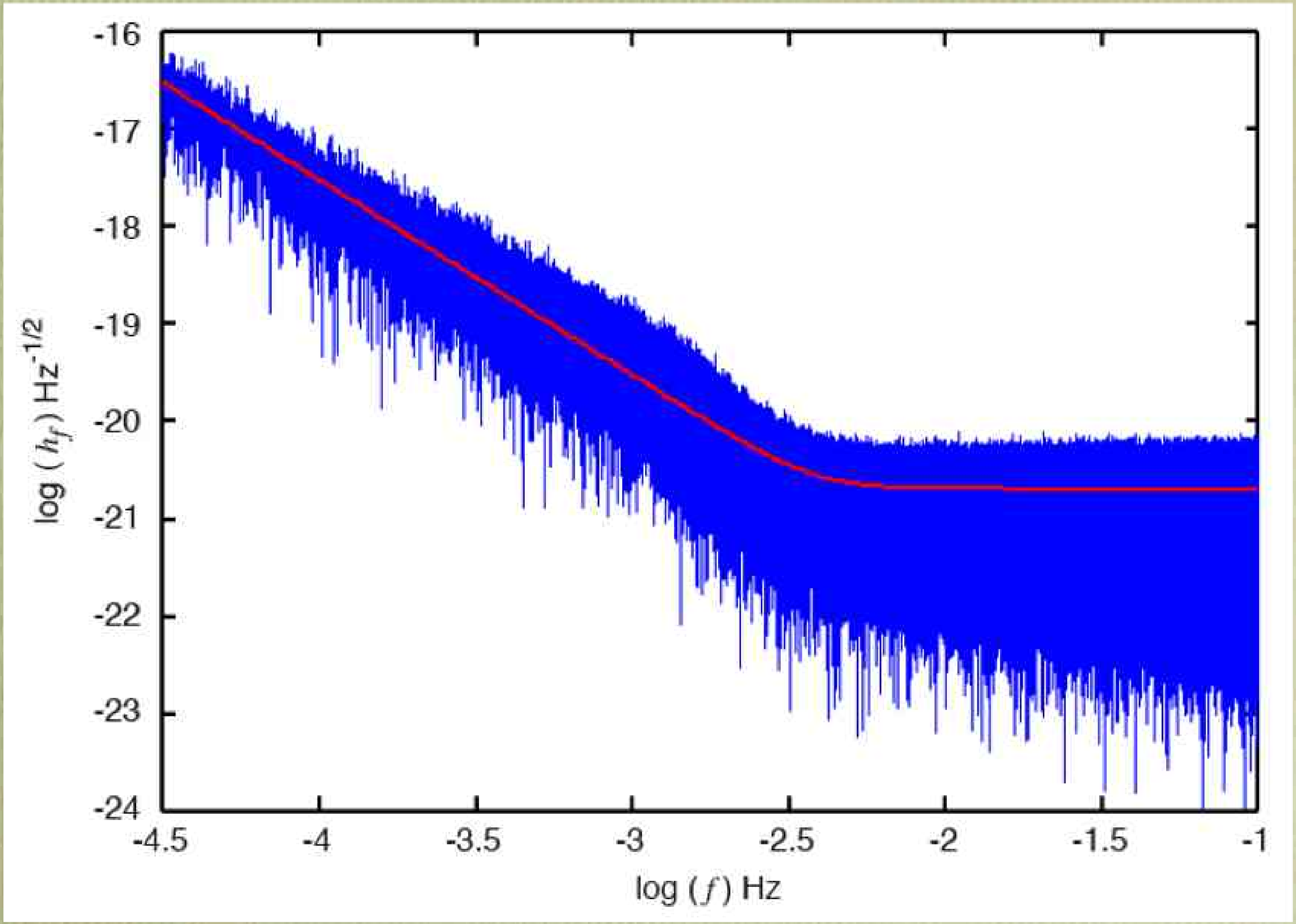
Source Density II



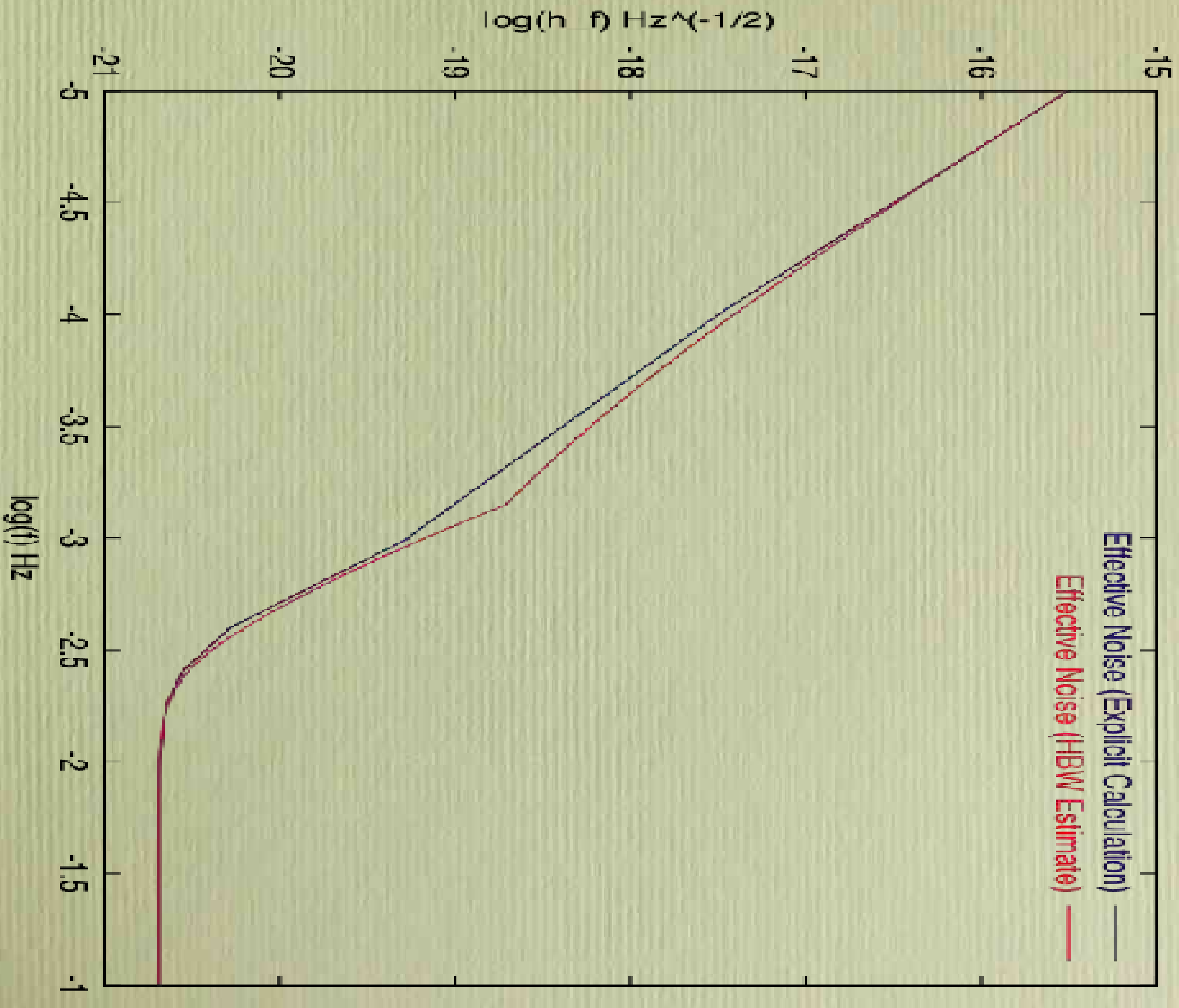
Source Density II



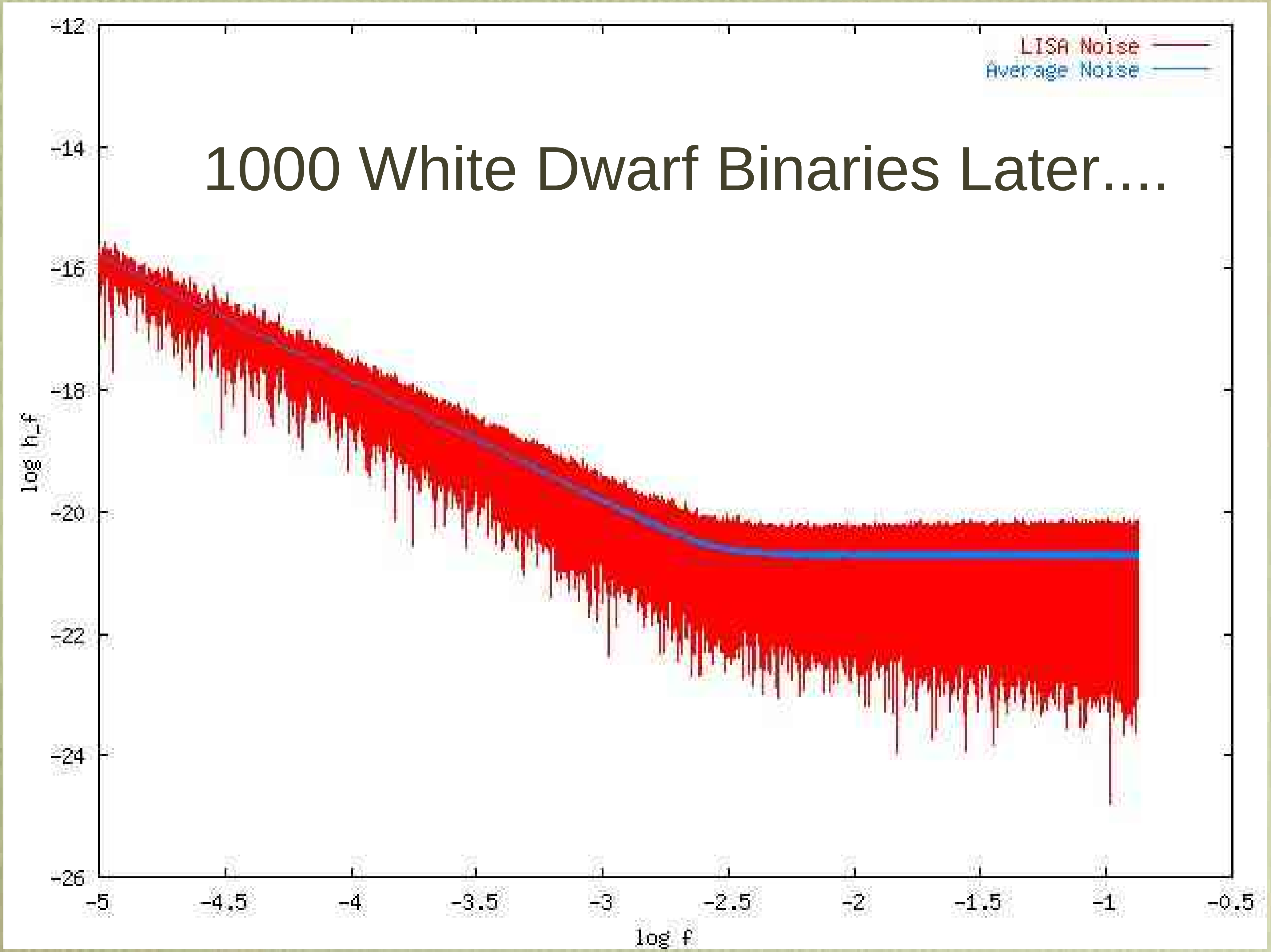
Bright Sources Removed



Revised Confusion Level



Throwing the Baby out with the Bath Water



Can’t remove the Galactic “noise” to produce a “cleaned” data stream

How do we do this?

Optimal Data Analysis = Matched Filtering

$$s(t) = h(t) + n(t) = \sum_{i=1}^{N} h_i(t) + n(t)$$

Optimal Filter includes all *N* resolvable sources

Source parameters $\vec{\lambda} = \vec{\lambda}^{(1)} + \vec{\lambda}^{(2)} + \dots$

Parameters per source $d_i = 7 \rightarrow 17$

Parameter Space Dimension $D = \sum_{i=1}^{N} d_i \sim 40,000$

Direct Search Cost $\sim \text{const.}^N$

Multi-Source Searches

Not as bad as it sounds...

$(h_i|h_j) \approx 0$ for galactic binaries *i, j* separated by $\sim \mu\mathrm{Hz}$

...But still pretty bad

Techniques that help:

- Fast Source Identification
  - $\mathcal{F}$ - Statistic Jaranowski, Krolak & Schutz (1998), Krolak, Tinto & Vallisneri (2004)
  - Demodulation Cornish & Larson (2002), Hellings (2003), Cornish (2004)
- Sub-Optimal Initial Guess
  - gCLEAN (iterative) Cornish & Larson (2003)
  - Markov Chain Monte Carlo (stochastic) Christensen, Dupuis, Woan & Meyer, (2004)
  - Genetic Algorithms (Darwinian)
- Refinement of Initial Guess
  - Linear Least Squares Cornish, Hellings & Rubbo (2004)
  - Simulated Annealing

Signal Analysis 101

Log Likelihood

$$\mathcal{L}(\vec{\lambda}') = \log L(\vec{\lambda}') = (s|h') - \frac{1}{2}(h'|h')$$

$$(x|y) = 2\int \frac{\tilde{x}\tilde{y}^* + \tilde{x}^*\tilde{y}}{S_n(f)}$$

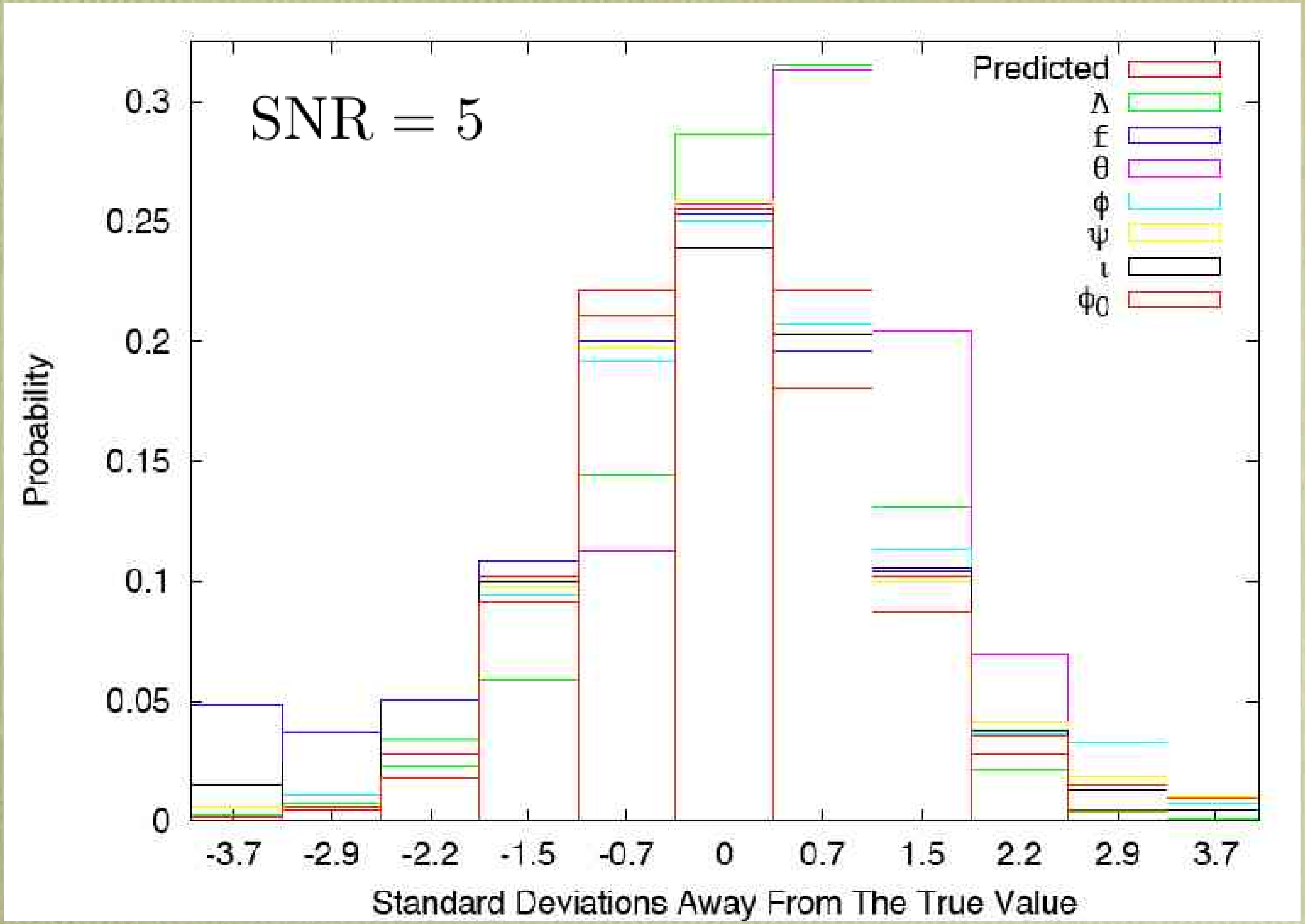
Expectation Values at Maximum Likelihood

$$\langle \mathcal{L}(\vec{\lambda}) \rangle = \frac{1}{2}(h|h) = \frac{1}{2}\mathrm{SNR}^2$$

$$\left\langle \frac{\partial \mathcal{L}(\vec{\lambda})}{\partial \lambda_i} \right\rangle = 0$$

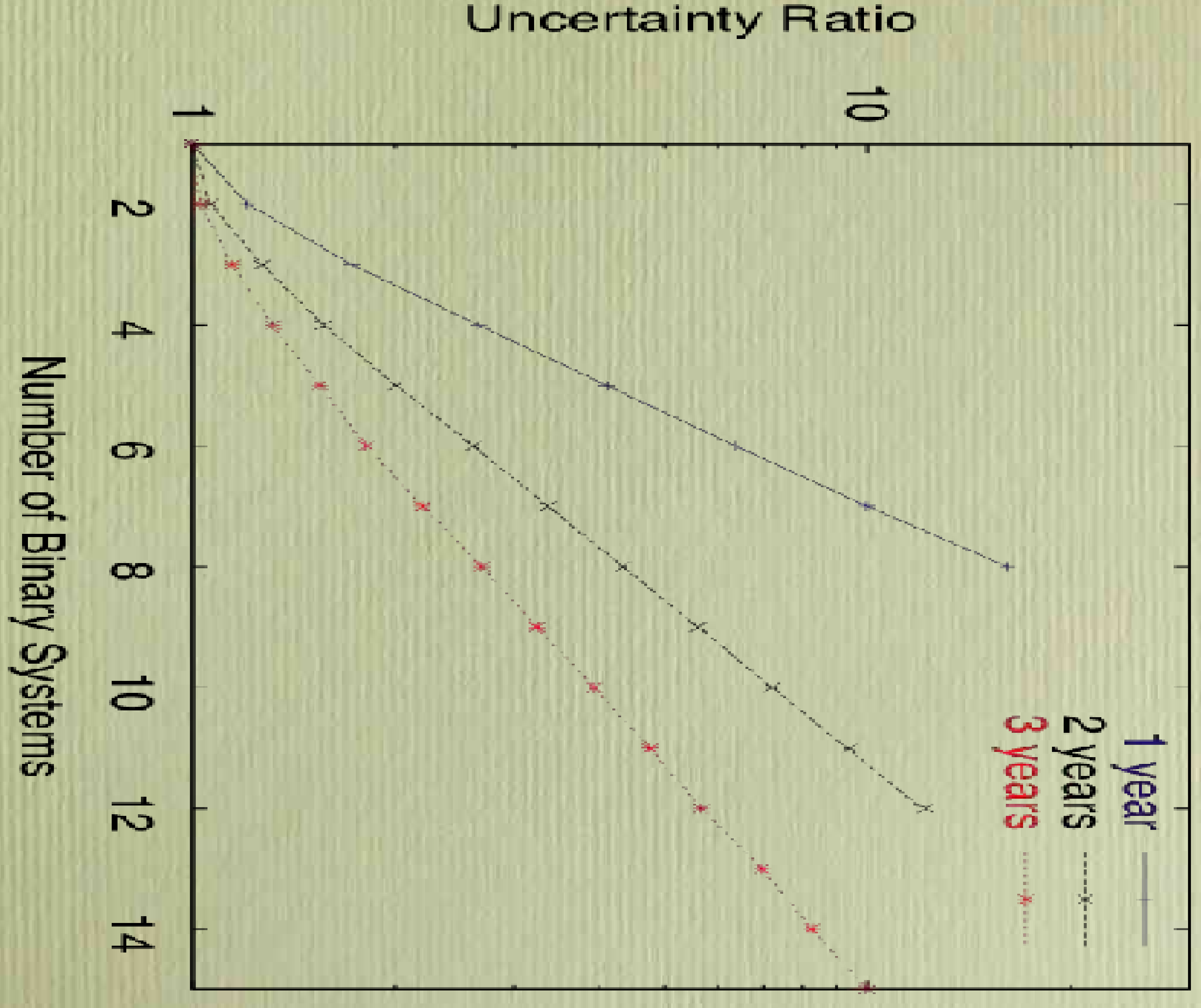
$$\left\langle \frac{\partial^2 \mathcal{L}(\vec{\lambda})}{\partial \lambda_i \partial \lambda_j} \right\rangle = -(h_{,i}|h_{,j}) = -\Gamma_{ij}$$

Parameter Uncertainity Estimation

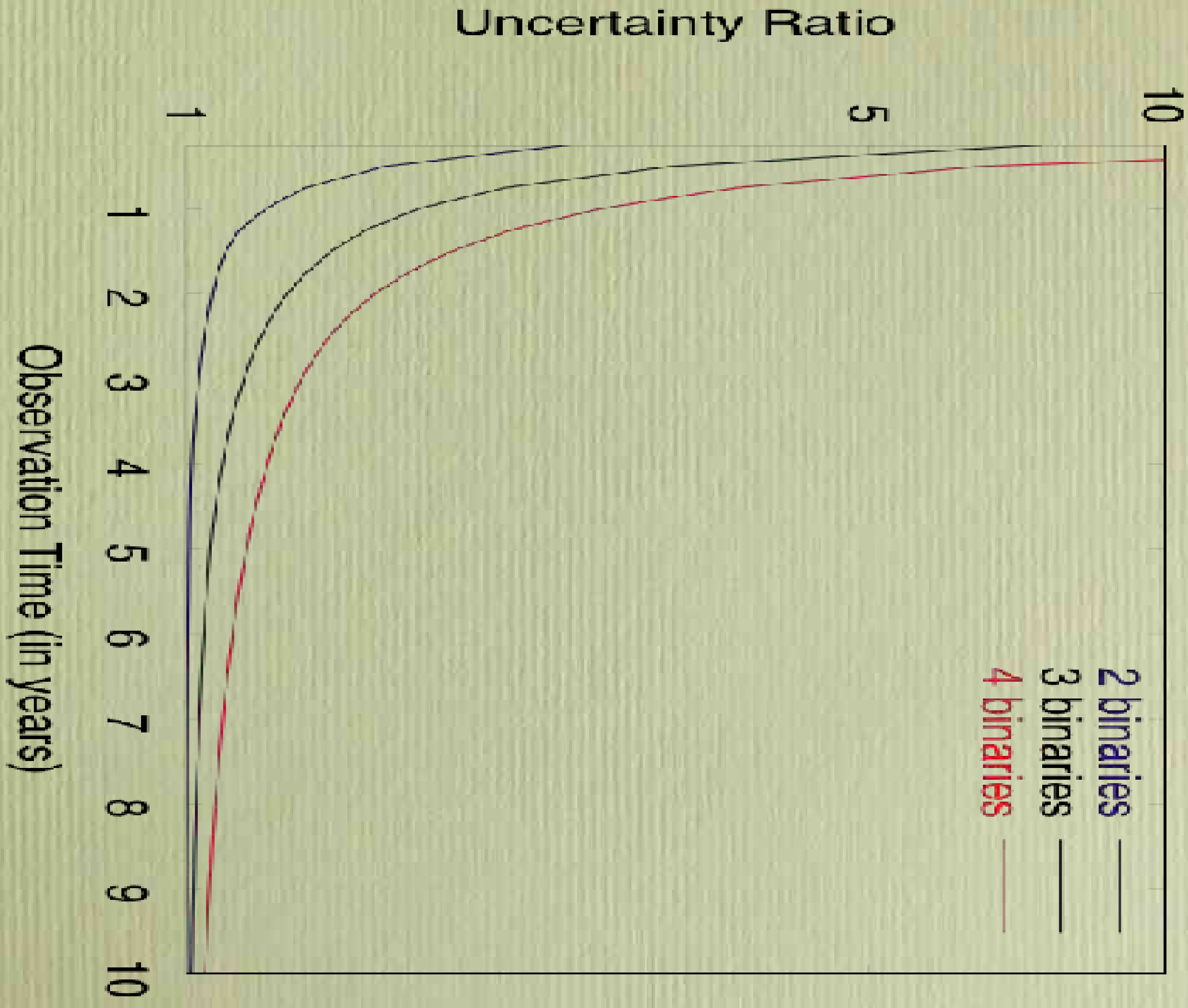


Fisher Matrix Prediction v.s. Template Matching

Source Confusion



Source Confusion



$\mathcal{F}$ - Statistic

$$s(t) = \sum_{i=1}^{4} a_i(A, \psi, \iota, \varphi_0) A^i(t; f_0, \theta, \phi)$$

$$\begin{aligned}
A^1(t) &= D^+(t;\theta,\phi)\cos\Phi(t;f_0,\theta,\phi) & a_1 &= A/2\left((1+\cos^2\iota)\cos\varphi_0\cos 2\psi - 2\cos\iota\sin\varphi_0\sin 2\psi\right)\\
A^2(t) &= D^\times(t;\theta,\phi)\cos\Phi(t;f_0,\theta,\phi) & a_2 &= -A/2\left(2\cos\iota\sin\varphi_0\cos 2\psi + (1+\cos^2\iota)\cos\varphi_0\sin 2\psi\right)\\
A^3(t) &= D^+(t;\theta,\phi)\sin\Phi(t;f_0,\theta,\phi) & a_3 &= -A/2\left(2\cos\iota\cos\varphi_0\sin 2\psi + (1+\cos^2\iota)\sin\varphi_0\cos 2\psi\right)\\
A^4(t) &= D^\times(t;\theta,\phi)\sin\Phi(t;f_0,\theta,\phi) & a_4 &= A/2\left((1+\cos^2\iota)\sin\varphi_0\sin 2\psi - 2\cos\iota\cos\varphi_0\cos 2\psi\right)
\end{aligned}$$

$$N^i = (s|A^i)$$

Four filters per source, depend only on three parameters $f_0,\ \theta,\ \phi$

$$N^i = a_j (A^j|A^i) = a_j M^{ji}$$

$$\mathcal{F} \simeq \log L = \frac{1}{2} M^{-1}_{ij} N^i N^j$$

Jaranowski, Krolak & Schutz, Phys.Rev.D58:063001,1998

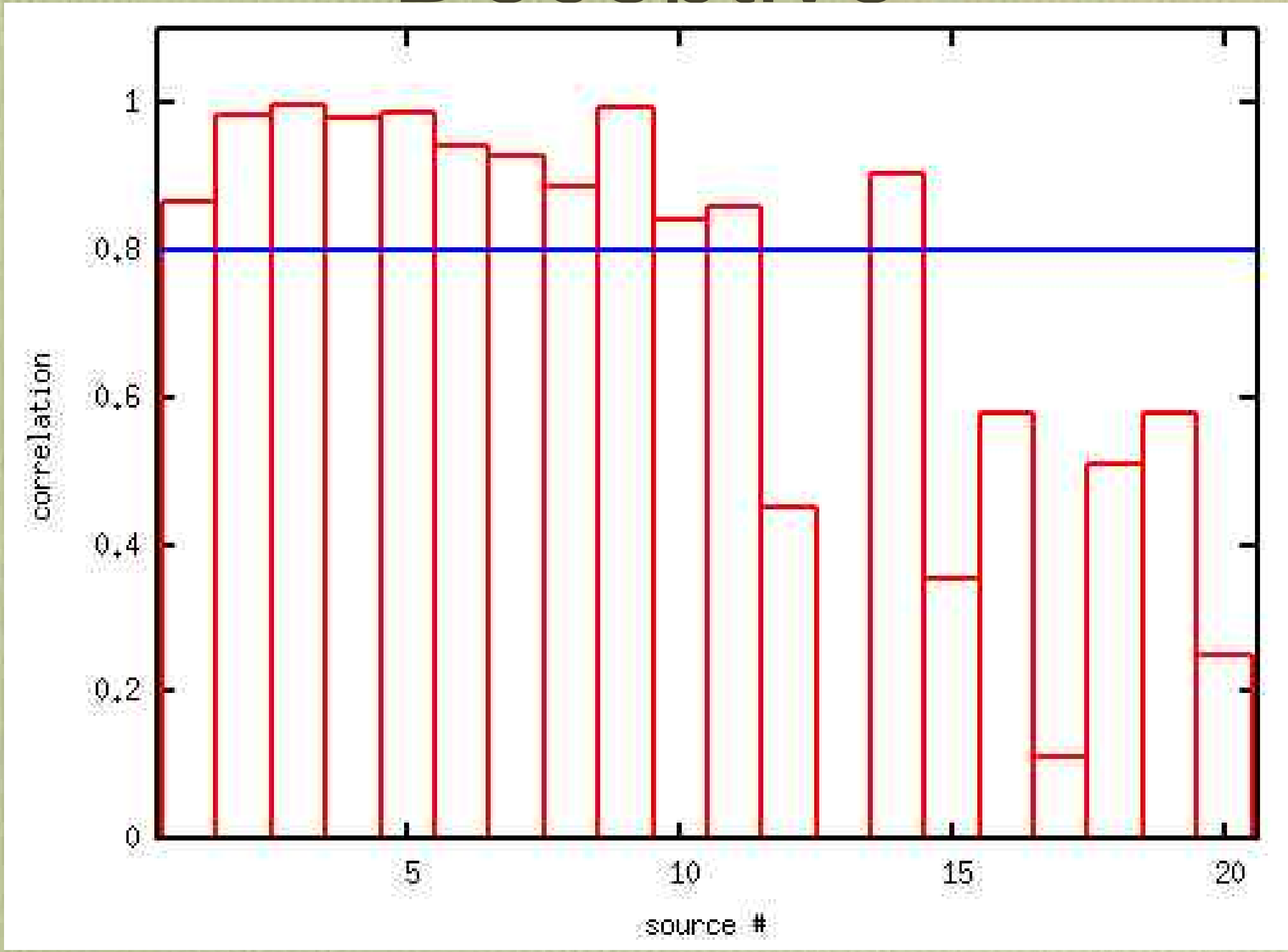
Krolak, Tinto & Vallisneri, gr-qc/0401108

Naive application of $\mathcal{F}$ - Statistic

QuickTimeª and a
GIF decompressor
are needed to see this picture.

Sequential Removal, Brightest to Dimmest

Appearances can be Deceptive



Only 12 real sources in fit. Fake sources all had $\mathrm{SNR} > 6$

Knowing when to Stop

More sources = More parameters = Better Fit

Question: If a filter built from 1000 Galactic Binaries gives same log likelihood as a filter describing one SMBH binary, which solution do we favour?

Bayesian Information Criteria

$$\mathrm{BIC} = -2\log L + D\log \mathcal{N}$$ (Schwarz 1978)

Penalty for using extra parameters

Example: Galactic Binary @ ∞ mHz

$$\mathrm{BIC} \simeq -\mathrm{SNR}^2 + D\log \mathcal{N}$$

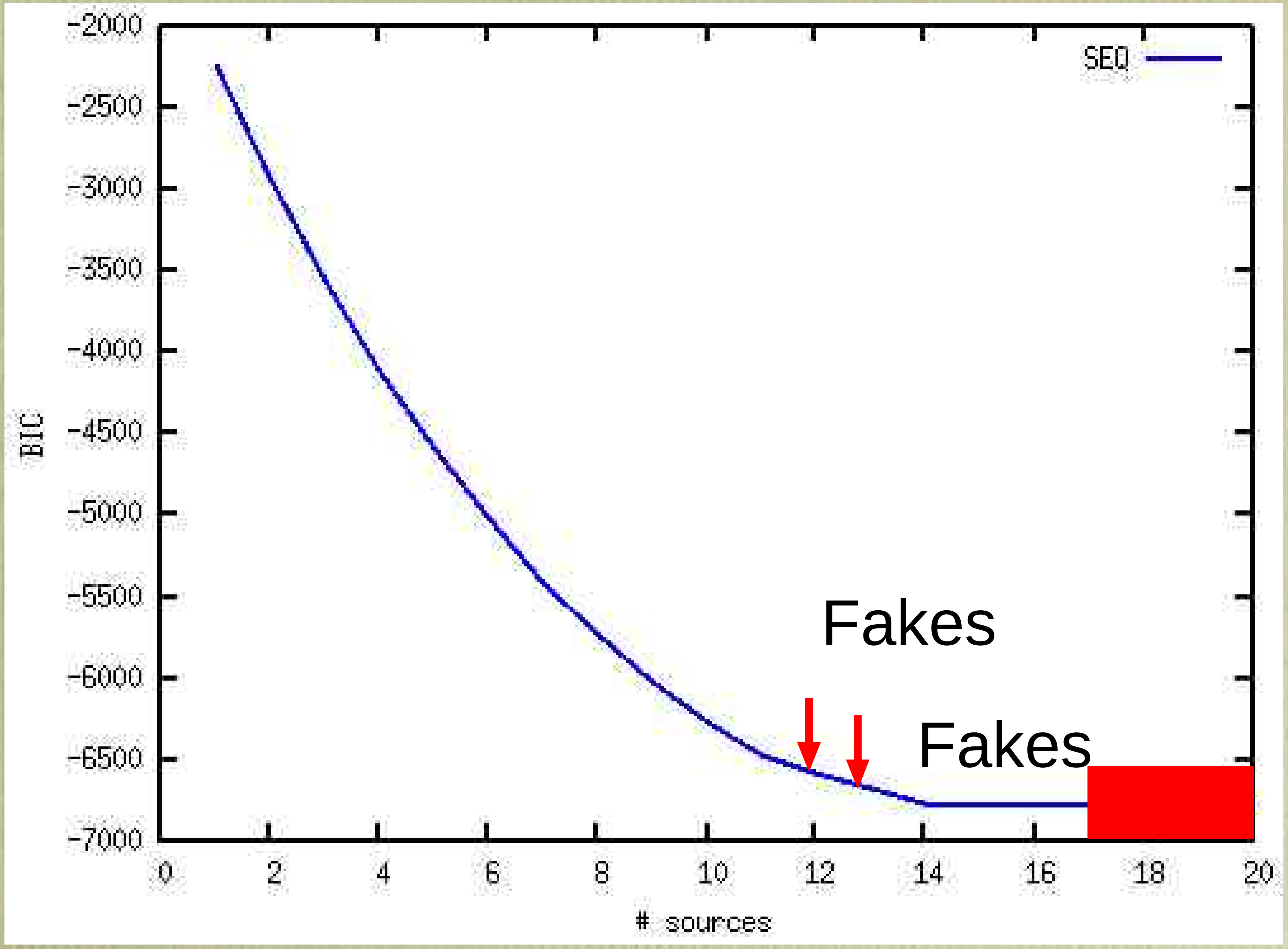
$$\mathcal{N} \simeq 2 \times 2 \times 10$$

$$D = 7$$

$$\text{Require BIC} < -2 \quad \Rightarrow \quad \mathrm{SNR} > 5.3$$

Stops the Dwarfs from eating the Giant

BIC for Sequential Removal



How do we do better?

- Iterative Initial Guess
- Global Re-Fit of Initial Guess

gCLEAN ©

Identify Brightest Source

Record what is Subtracted

SNR > 5 ?

Yes

Remove of brightest (to level of second brightest)

No

Group subtraction record into reconstituted sources

Report average parameters of sources with $\Delta \text{BIC} < -2$

Yields a good initial guess for the resolvable sources

© Cornish & Larson, Phys.Rev.D67:103001,2003

Linear Least Squares Refinement

$$\text{Initial Guess} \;\Rightarrow \text{Optimal Solution}$$

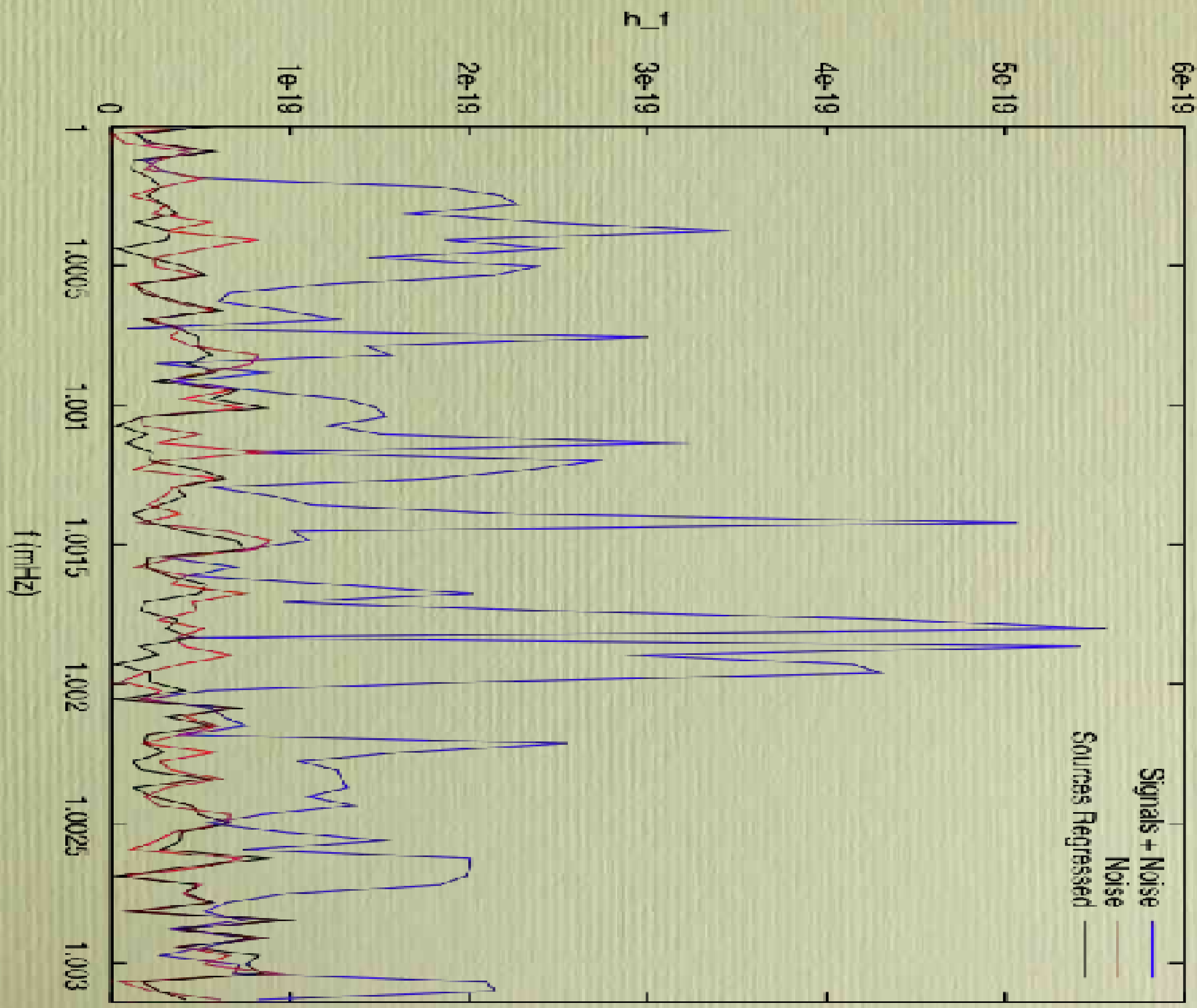
$$\Delta s(t) = s(t) - h(t) = \frac{\partial h(t)}{\partial \lambda_i} \Delta \lambda_i + z(t)$$

$$\text{Minimize the residual } (z|z)$$

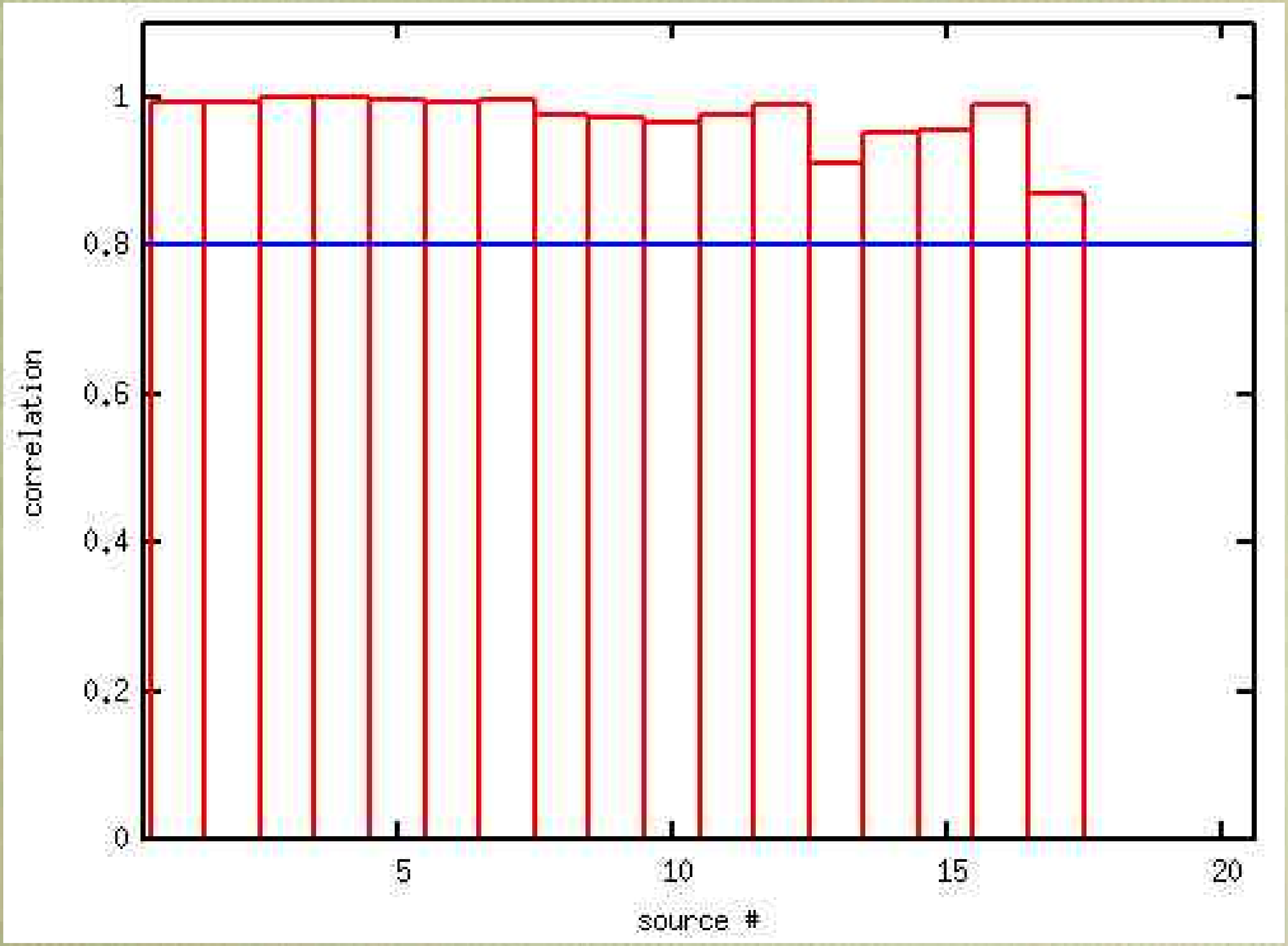
$$\text{Solution: } \; \Delta \lambda_i = \Gamma^{-1}_{ij} (\Delta s | \partial_j h)$$

$$\Gamma_{ij} \text{ full } D \times D \; \text{ dimensional Fisher matrix}$$

Least Squares Fitting



Quality of Fit



Current Work

- Full simulation using gCLEAN + LSF
- MCMC
- Including SMBH binaries