

A. Sesana

*Low-Frequency Gravitational Radiation  
from Coalescing Massive Black Holes*

A. Sesana<sup>1</sup>, F. Haardt<sup>1</sup>, P. Madau<sup>2</sup> & M. Volonteri<sup>2</sup>, ApJ in press, astro-ph/040154  
A. Sesana, F. Haardt & P. Madau, in preparation

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## OUTLINE

-*Seed BHs Formation*

-*From Seed BHs to SMBHs: Hierarchical Clustering*

-*MBHs Mass Growth: Accretion vs Merging*

-*Dynamical Evolution of BHs*

-*Cosmic Gravitational Wave Background*

-*Resolved vs Unresolved signal: GW detection with LISA*

## **THE MODEL**

*SMBHs are grown from **seeds** pregalactic BHs, endproduct of the **first stars**. These seeds are incorporated in larger and larger halos, **accreting gas** and **interacting each with other** after mergers.*

### **OBSERVATIONAL CONSTRAINTS:**

- 1. LF of quasars**
- 2.  $M_{BH}$ - $\sigma$  relation**
- 3. Galactic cores**

# ORIGIN OF SEED BHs

$H_2$  cooling  $\rightarrow T=10^3$  K  $\rightarrow z=30$   $M_{DM}=10^6 M_\odot$

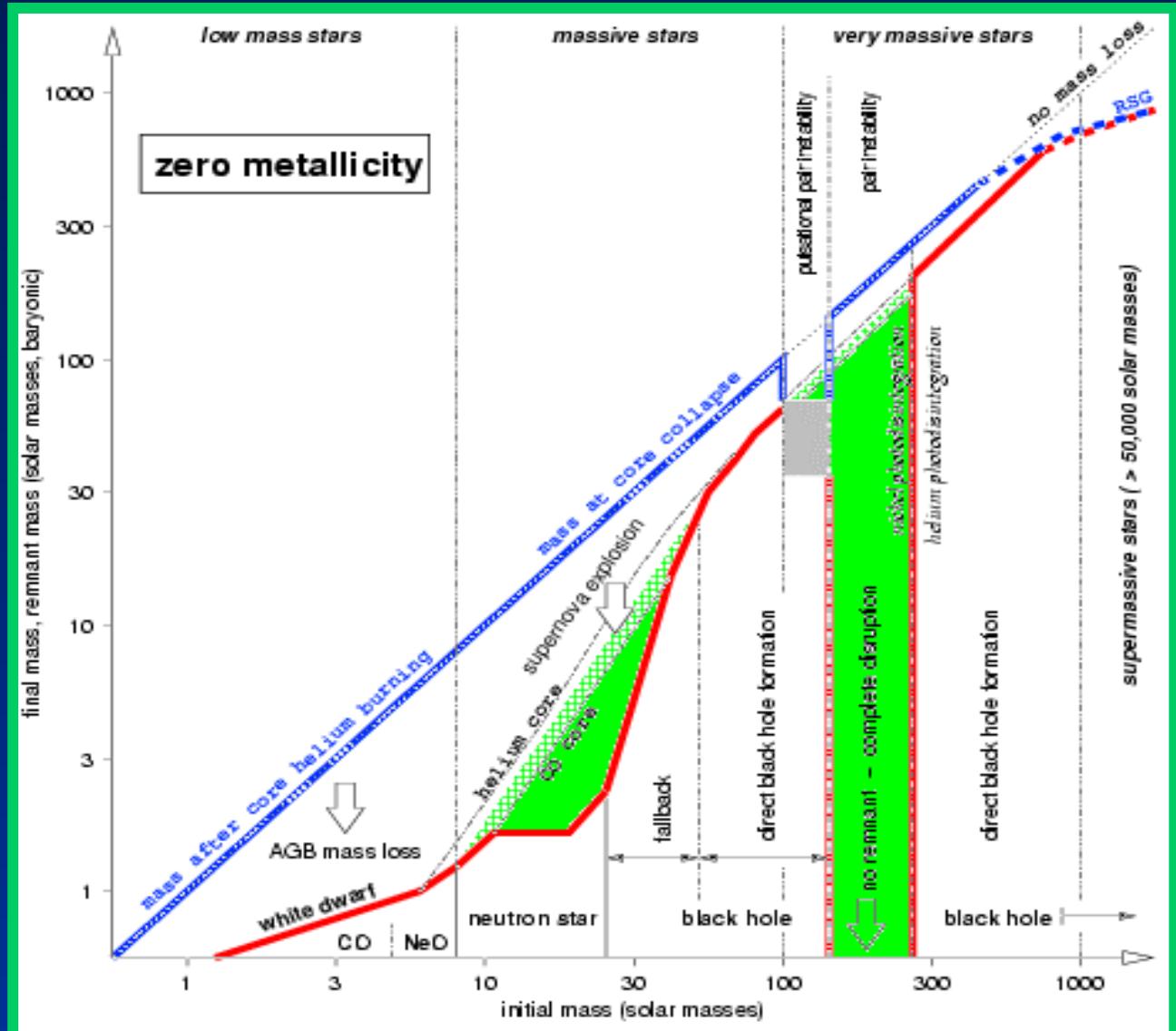
$\downarrow$   
Subsonic collapse

$\downarrow$   
No fragmentation

$\downarrow$   
Formation of VMSs

$\downarrow$   
Intermediate mass  
seed BHs

Abel, Bryan & Norman 2000;  
Iapamonti et al. 2002)



# MERGING HISTORY OF SMBHs

## ASSUMPTIONS:

Seeds  $M_{BH} \sim 150 M_\odot$  in  $3.5\sigma$  fluctuations at  $z=20$

(Madau & Rees 2001)

Halo density profile:

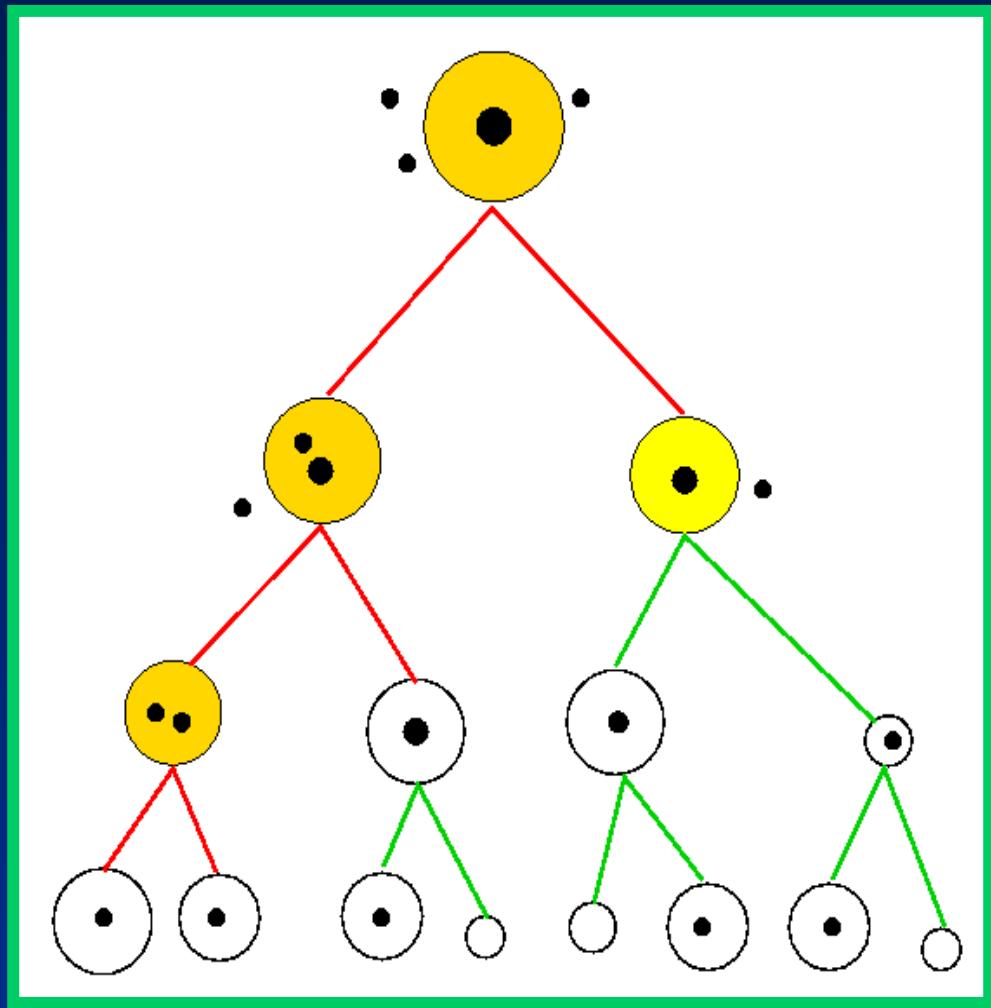
DM: NFW

Baryons: SIS

The semi-analytic code follows the accretion and the dynamical history of BHs in every single branch of the tree

The adopted threshold for density peaks hosting a seed insures an occupation fraction of order unity today for halos more massive than  $10^{11} M_\odot$

(Volonteri, Haardt & Madau 2003)



Binary merger trees starting at  $z=20$  in a  $\Lambda$ CDM cosmology

# SMBHs ASSEMBLY

## 1. dynamical friction (Lacey & Cole 1993, Colpi et al. 2000)

from the interaction between the DM halos to the formation of the BH binary  
determined by the global distribution of matter  
efficient only for ***major mergers*** against mass stripping

## 2. hardening of the binary (Quinlan 1996, Merritt 1999, Milosavljevic & Merritt 2001)

***3 bodies interactions*** between the binary and the surrounding stars  
the binding energy of the BHs is larger than the thermal energy of the stars  
the SMBHs create a ***stellar density core ejecting the background stars***

## 3. emission of gravitational waves (Peters 1964)

takes over at subparsec scales  
leads the binary to coalescence

## WARNING!

- ★ Triple BHs interactions
- ★ Gravitational rocket

# ARE MERGERS EFFICIENT IN LEADING MBHs BINARIES TO COALESCENCE?

Dynamical friction

Chandrasekhar timescale:

$$t_{df} = 1.65 \frac{1+P}{P} \frac{1}{H(z)\sqrt{\Delta_{vir}} \ln \Lambda} \Theta$$

Hardening and GW emission

typically  $t_h >> t_{gr}$ . Gravitational slingshot acts on a timescale:

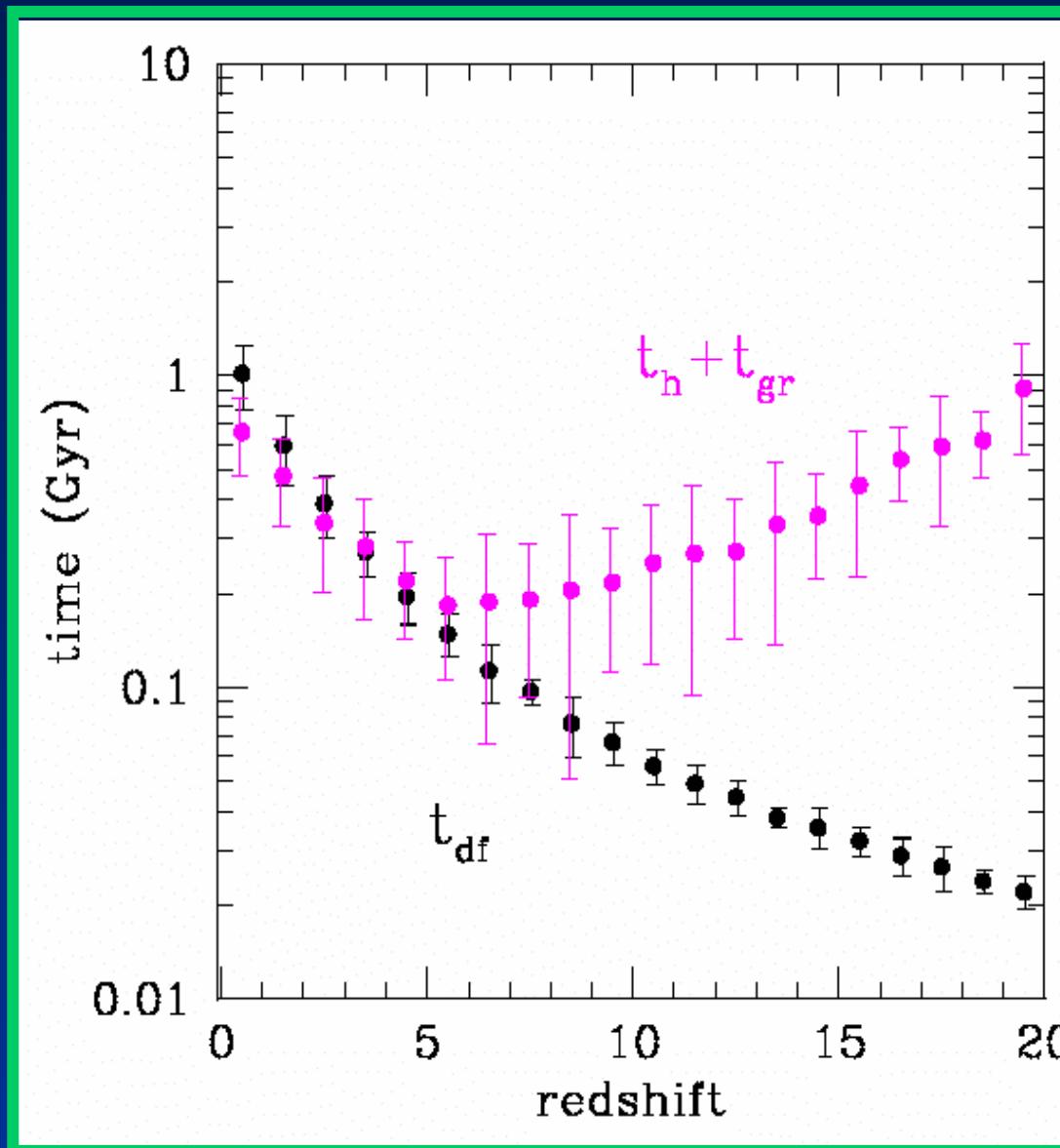
$$t_h \propto \frac{r_c(t)^2}{\sigma a}$$

(Volonteri, Madau & Haardt 2003)

$t_{df} + t_h + t_{gr} < t_{Hubble}$  if  $z < 15$



BH binaries can coalesce!



# How does the SMBHs mass grows along the cosmic history?

Mergers

Accretion

To recover the local  $M_{BH}-\sigma$  & the quasar LF:

**ACCRETION**

(cfr Kauffmann & Haehnelt 2000, Wyithe & Loeb 2002)

- \* only during major mergers  
 $P=M_s/M>0.3$

- \* the accreted mass is a fixed fraction of the  $M_{BH}-\sigma$  relation

$$\Delta m = k \sigma^{4.6}$$

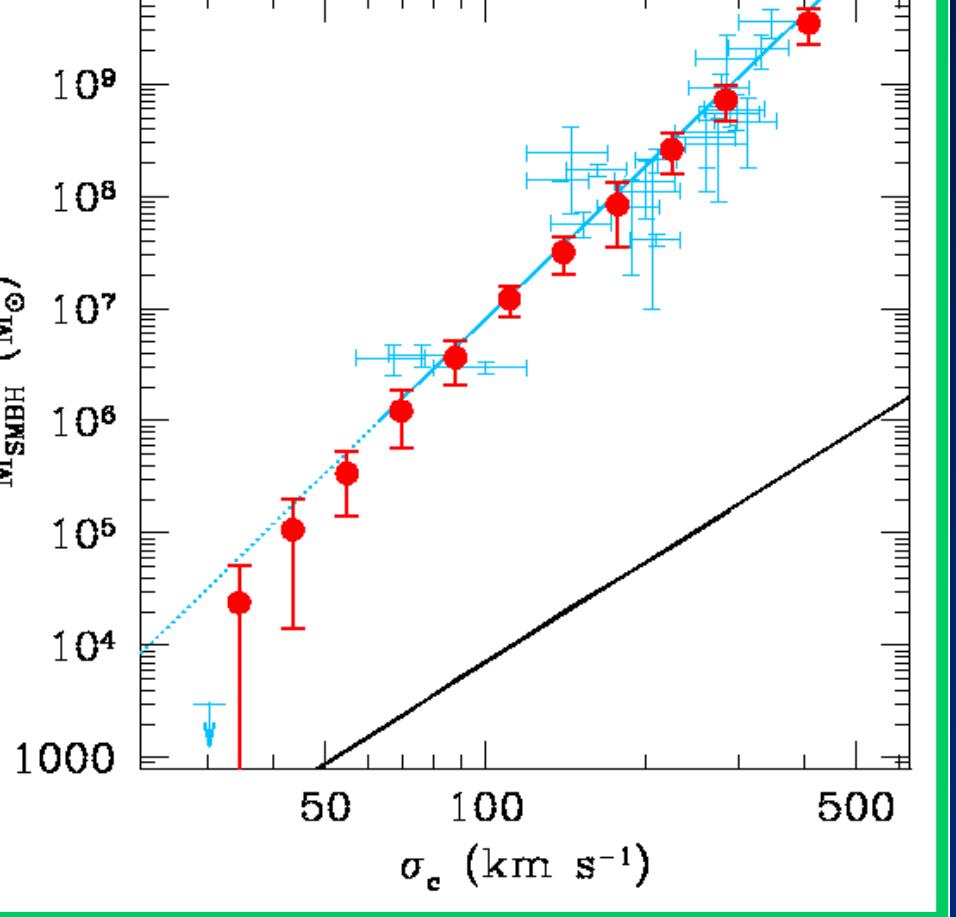
(cfr King 2003, Silk & Rees 1998)

- \* Eddington accretion rate

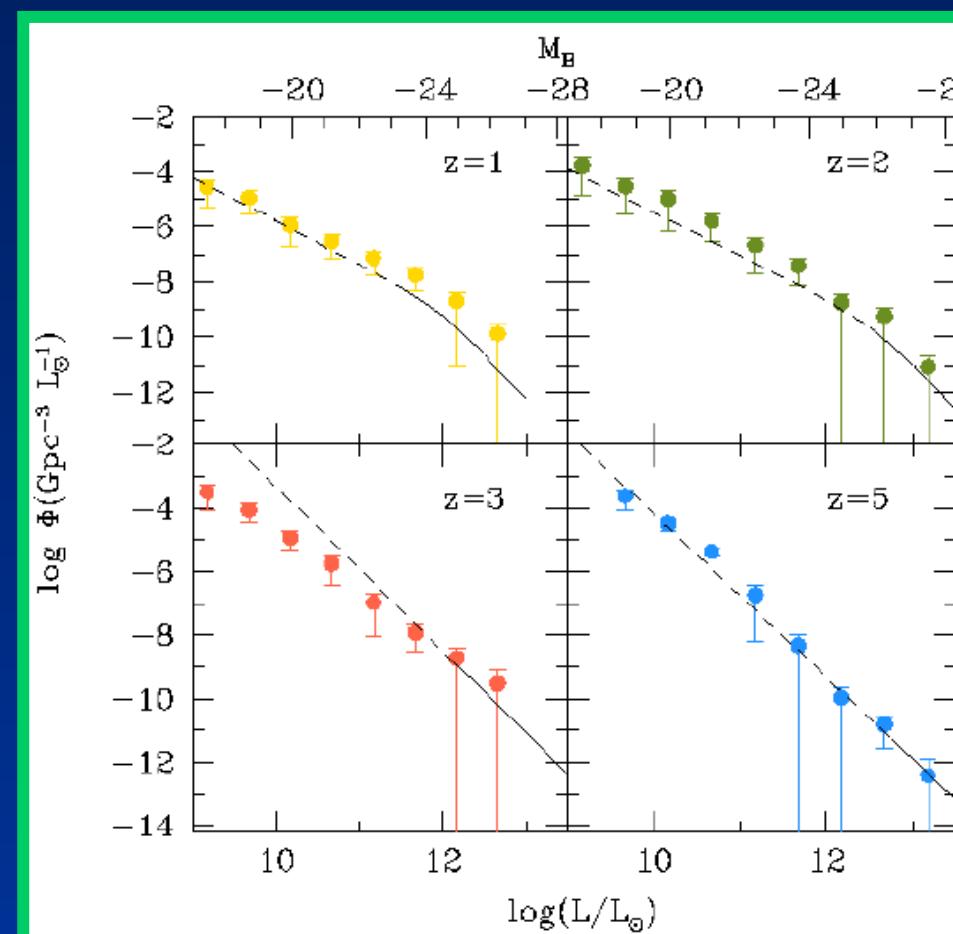


*final mass of SMBHs  
is dominated by accretion,  
with mergers playing a  
secondary role*

$$M_{BH,acc} \gg M_{BH,merg}$$

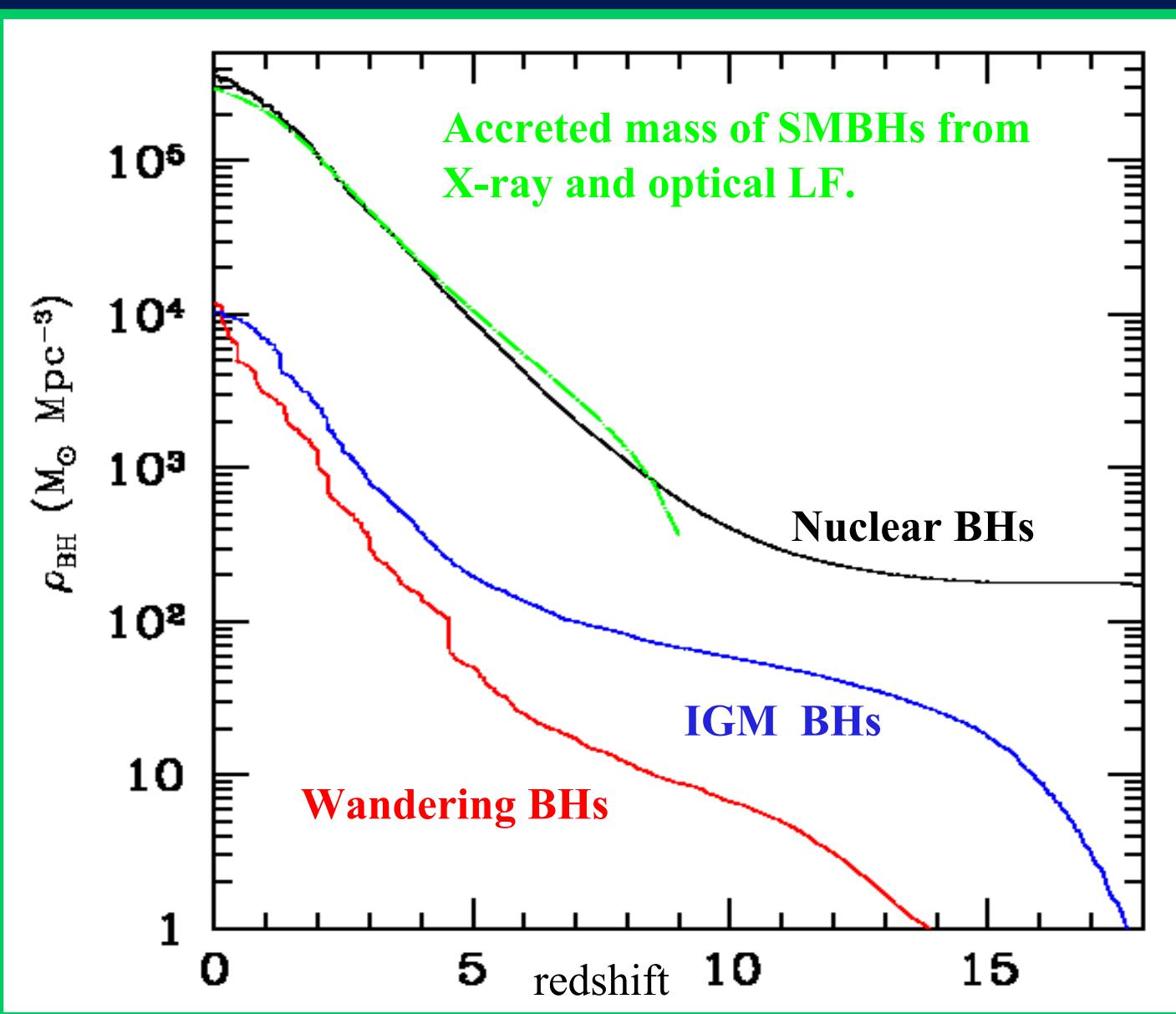


*BHs in halos which have experienced major mergers only at high z have MUCH lower masses*

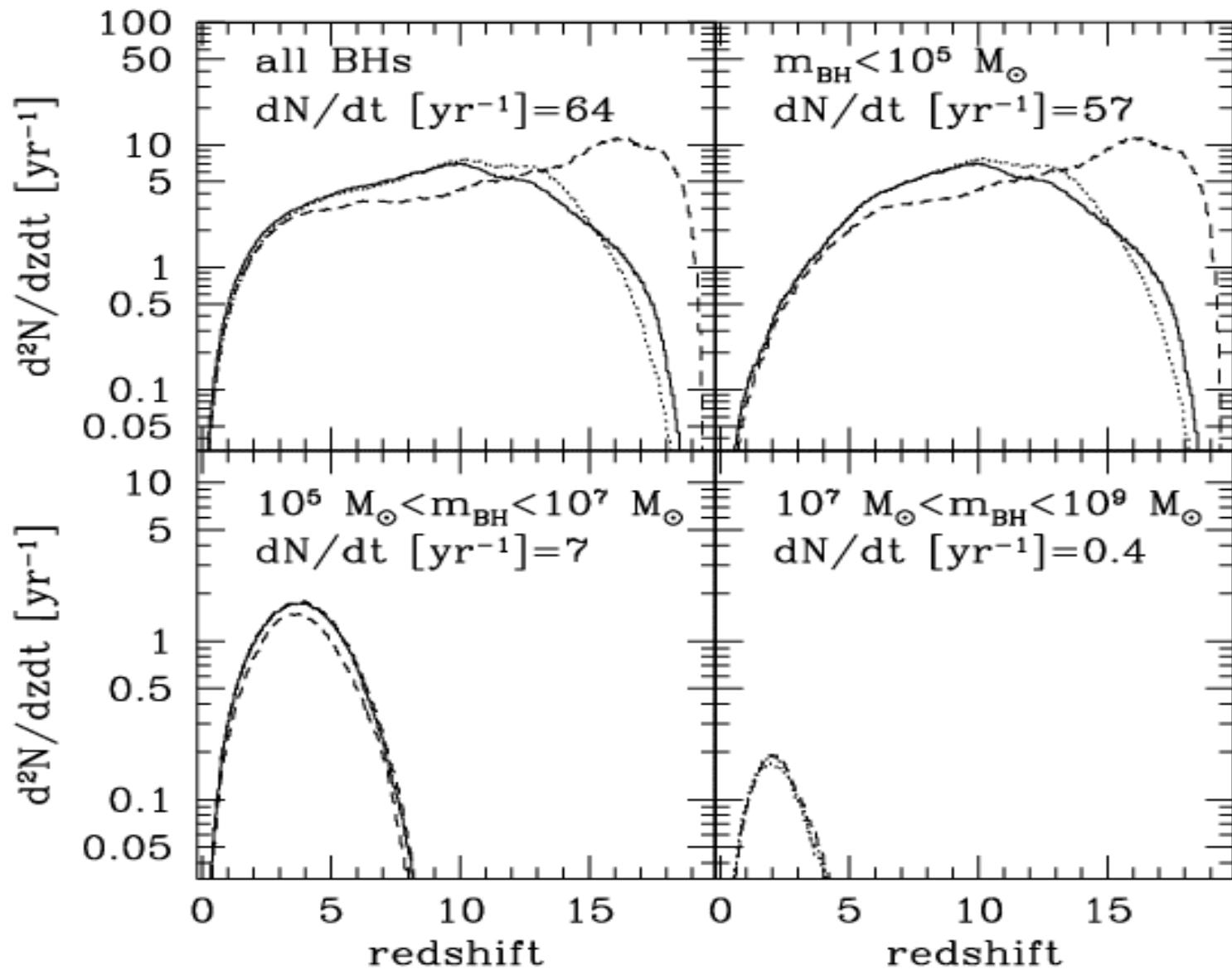


*...converting the accretion rate into luminosity ...*

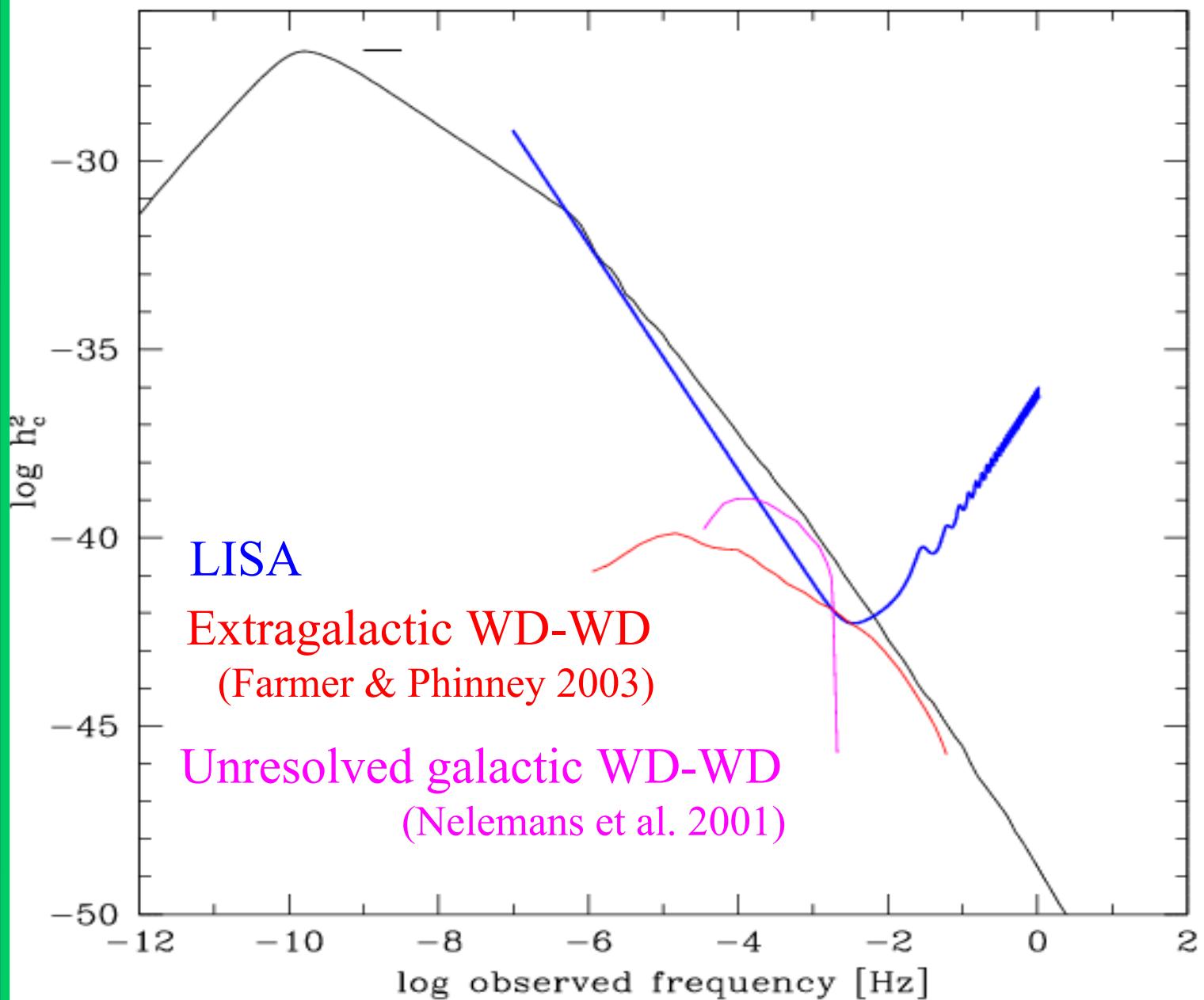
# EVOLUTION OF MBH DENSITY FUNCTION



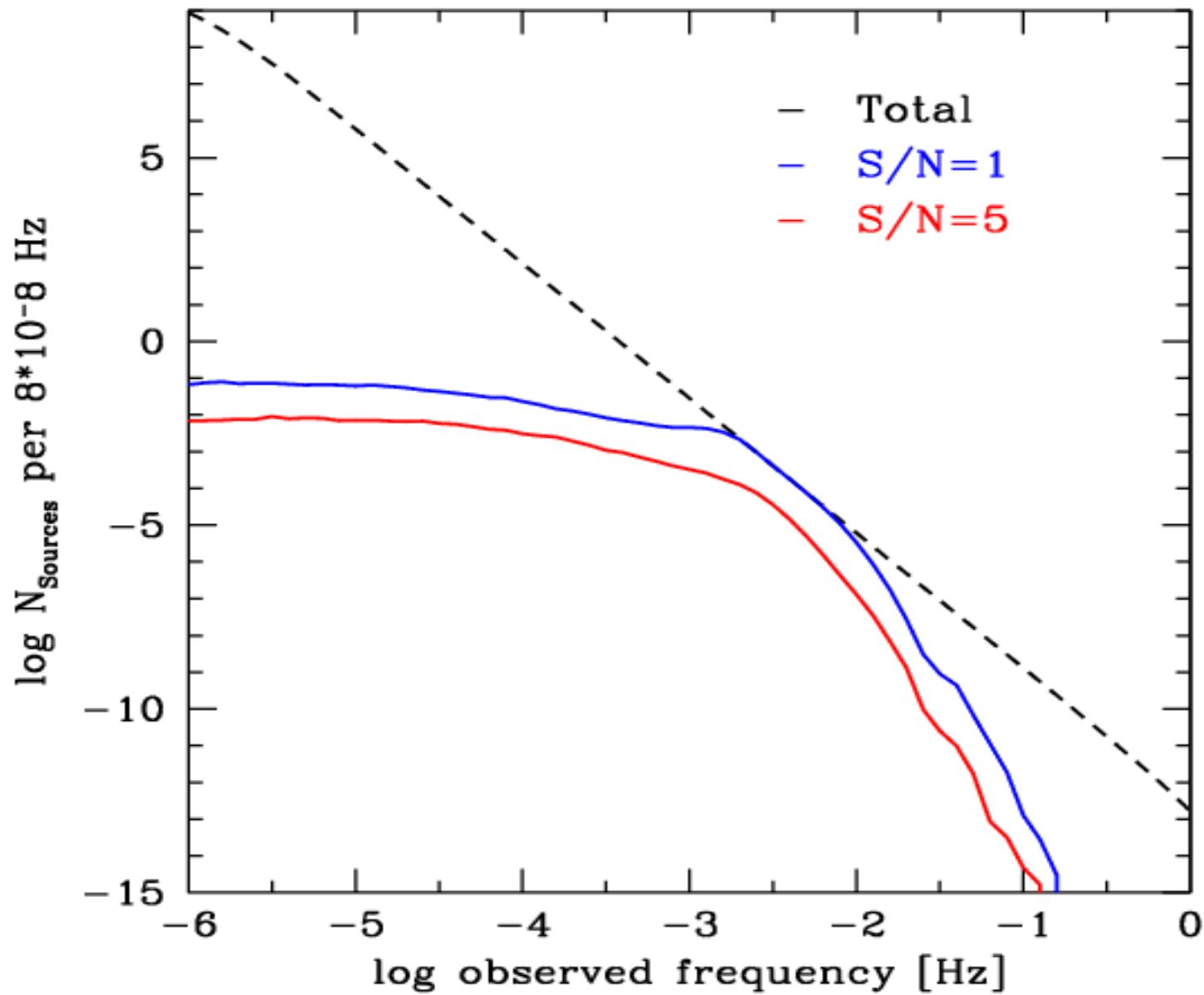
# COALESCENCE RATE



# GW BACKGROUND



# RESOLUTION OF SIGNAL



# CHARACTERISTIC STRAIN $h_c$ FOR SINGLE EVENTS

$f$ ) is roughly  $h\sqrt{n}$ , where  $n$  is the number of cycles spent by the source in a frequency interval  $\sim f$ . (e.g. Thorne 1995, Hughes et al. 2001)

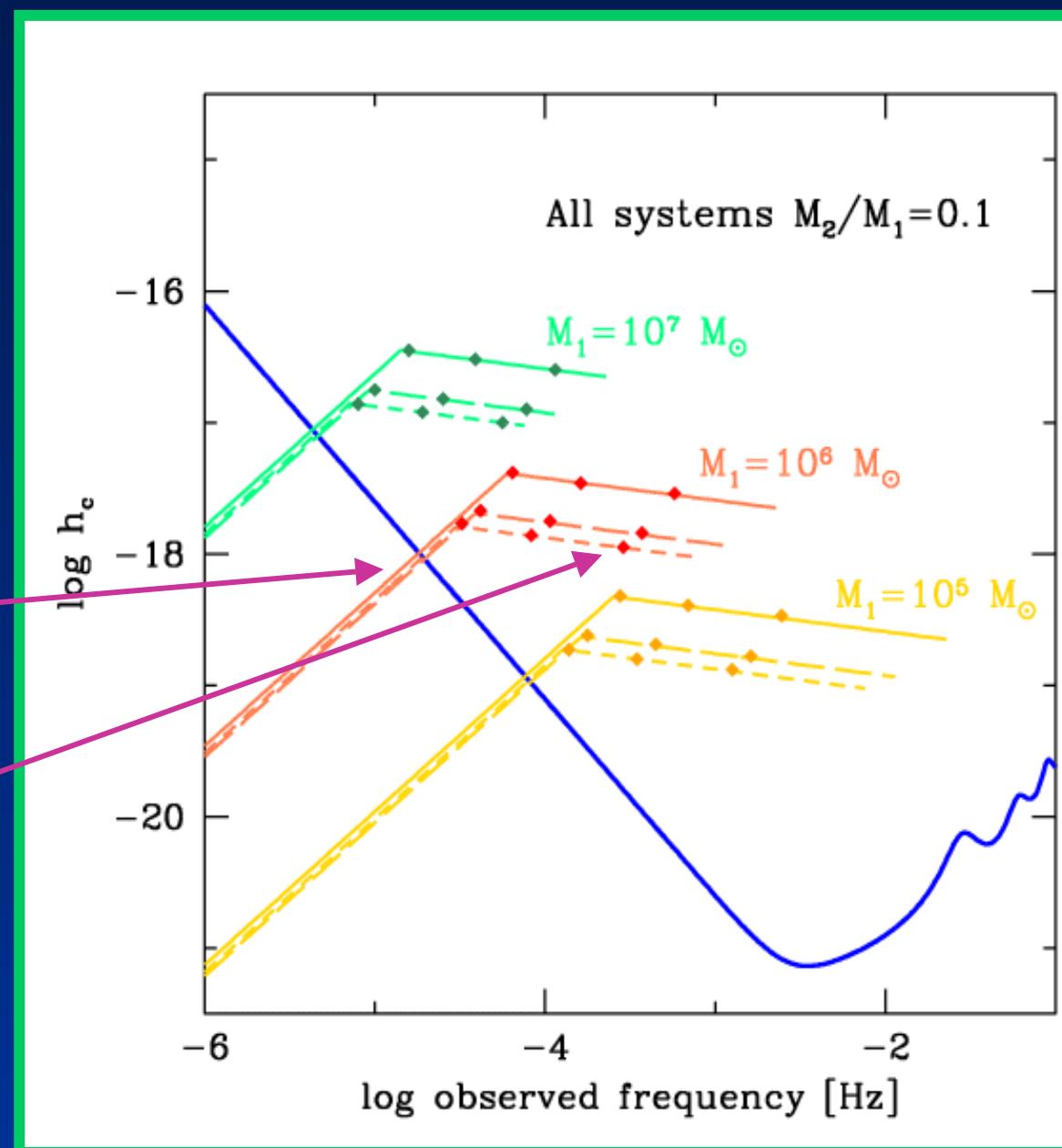
For an observation of duration  $T$ :

$$f n < f T \rightarrow h_c \propto f^{7/6}$$

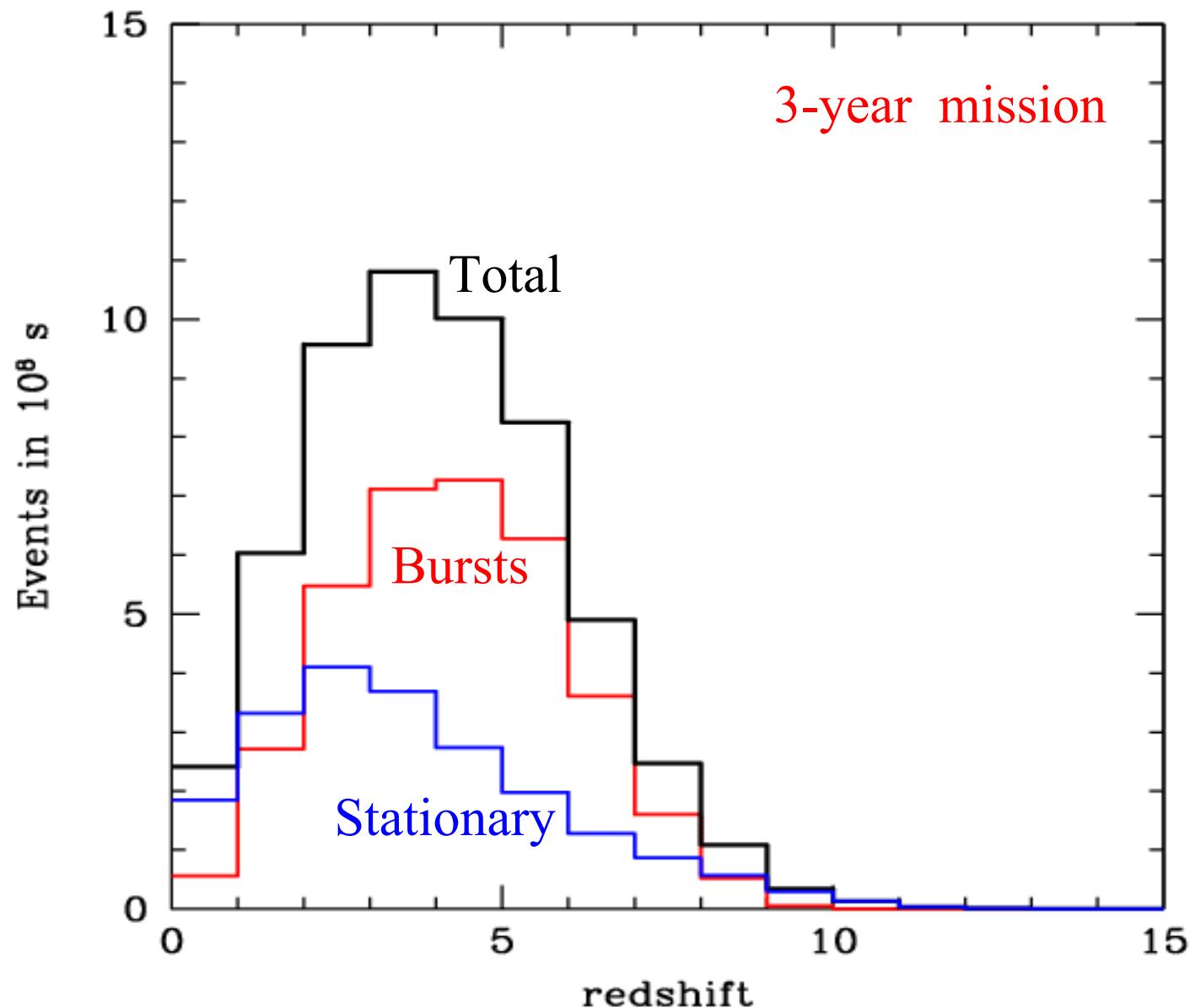
Stationary Source

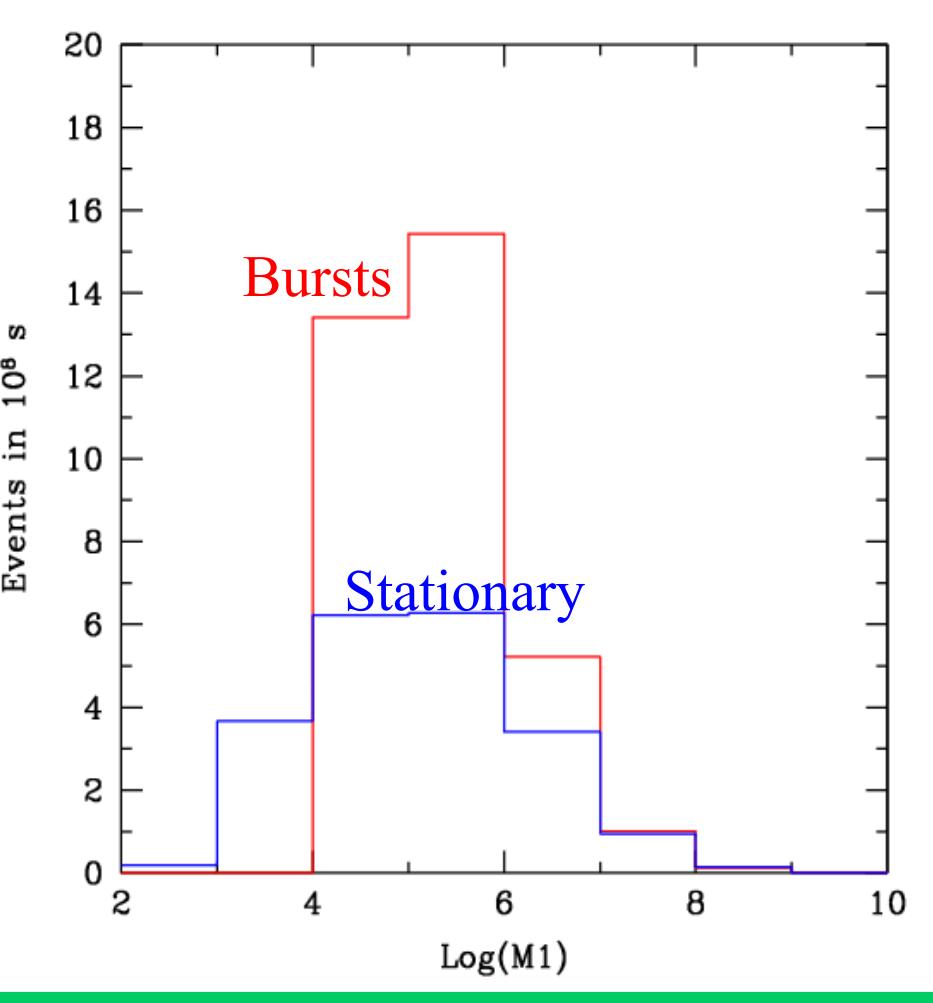
$$f n > f T \rightarrow h_c \propto f^{-1/6}$$

Shifting Source (Burst)

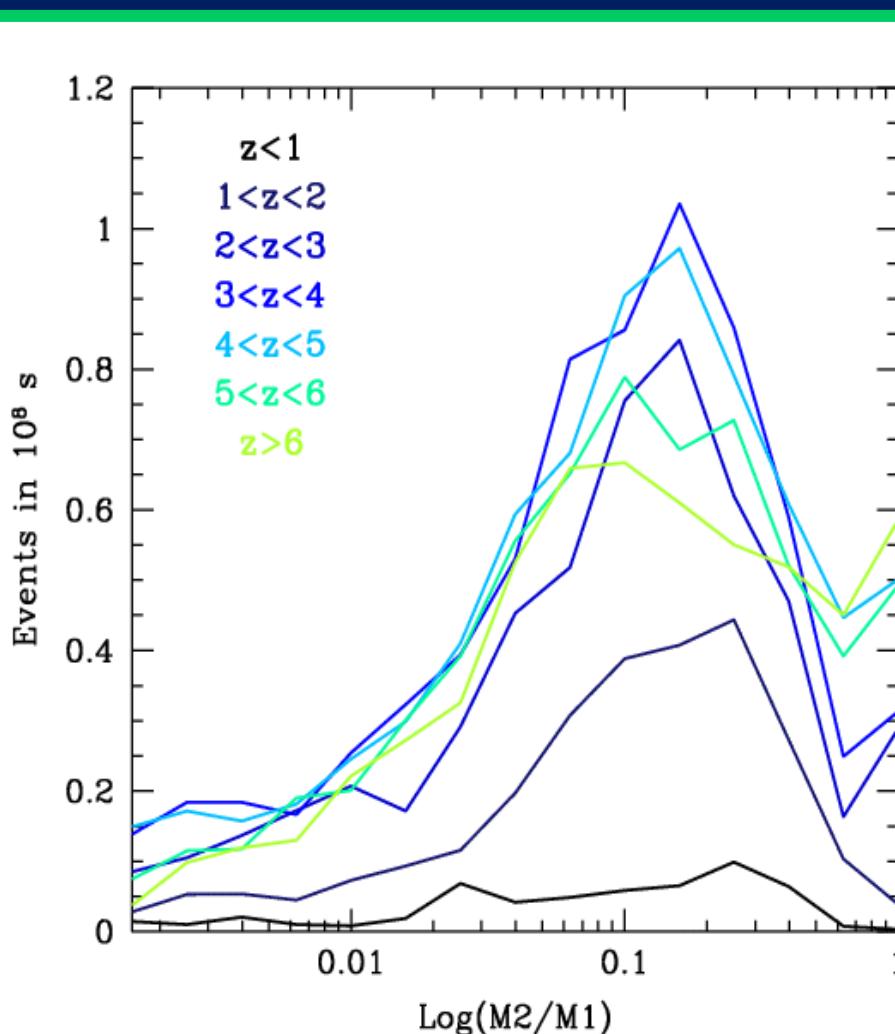


# SOURCE NUMBER COUNTS



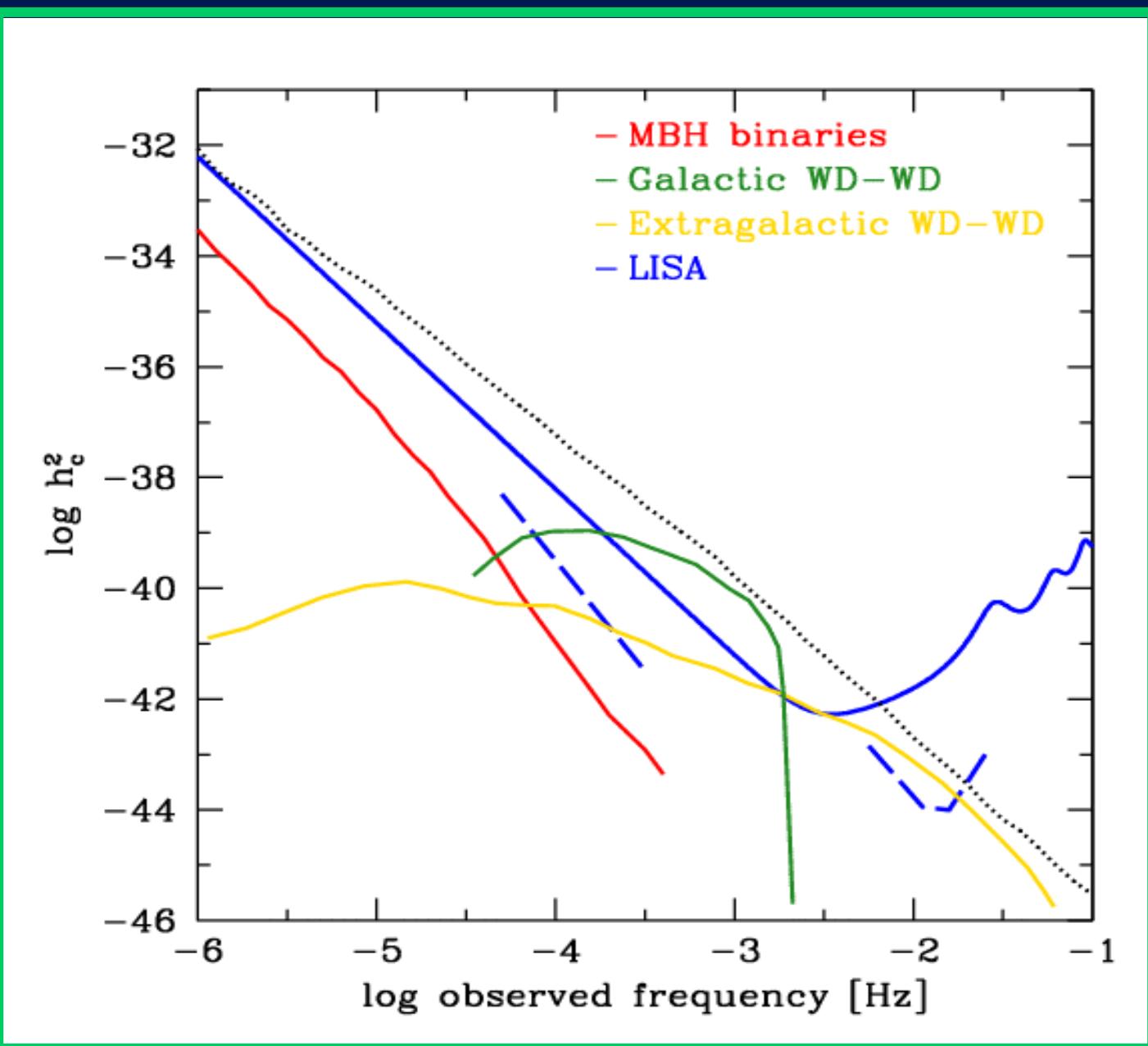


Mass function of detected events.  
 observable binary systems  
 have masses in the range  
 $10^4 < M_1 < 10^7 M_\odot$



Mass ratio of bursts for different redshift intervals: at low-ish  $z$  the typical mass ratio is  $M_2/M_1 \sim 0.1$

# UNRESOLVED SIGNAL: MBH CONFUSION NOISE



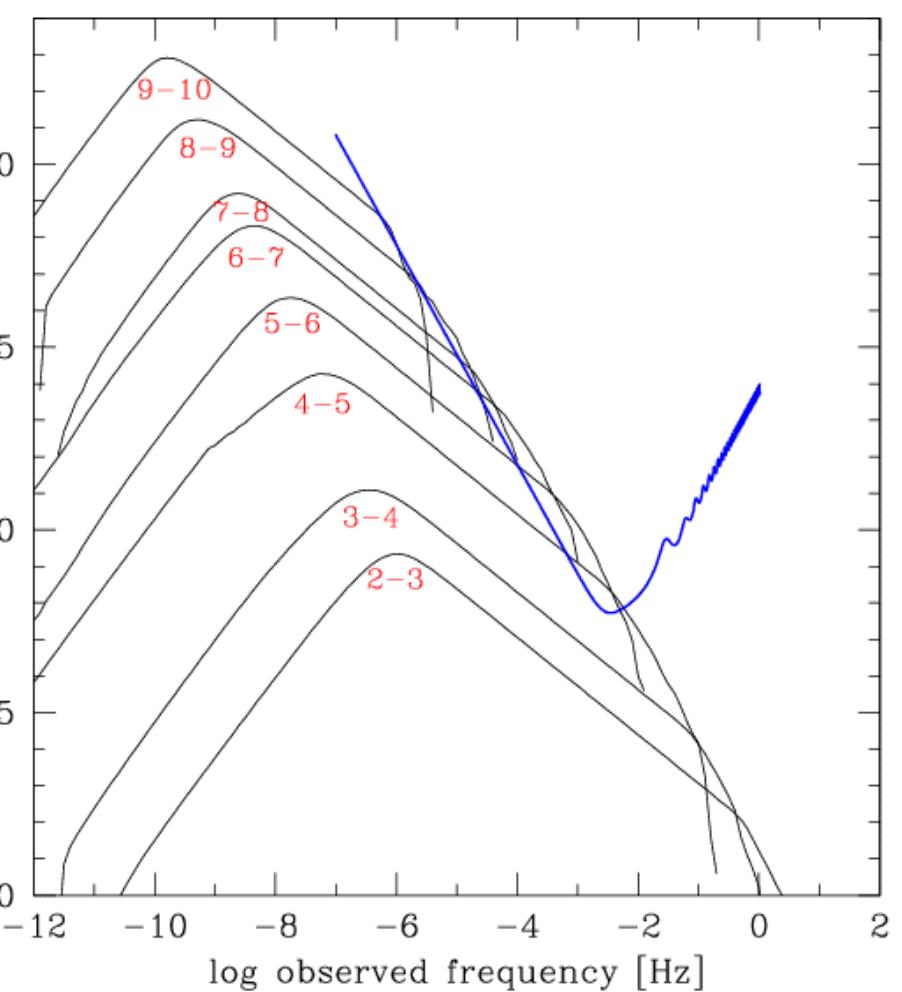
Confusion noise due to MBH-MBH binaries is well below LISA threshold

## SUMMARY

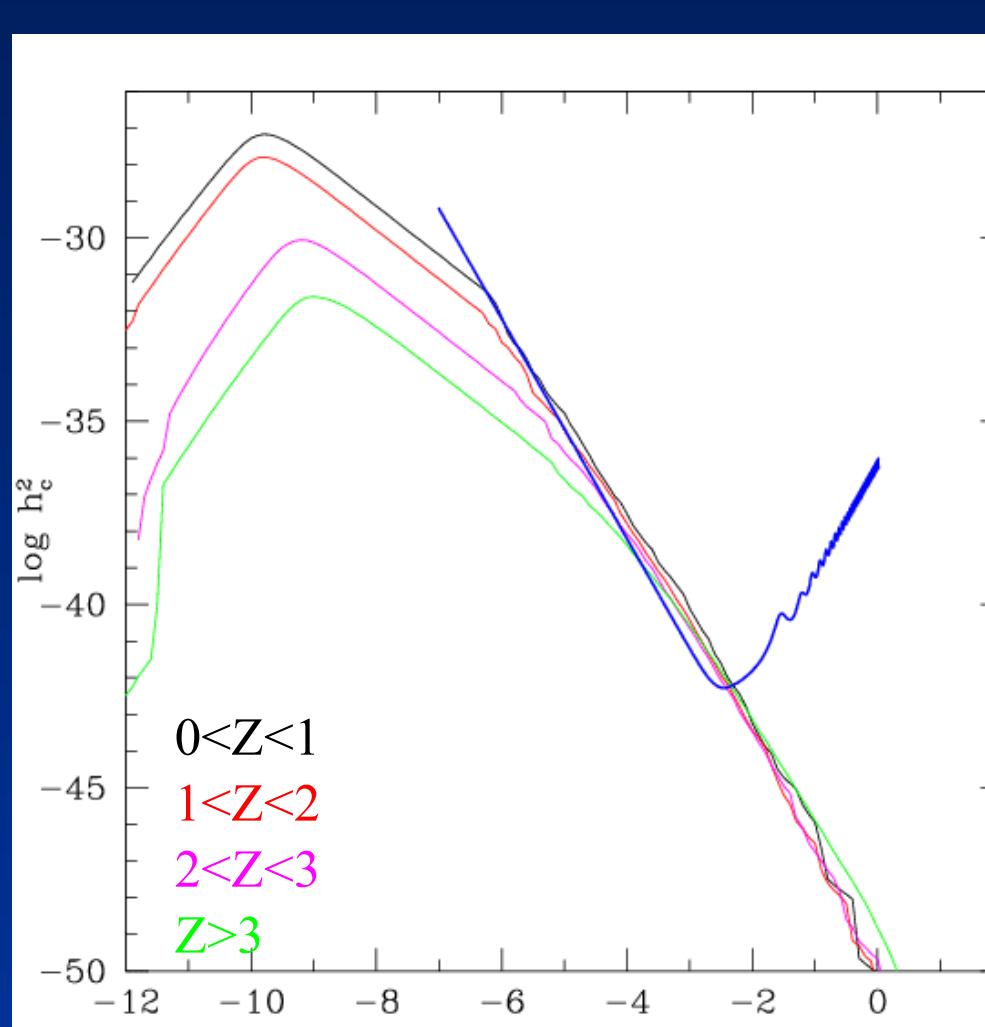
Starting from  $150 M_{\odot}$  seed BHs forming at  $z \sim 20$

- 1) The evolution of SMBHs and host galaxies can be followed in a semi-analytic way through merger trees
- 2) Modelling the accretion history and BH dynamics we can satisfy observational constraints (LF, etc.)
- 3) We expect a **low-frequency GW background**
- 4) LISA would be able to resolve the GWB into single sources: **~20 stationary events; ~35 bursts** with  $S/N > 5$  in a 3-year mission
- 5) For detected sources we expect:  **$10^4 < M_1 < 10^7 M_{\odot}$ ;  $M_2/M_1 \sim 0.1$**
- 6) Subtracting resolvable sources, the resulting **confusion noise is well below LISA sensitivity threshold**





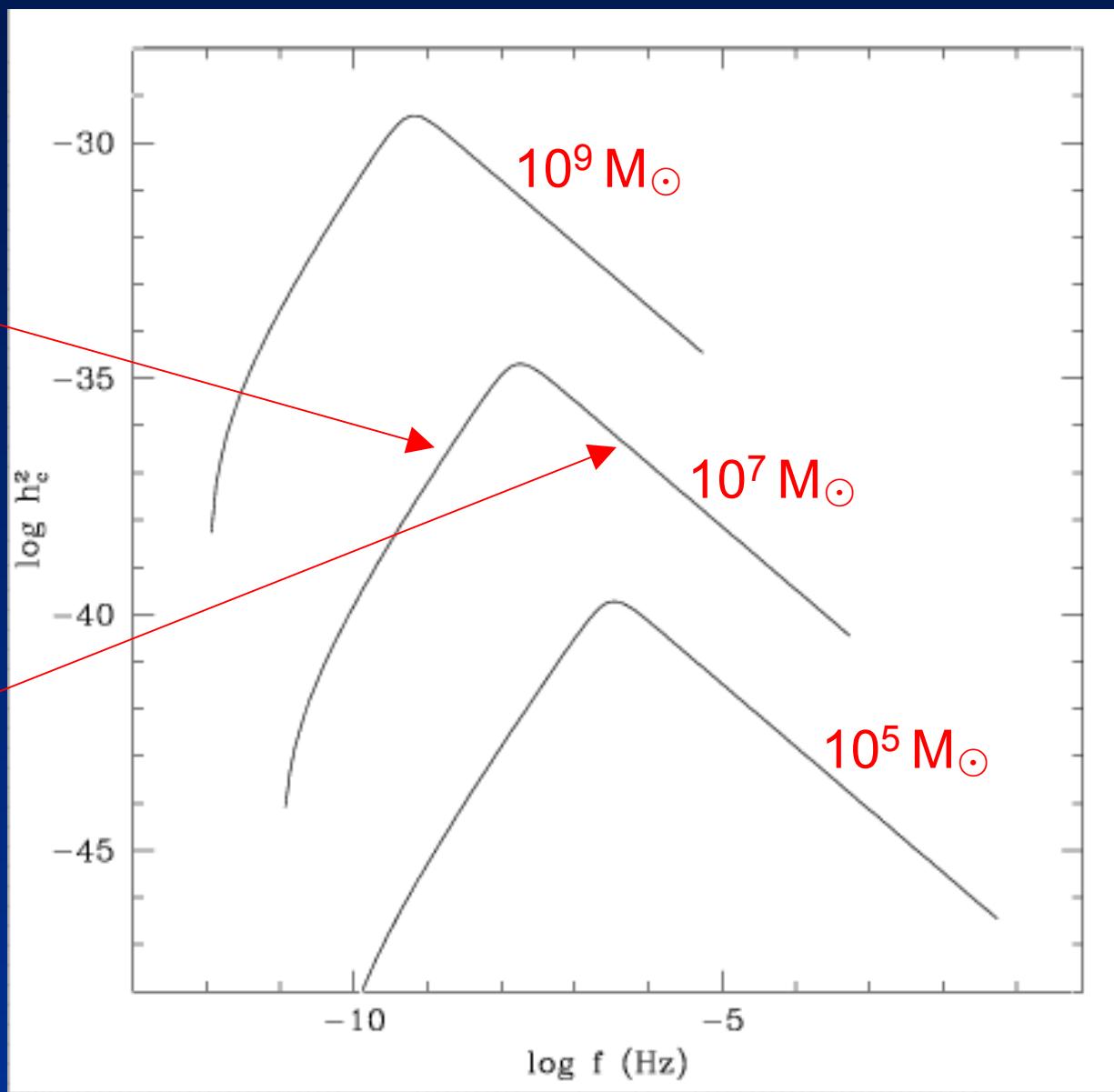
*Mass contribution*

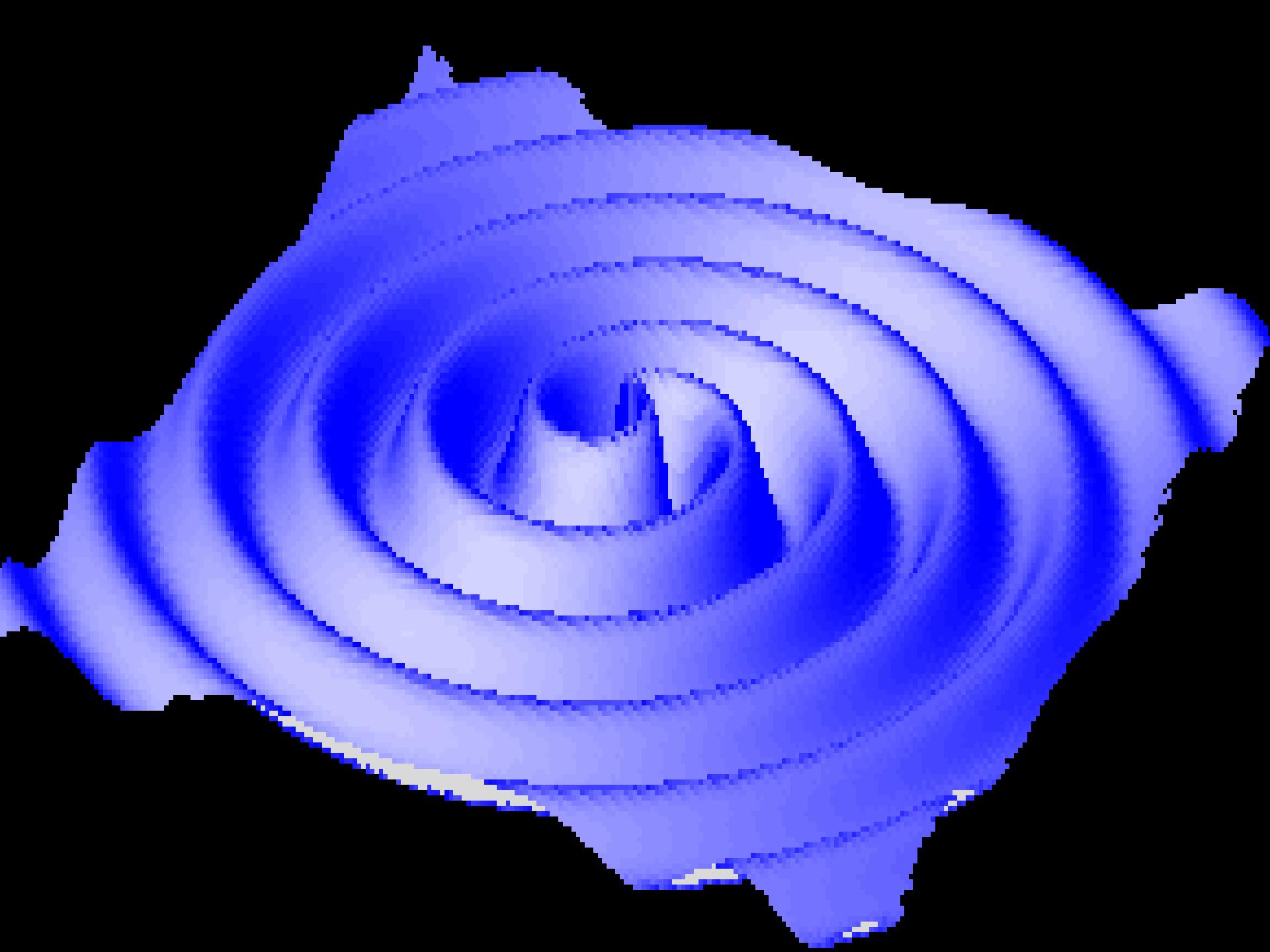


# GW STRAIN SPECTRUM FROM BINARY SYSTEMS

$$h_c^2(f)_{sl} \propto f^2$$

$$h_c^2(f)_{gw} \propto f^{-4/3}$$



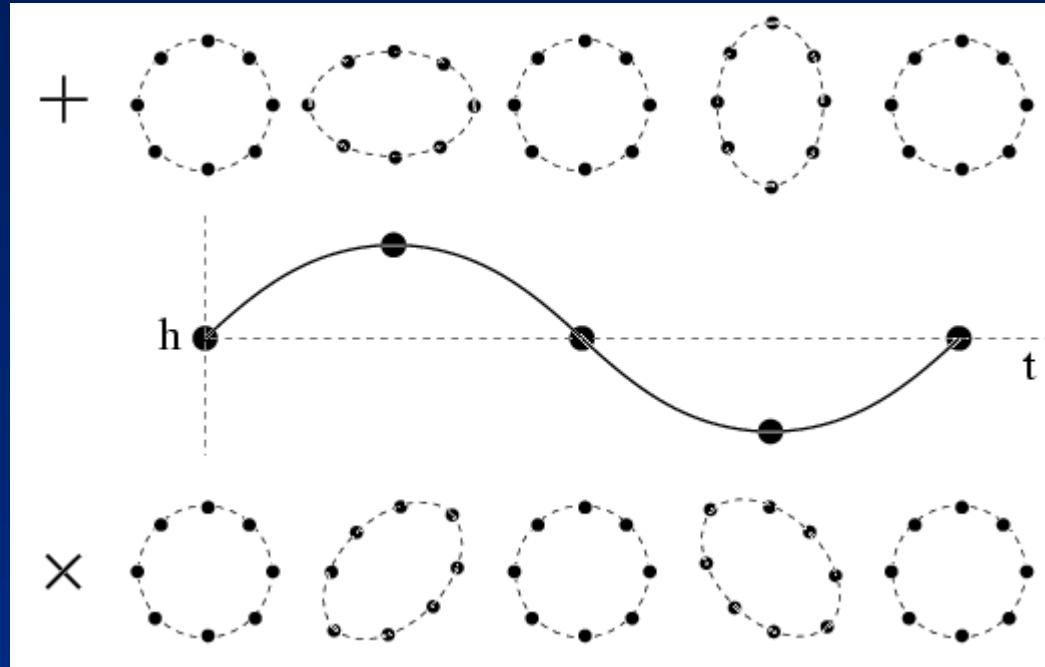


# LINEARIZED THEORY OF GW: THE TT GAUGE

Perturbation perpendicular to wave propagation direction

The metric becomes

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + h_+^{TT} & h_\times^{TT} \\ 0 & 0 & h_\times^{TT} & 1 - h_+^{TT} \end{pmatrix}$$



Luminosity of the source

$$L_{gw} = \frac{G}{5c^5} \left\langle \sum_{ij} \frac{d^3}{dt^3} Q_{ij} \left( t - \frac{x}{c} \right) \frac{d^3}{dt^3} Q^{ij} \left( t - \frac{x}{c} \right) \right\rangle$$

# CHARACTERISTIC STRAIN $h_c$ FOR SINGLE EVENTS

$f$ ) is roughly  $h\sqrt{n}$ , where n is the number of cycles spent by the source in a frequency interval  $\sim f$ . (e.g. Thorne 1987, Hughes et al. 2001)

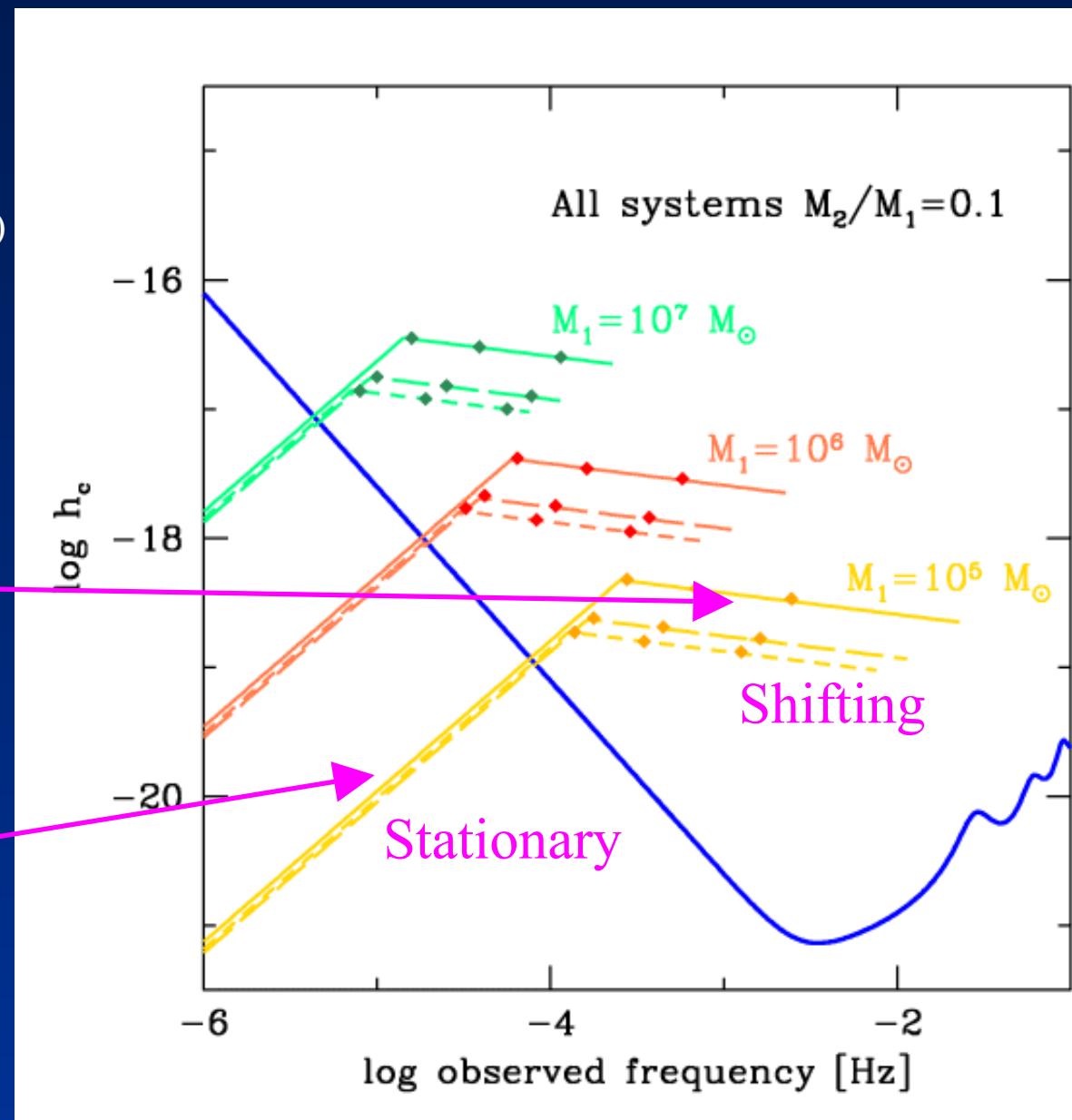
or an observation time  $\kappa$ :

$$\text{if } n < f\kappa \quad n \approx f^2/f \quad \rightarrow \quad h_c \propto f^{-1/6}$$

(shifting phase)

$$\text{if } n > f\kappa \text{ we set} \quad n = f\kappa \quad \rightarrow \quad h_c \propto f^{7/6}$$

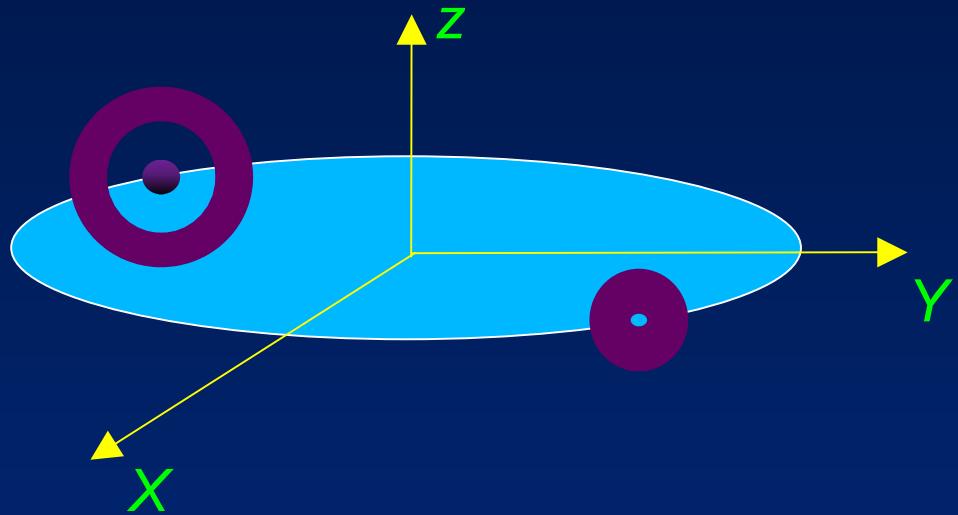
('stationary' phase)



# GW FROM SMBHs BINARIES

Energy spectrum

$$\frac{dE_{gw}}{df} f = \frac{dE_{gw}}{dt} \frac{dt}{dr} \frac{dr}{df} f.$$



Where

$$L_{gw} = \frac{32 G^{7/3} \pi^{10/3}}{5 c^5} (\mathcal{M} f)^{10/3}$$

$$\frac{dr}{df} f = -\frac{2}{3} \left[ \frac{G (M_1 + M_2)}{\pi^2} \right]^{1/3} f^{-2/3}$$

$dt/dr$  is determined by the dominant hardening process:

- ejection of background stars in the early stage
- GW emission in the late stage

# MODELLO COSMOLOGICO GERARCHICO

Applicazione della CR alla cosmologia

Big Bang

Fluttuazioni quantistiche durante l'inflazione producono perturbazioni di densità con varianza

$$\sigma_R^2(t) = \frac{\langle \delta M^2(R, t) \rangle}{\langle M(R, t) \rangle^2} \propto M^{-2/3}(t)$$

Sviluppo lineare indipendente dalla scala di massa secondo il fattore di crescita

$$D_c(t) \propto t^{2/3}$$

Le prime perturbazioni ad entrare in regime non lineare e di conseguenza a collassare sono quelle più piccole.

Quali sono le piu` piccole strutture che possono collassare?

Meccanismo di *free streaming*

CDM:  $M_{\min} \square 10^6 M_{\odot}$ :  
scenario *bottom-up*

HDM:  $M_{\min} \square 10^{14} M_{\odot}$ :  
scenario *top-down*

## CLUSTERING GERARCHICO

Formalismo Press & Schechter per la ricostruzione  
dell'assemblamento degli aloni

$$F(R, z) = \int_{\delta_c}^{\infty} \frac{d\delta}{\sqrt{2\pi} D(z) \sigma_R(z)} \exp \left[ -\frac{\delta^2}{2D^2(z) \sigma_R^2(z)} \right]$$

$$\frac{dN}{dM}(M, z = z_0) = \frac{1}{\sqrt{2\pi} S^{3/2}} \frac{M_0}{M} \frac{d\delta_c}{dz} \frac{d\sigma_M^2}{dM} dz$$

→ MERGER TREES

# EVIDENZE DELLA PRESENZA DI SMBH NELLE GALASSIE

## MISURE Sperimentali

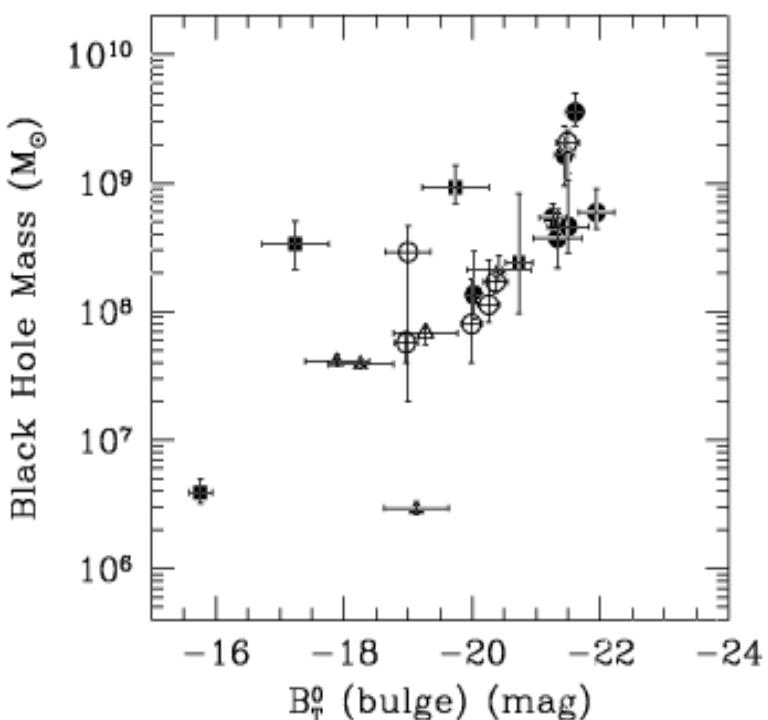
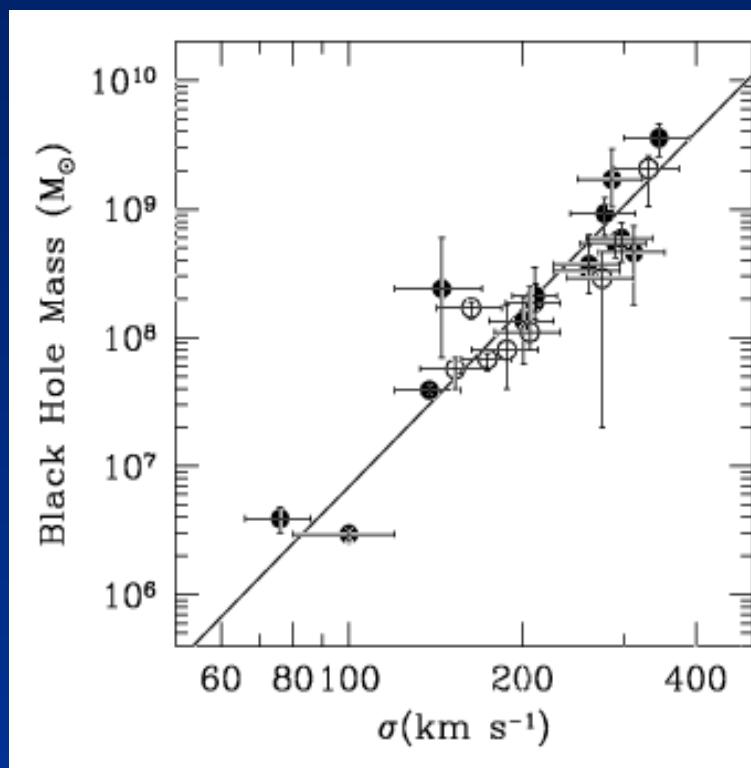
- misure di dinamica stellare
- misure dinamiche di massa
- misure dinamiche di gas
- reverberation mapping



## RELAZIONI TRA SMBH E GALASSIE

$$m_{BH} = (4.4 \pm 0.9) \times 10^7 M_{\odot} \sigma_{c,150}^{4.58 \pm 0.52}$$

$$m_{BH} = 0.78 \times 10^8 M_{\odot} \left( \frac{L_{B,bulge}}{10^{10} L_{\odot}} \right)^{1.08}$$



## **EVOLUZIONE DEI SMBH**

### **FORMAZIONE DEI PRIMI BH**

Aloni con  $M \square 10^6\text{-}10^7 M_{\odot}$  a  $Z \square 30$



Cooling H<sub>2</sub>



Formazione di VMS



Collasso con formazione di BH di massa intermedia ( $M \square 150 M_{\odot}$ )

### **MERGING TRA DUE ALONI CHE OSPITANO BH**



Attività di Quasar



Formazione di un sistema binario di BH

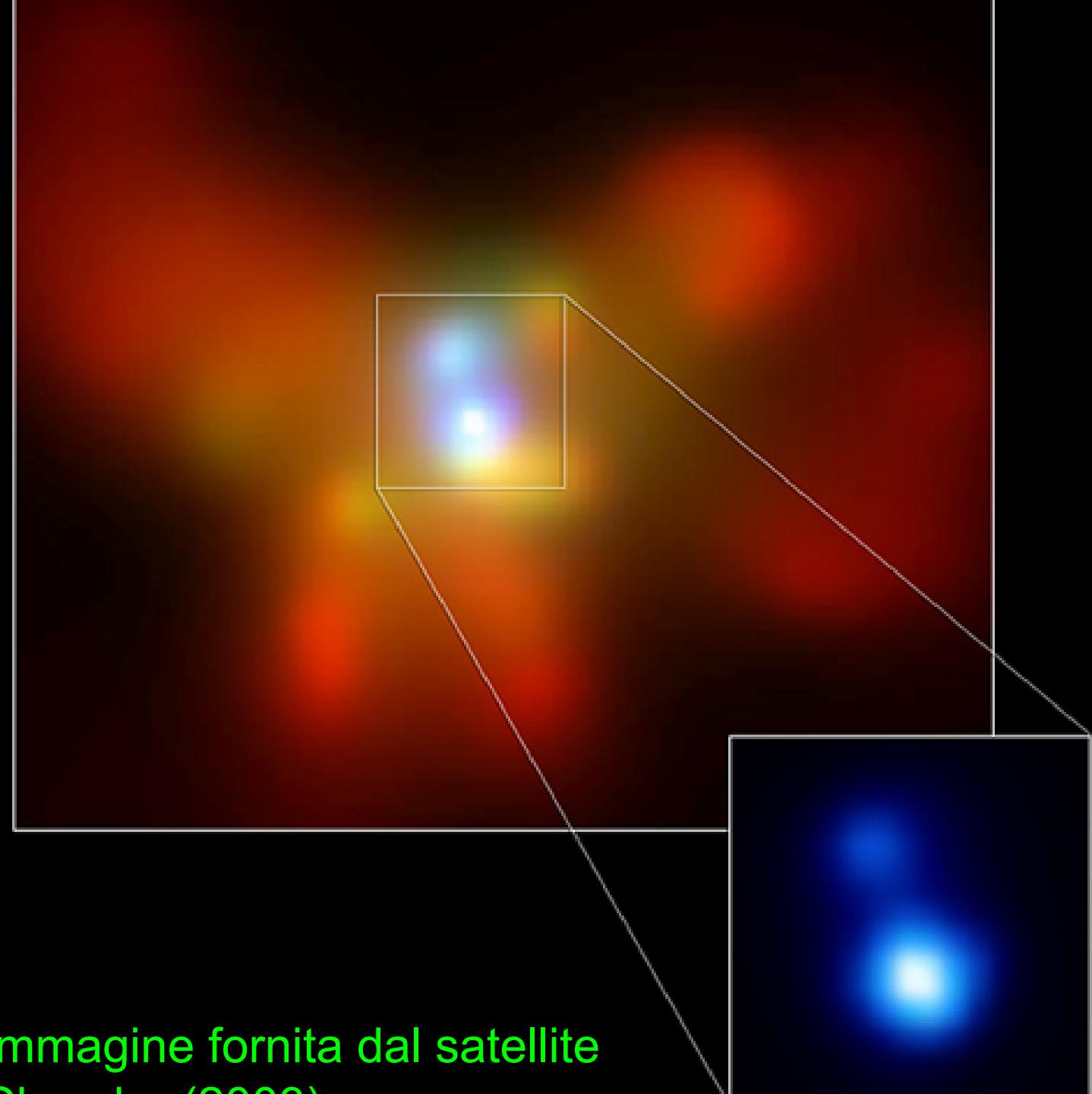


Immagine fornita dal satellite  
Chandra (2002)

# **DINAMICA DI UN SISTEMA BINARIO DI BH**

Si distinguono tre fasi evolutive successive coincidenti con tre diversi meccanismi di restringimento dell'orbita:

1) Frizione dinamica → non si puo` ancora parlare  
propriamente di sistema binario

2) Hardening → Slingshot  
incertezze: - eccentricita` dell'orbita  
- accrescimento di gas  
- *wandering* della binaria

→ fattore moltiplicativo per  $t_H$

3) Emissione di GW coalescenza in tempi brevi

Altri fattori di complicazione: 1) Interazioni triple  
2) Gravitational rocket

# TEORIA LINEARE DELLE GW

Metrica debolmente perturbata:  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$  con  $|h_{\mu\nu}| \ll 1$



equazioni di Einstein linearizzate per la perturbazione  $h_{\mu\nu}$

$$\square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

E` un'equazione d'onda!

Approssimazione di *slow motion* :  $v_s \ll c$



Soluzione in approssimazione di quadrupolo :

$$\begin{cases} \bar{h}^{\mu 0}(t, r) = 0 \\ \bar{h}^{ij}(t, r) = \frac{2G}{c^4 r} \left[ \frac{d^2}{dt^2} q^{ij} \left( t - \frac{r}{c} \right) \right] \end{cases}$$

- N.B. - La soluzione e` un'onda che propaga alla velocita` della luce  
- Sono coinvolte solo le componenti spaziali della metrica  
- La perturbazione e` strettamente legata al momento di

# CALCOLARE UN FONDO COSMICO DI GW

Si definisce l'ampiezza caratteristica della perturbazione  $h_c$

$$\langle h_{ij}(t) h^{ij}(t) \rangle = 2 \int_0^\infty d(\log f) h_c^2(f)$$

Spettro di potenza

$$h_c^2(f) = 2 f S_h(f)$$

Legame diretto con



Densita` di  
energia

$$\frac{d\rho_{gw} c^2}{d \log f} = \frac{\pi}{4 G} f^2 h_c^2(f)$$

Calcolabile come :

$$\rho_{gw}(f) c^2 = \int_0^\infty dz \int_0^\infty dM_1 \int_0^{M_1} dM_2 \frac{1}{1+z_e} \frac{d^3 N}{dz dM_1 dM_2} \left( \frac{dE_{gw}}{df_r} f_r \right)$$

Considerando : - sorgenti isotropicamente distribuite  
- eventi di durata cosmologica

E necessario conoscere : - l'evoluzione temporale della funzione di massa  
- lo spettro di emissione della singola sorgente

# **DERIVAZIONE DELLO SPETTRO DI EMISSIONE**

$$\frac{dE_{gw}}{df} f = \frac{dE_{gw}}{dt} \frac{dt}{dr} \frac{dr}{df} f.$$

a) Frequenze limite di emissione

- Frequenza al raggio di hardening :

$$f_H = \frac{8 \sigma_\star^3}{\pi G} \frac{(M_1 + M_2)^{1/2}}{M_2^{3/2}}$$

- Frequenza all'ultima orbita stabile :

$$f_{uos} = \frac{c^3}{\pi (2\alpha)^{3/2}} \frac{(M_1 + M_2)^{1/2}}{M_1^{3/2}}$$

b) Termini differenziali che producono lo spettro

1) Rapporto tra separazione  
e frequenza emessa

$$\frac{dr}{df} f = -\frac{2}{3} \left[ \frac{G (M_1 + M_2)}{\pi^2} \right]^{1/3} f^{-2/3}$$

2) Luminosita` della sorgente

$$L_{gw} = \frac{32 G^{7/3} \pi^{10/3}}{5 c^5} (\mathcal{M} f)^{10/3}$$

## Evoluzione temporale dell'orbita

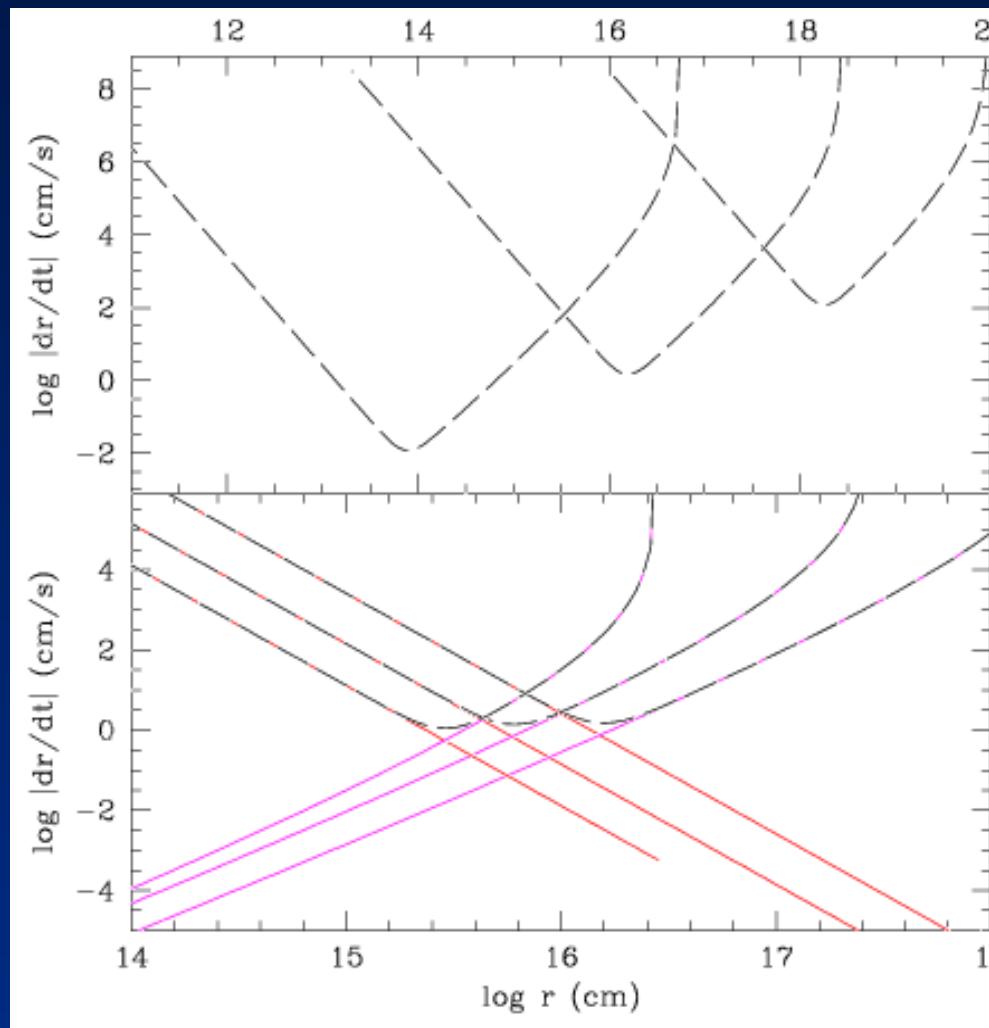
) Fase dominata da slingshot

$$\frac{dr}{dt} \Big|_{sl} = -\frac{8H\sigma_*^5}{9\pi G^2 (M_1 + M_2)^2} \frac{r^2}{\ln^2\left(\frac{r_h}{r}\right)}$$

) Fase dominata da emissione di GW

$$\frac{dr}{dt} \Big|_{gw} = -\frac{64}{5} \frac{G^3}{c^5 r^3} M_1 M_2 (M_1 + M_2)$$

Il termine che ci interessa lo possiamo scrivere come :

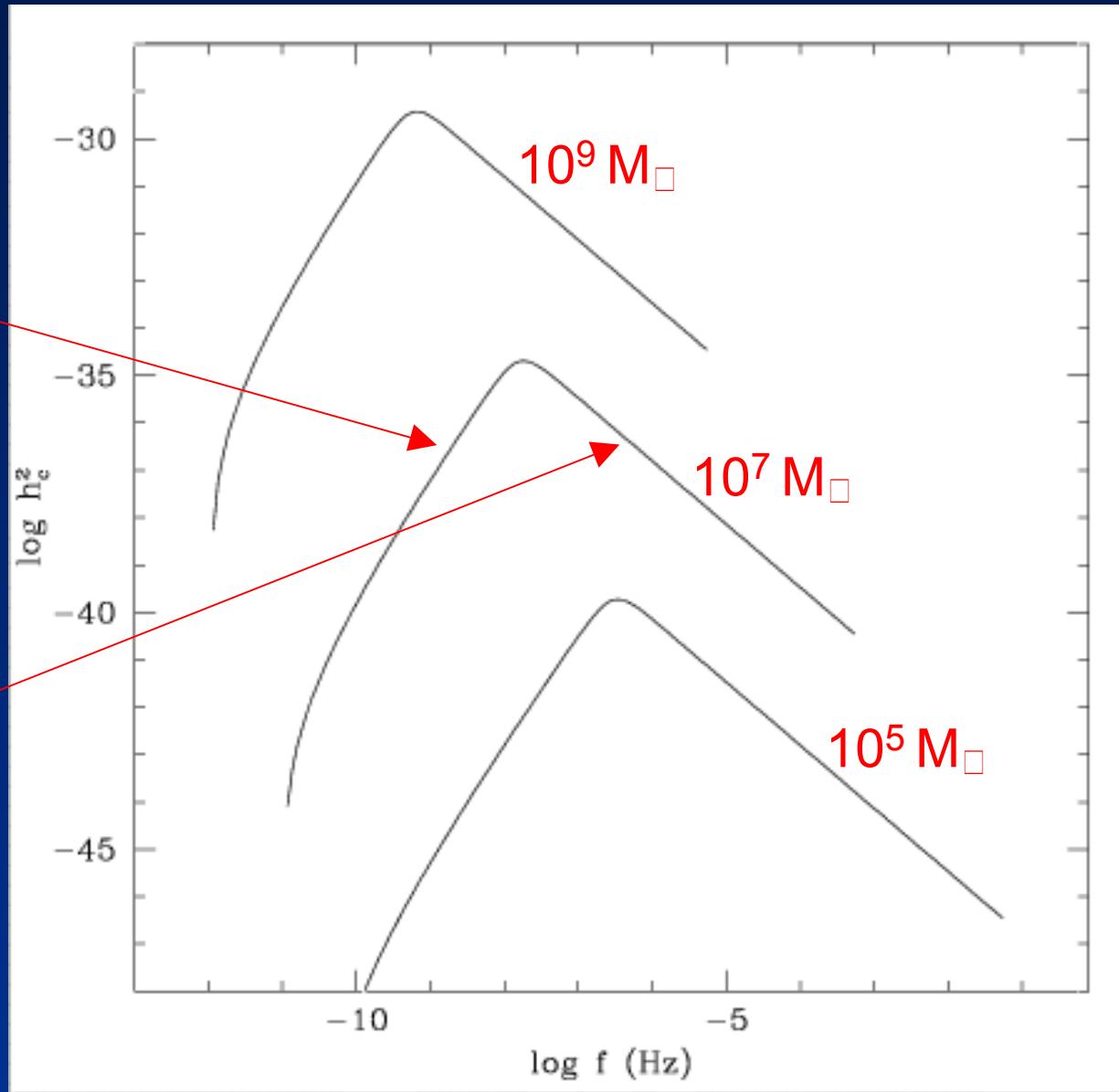


$$\frac{dt}{dr} = \frac{1}{\frac{dr}{dt} \Big|_{sl} + \frac{dr}{dt} \Big|_{gw}}$$

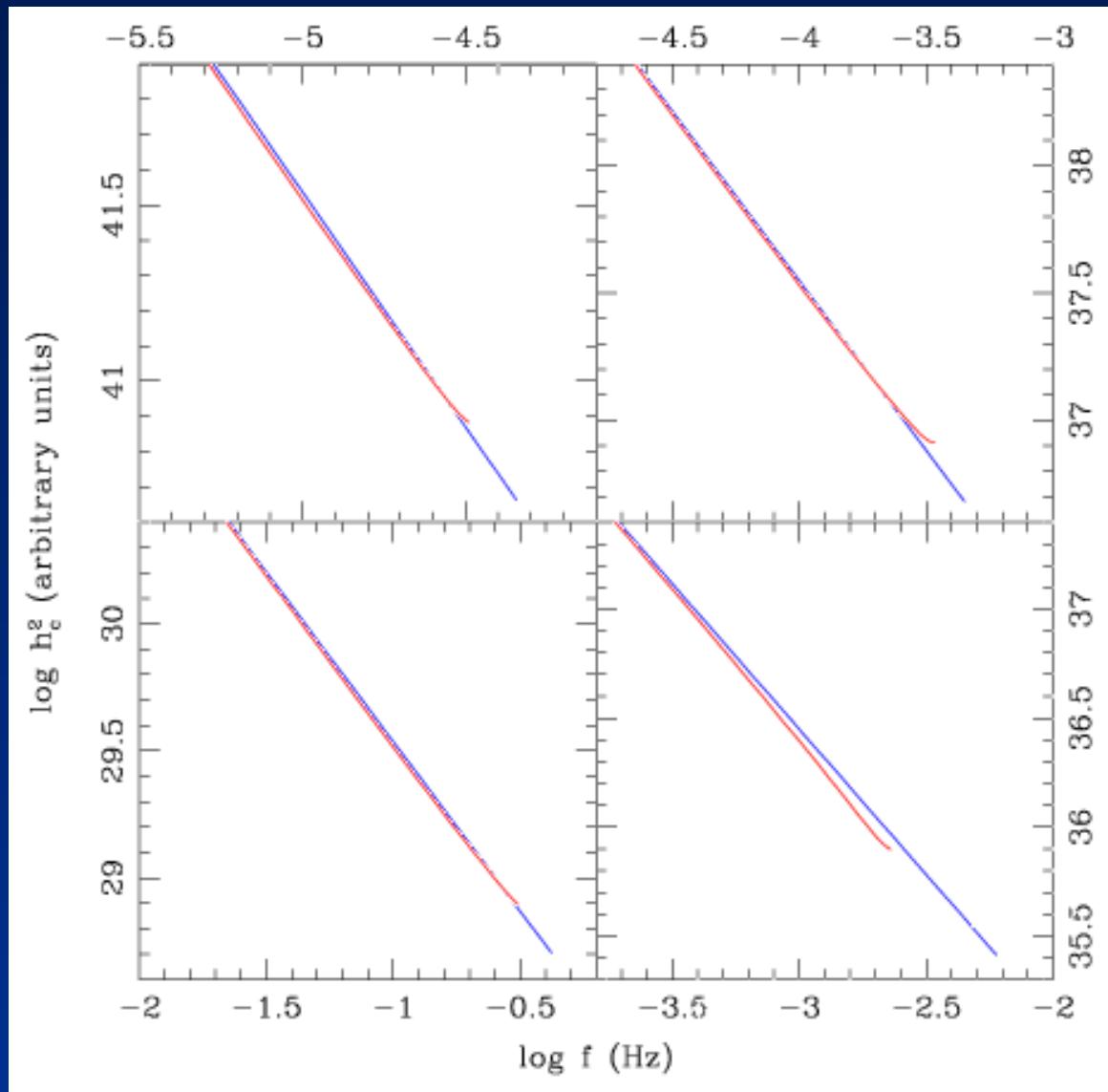
# ANDAMENTO DELLO SPETTRO DEL SINGOLO EVENTO

$$h_c^2(f)_{sl} \propto f^2$$

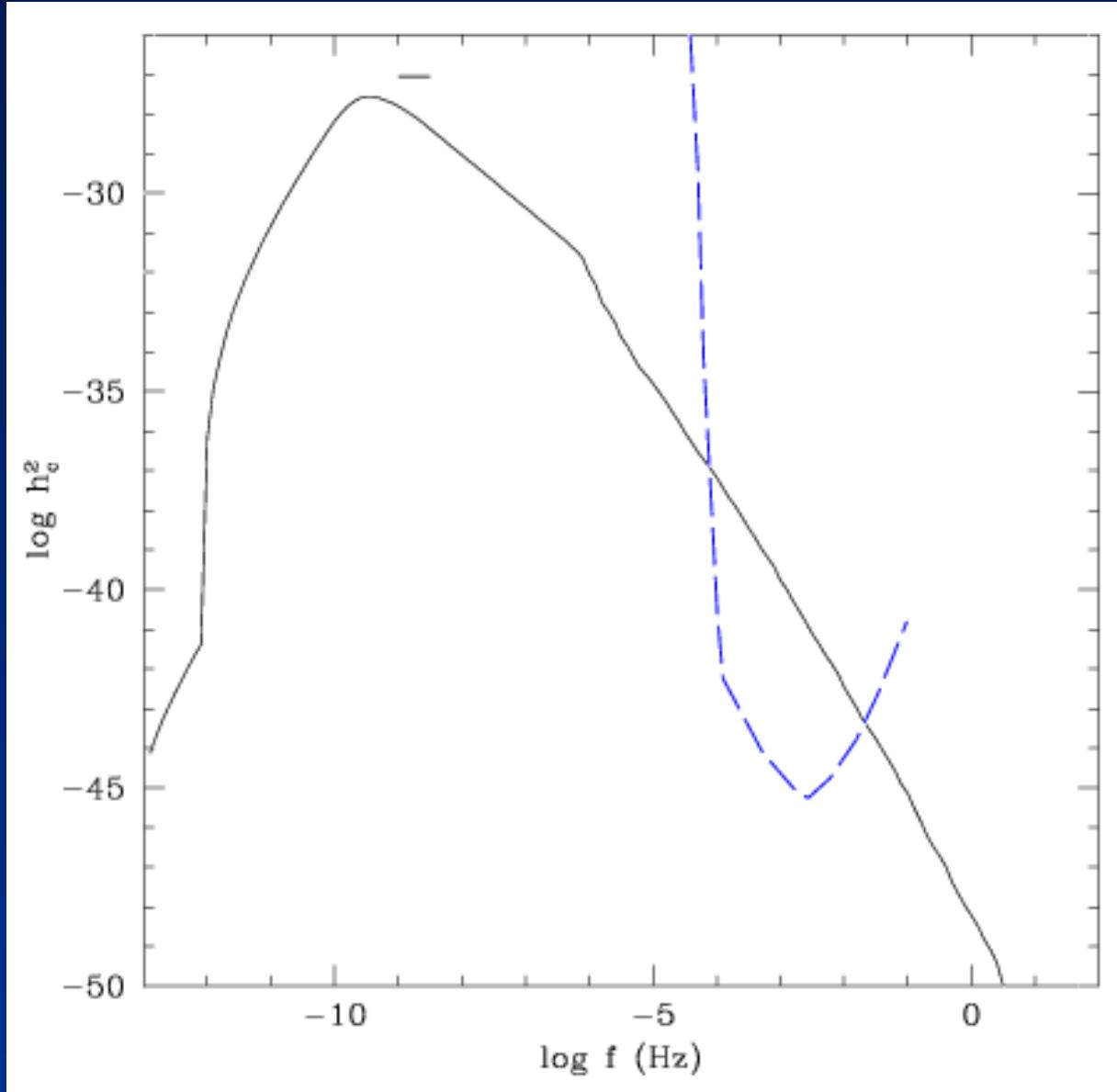
$$h_c^2(f)_{gw} \propto f^{-4/3}$$



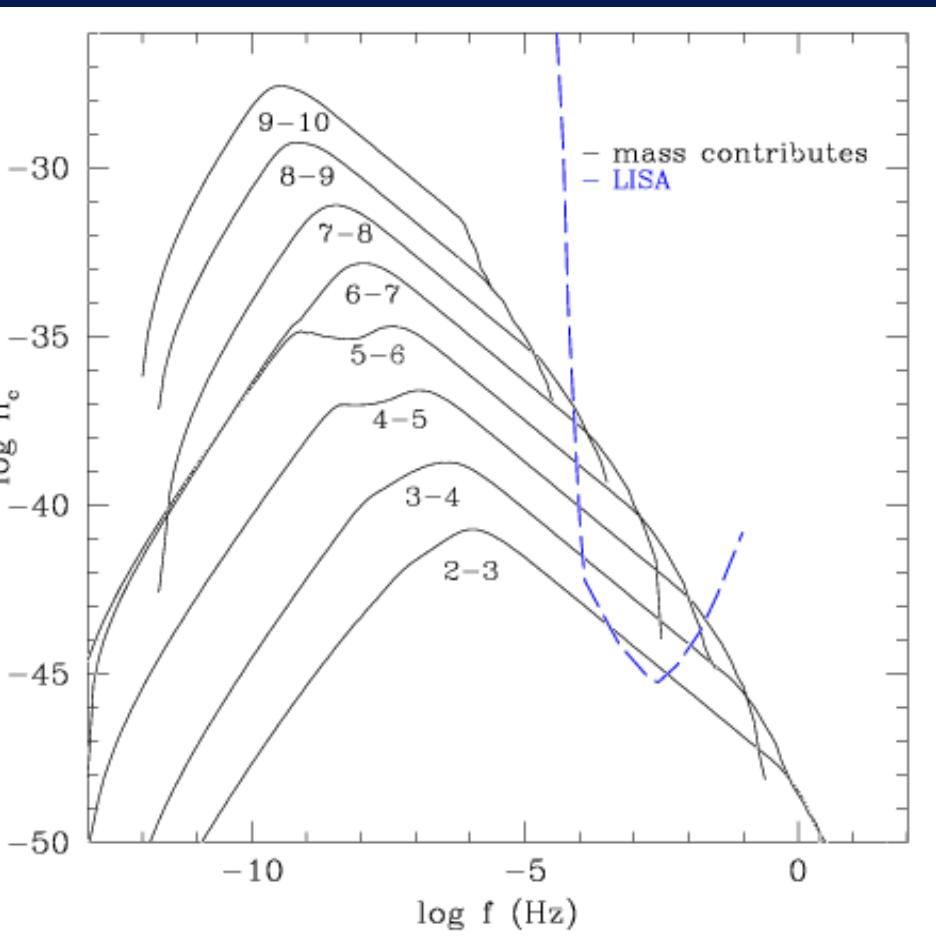
# CORREZIONI POST NEWTONIANE (ORDINE 2P.N.)



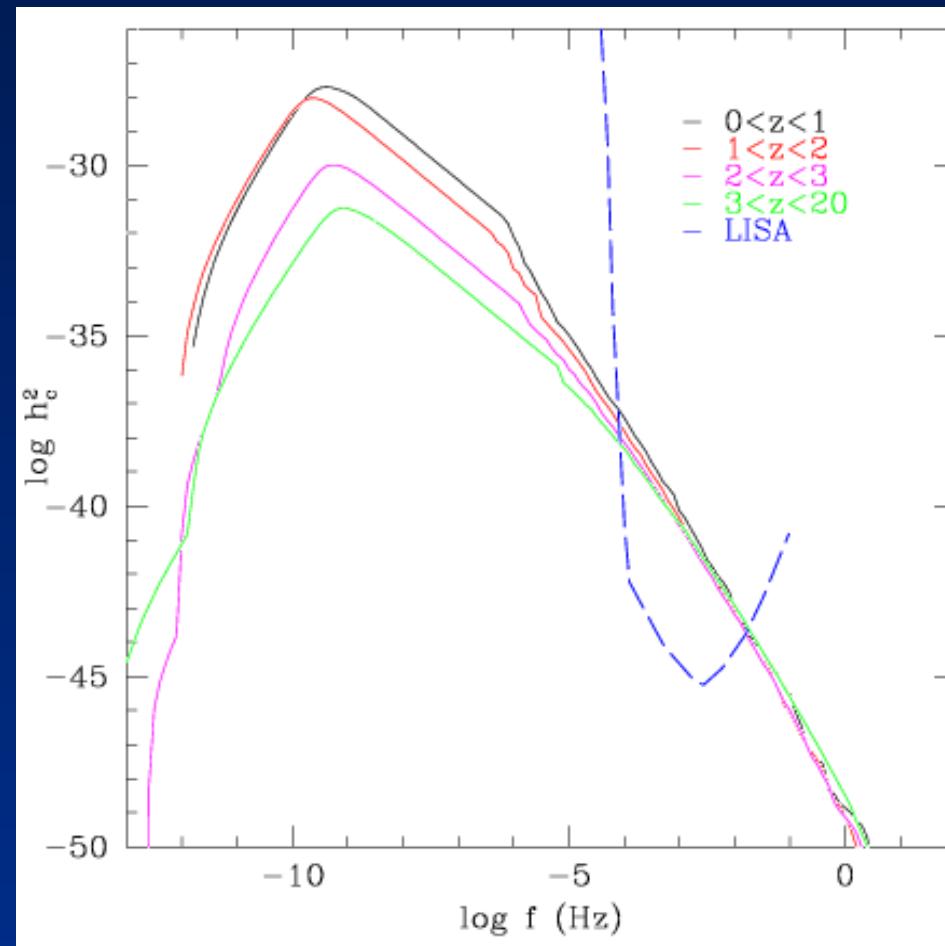
# IL FONDO COSMICO



# CONTRIBUTI IN INTERVALLI DI MASSA E REDSHIFT

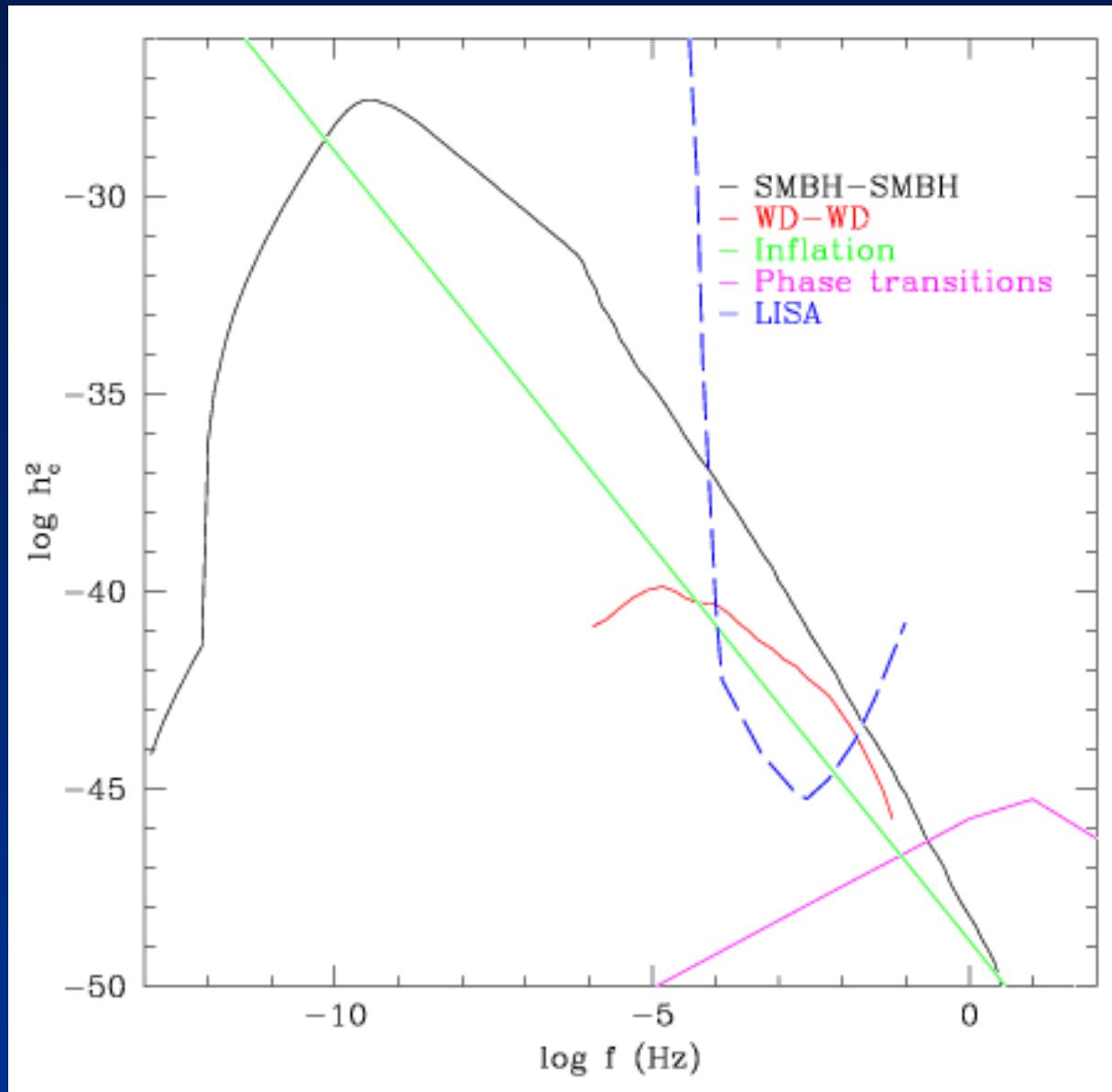


Contributi da diversi intervalli  
di massa

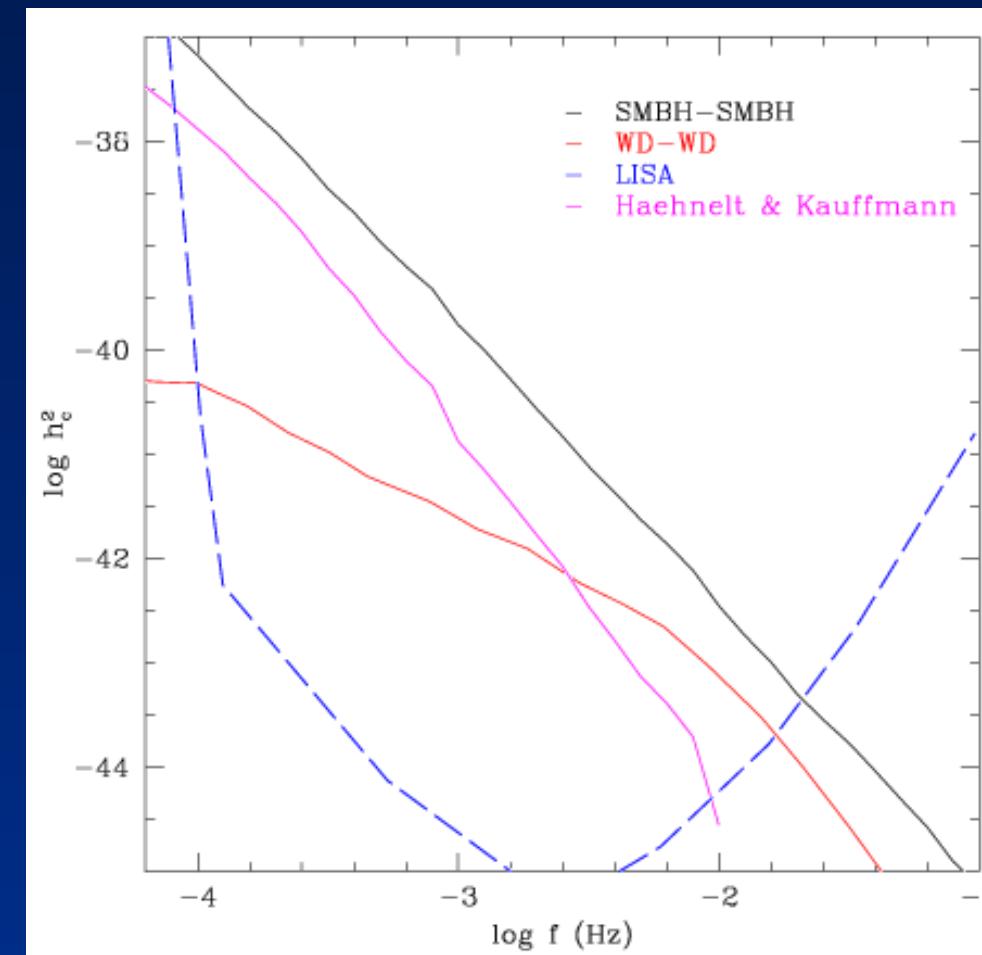
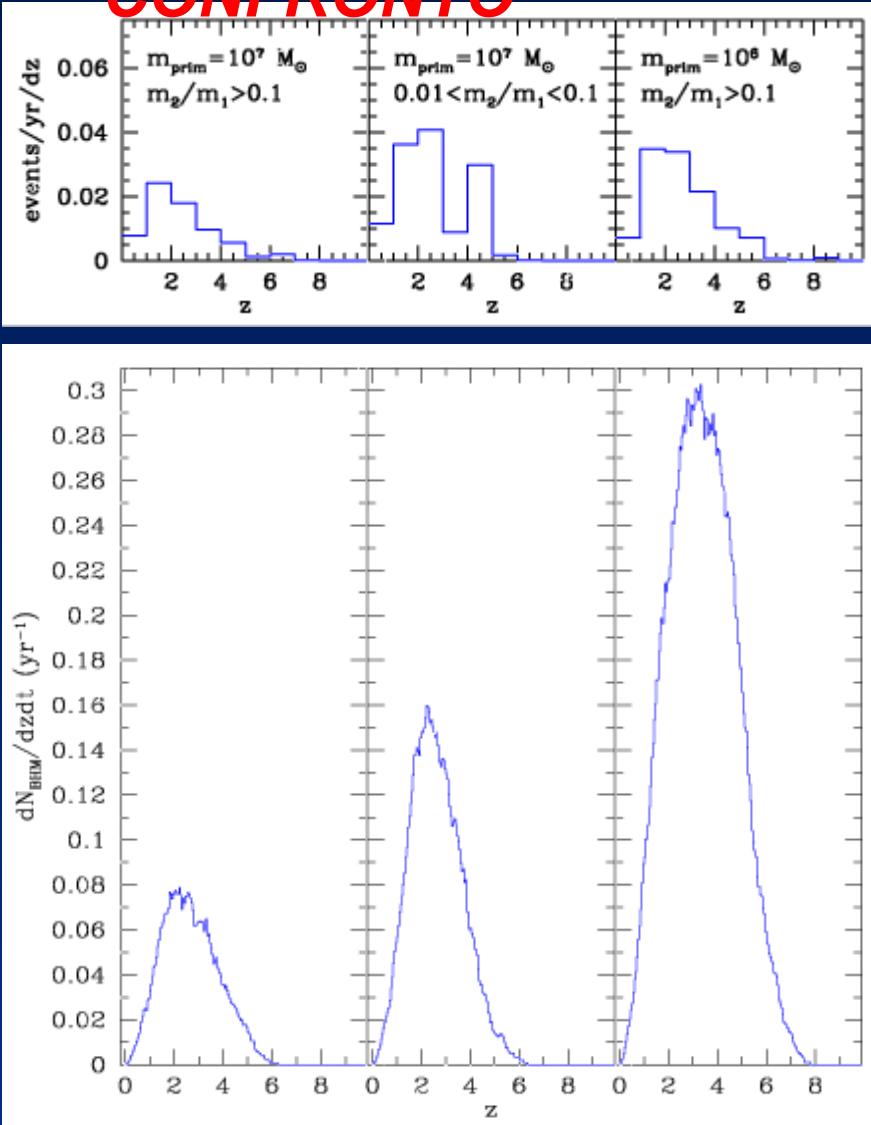


Contributi da diversi intervalli  
di redshift

# **CONFRONTO CON ALTRI FONDI COSMICI**



# DUE MODELLI DI EVOLUZIONE DI SMBH A CONFRONTO



Confronto tra coalescing rate nei modelli di Volonteri (sotto) e di Haehnelt (sopra)

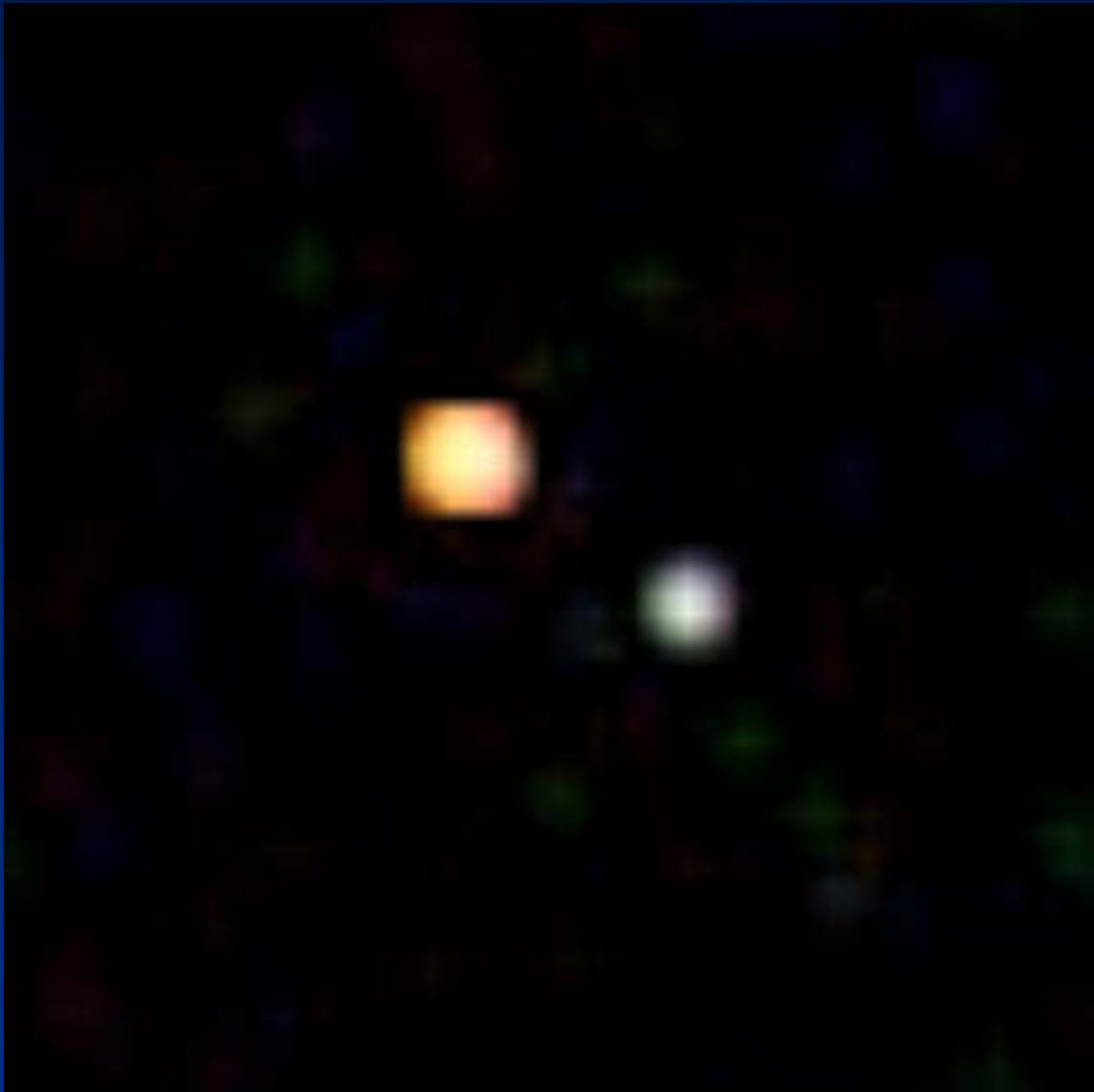
I due fondi cosmici prodotti

## **IN CONCLUSIONE:**

- Modello gerarchico di formazione delle strutture
- Prove dell'esistenza di SMBH
- SMBH binario: naturale sorgente di GW
- Fondo cosmico isotropo
- Sorgente di segnale dominante per LISA

GW movie





# FORMULA GENERALE PER LO SPETTRO

$$\begin{aligned}\frac{dE_{gw}}{df} f &= \frac{dE_{gw}}{dt} \times \frac{dt}{dr} \times \frac{dr}{df} f \times \Theta(f - f_H) \times \Theta(f_{uos} - f) \\ &= -\frac{32 G^{7/3} \pi^{10/3}}{5 c^5} (\mathcal{M} f)^{10/3} \times \frac{r^3 \ln^2 \left(\frac{r_h}{r}\right)}{A r^5 + B \ln^2 \left(\frac{r_h}{r}\right)} \times \frac{2}{3} \left[ \frac{G (M_1 + M_2)}{\pi^2} \right]^{1/3} f^{-2/3} \\ &\quad \times \Theta \left[ f - \frac{8 \sigma_\star^3}{\pi G} \frac{(M_1 + M_2)^{1/2}}{M_2^{3/2}} \right] \times \Theta \left[ \frac{c^3}{\pi (2 \alpha)^{3/2}} \frac{(M_1 + M_2)^{1/2}}{M_1^{3/2}} - f \right].\end{aligned}$$

Dalla relazione che lega  $h_c$  alla densita` di energia segue direttamente

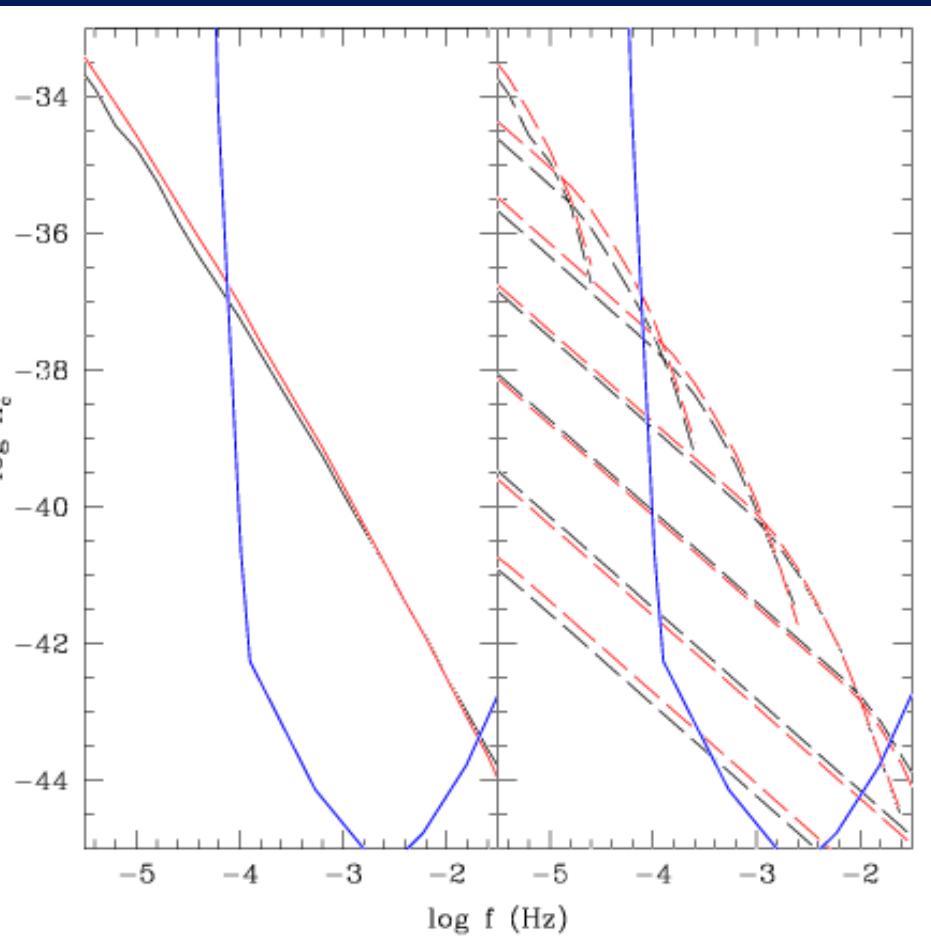
$$h_c^2(f) \propto \left( \frac{dE_{gw}}{df} f \right) f^{-2}$$

Lo spettro e` diviso in due parti:

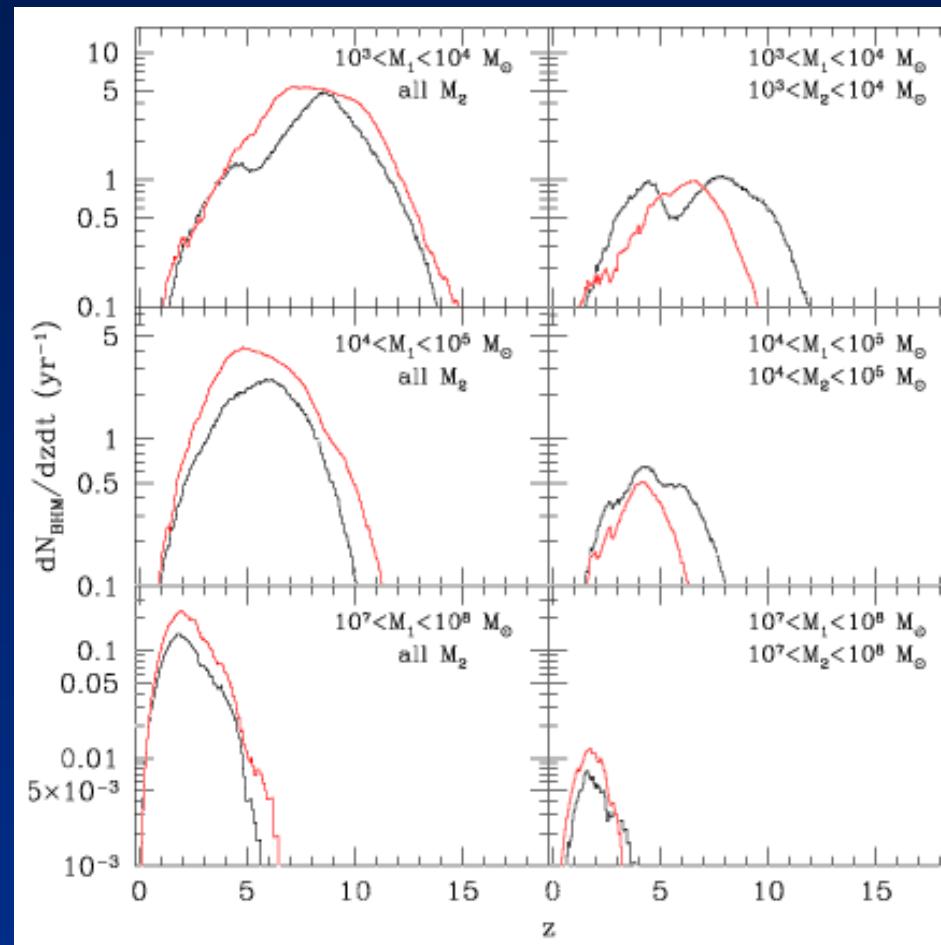
- Fase dominata da slingshot  $h_c^2(f)_{sl} \propto \sigma_\star^{-5} M_1^2 M_2^2 (M_1 + M_2) f^2$

- Fase dominata da emissione di GW  $h_c^2(f)_{gw} \propto \mathcal{M}^{5/3} f^{-4/3}$

# IMPATTO DI INTERAZIONI TRIPLE E GRAVITATIONAL ROCKET

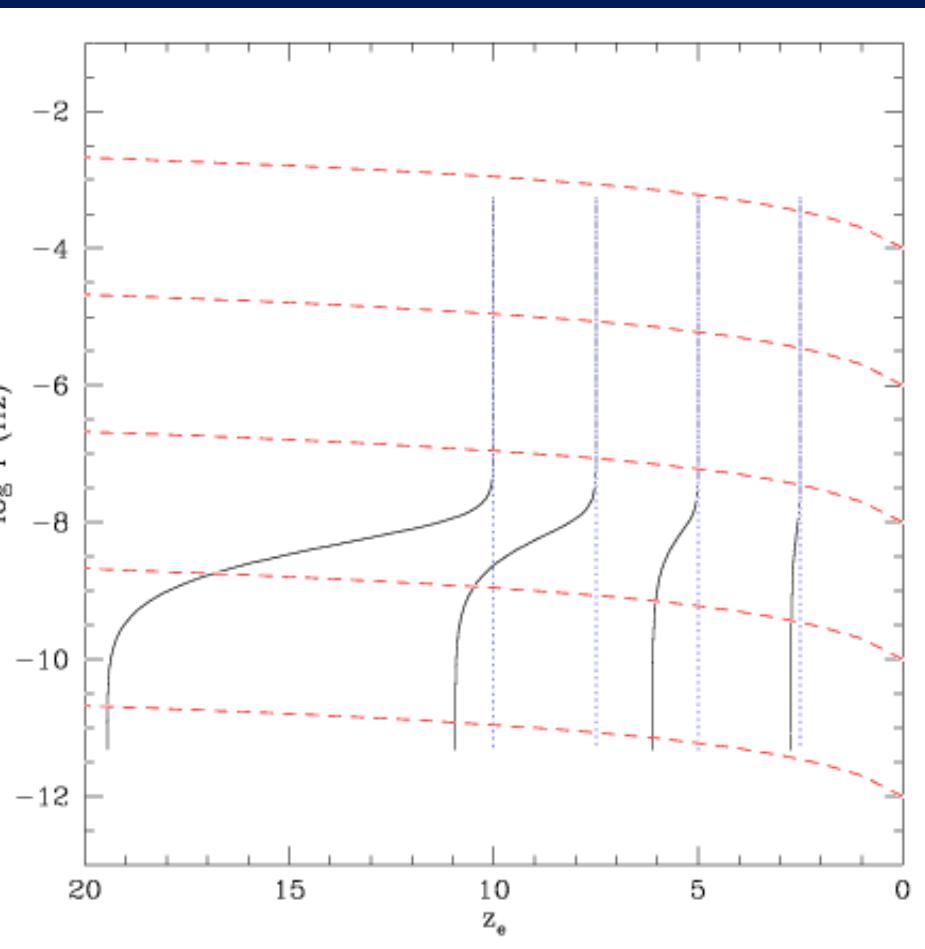


Confronto tra i due fondi cosmici  
(a sinistra) e relativi contributi in  
massa (a destra)

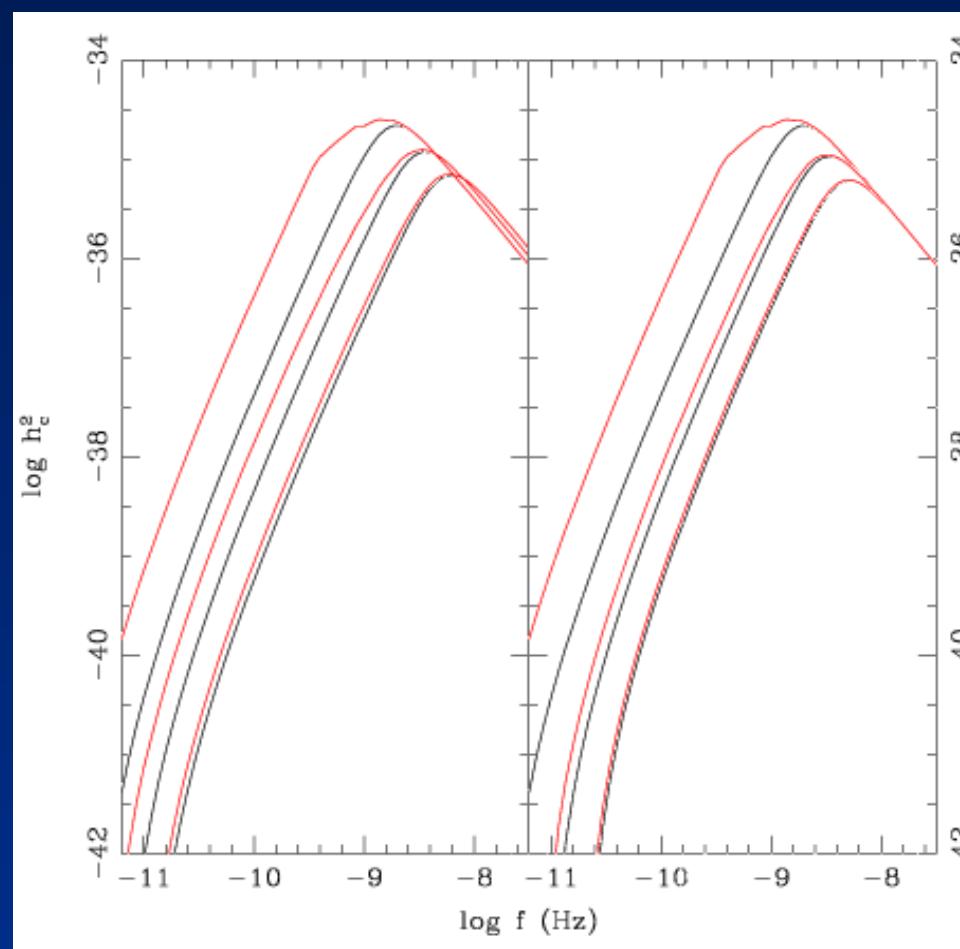


Andamento del coalescing rate  
nei due casi per diversi intervalli  
di massa

# ALLARGAMENTO DELLO SPETTRO



Frequenza emessa in funzione  
del redshift



Effetti prodotti sullo spettro

# ANDAMENTO DELLO SPETTRO DEL SINGOLO EVENTO

$$f_p \propto \sigma_{\star}^{3/2} [M_1^2 M_2^2 (M_1 + M_2)]^{-3/10}$$

$$h_c^2(f)_{sl} \propto f^2$$

$$h_c^2(f)_{gw} \propto f^{-4/3}$$

$$h_c^2(f) \propto (1+z)^3$$

$$h_c^2(f) \propto (1+z)^{-1/3}$$

