Gravitational Waves from Black Holes in the extreme mass ratio regime

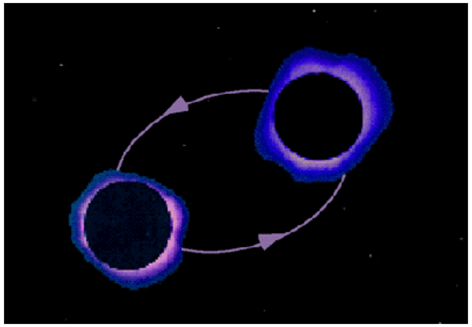
Carlos O. Lousto

Department Physics & Astronomy

The University of Texas at Brownsville

Center for Gravitational Wave Astronomy

Why BBHs is an interesting problem?



- Most of the galaxies host SMBH in their cores. The most likely and energetic event is the plunge / inspiral of normal stars.
- The two body problem remains one of the most interesting theoretical ones in GR.
- The comparable masses case in its most interesting stage (in the plunge radiates as much as in the whole binary history) requires solving the full nonlinear GR equations. This is a very difficult problem to solve!
- The small mass ratio regime seems more accessible to be solved using perturbative methods.

So far so good…

UTB TSC

The Problem

- Compute the metric perturbations for a particle orbiting a black hole from the field equations with a source term

$$G_{\mu\nu}[h_{\alpha\beta}] = T_{\mu\nu} = m \int u_\mu \, u_\nu \, \delta[x^\alpha - x^\alpha_p(\tau)]/\sqrt{-g} \, d\tau$$

- Compute the corrected trajectory of the particle from the geodesic equation

$$(d^2x^\mu/d\tau^2) + \Gamma^\mu{}_{\alpha\beta} \, (dx^\alpha/d\tau) \, (dx^\beta/d\tau) = 0$$

- The problem is that the metric near the particle has the form

$$h_{\mu\nu} \sim m \,/\, | \, x^\alpha - x^\alpha_p(t)|$$

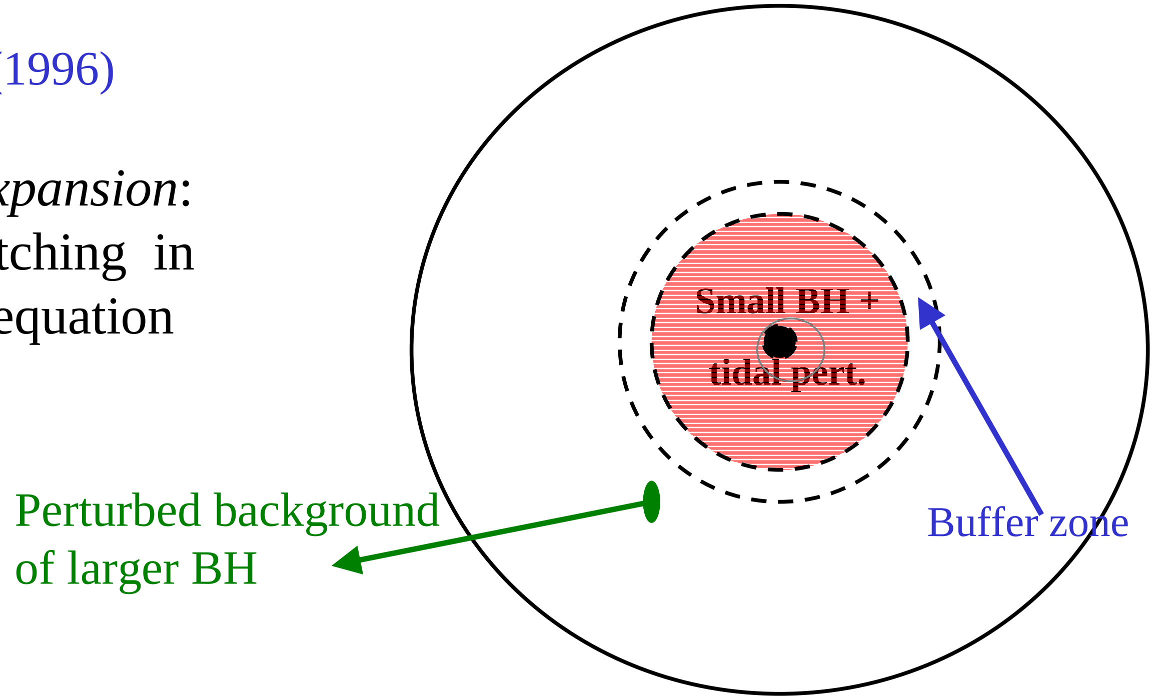
- We need to regularize the problem to extract any meaningful result.
This proved to be one of the most challenging problems for 25 years,

Until…

The Solution I

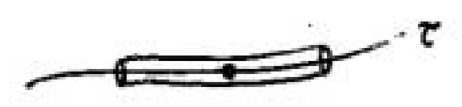
Mino, Sasaki & Tanaka (1996)

- *Matched asymptotic expansion*: Consistency of the matching in a buffer zone leads to equation of motion



•Alternative derivation via Brehme-DeWitt approach:

Derive “conserved” rank-two symmetric tensor integrate its divergence over the interior of the world tube surrounding the particle to derive the equations of motion



The Solution II

Quinn & Wald (1996)

- Axiomatic approach

1. Comparison axiom: *Identify* u^{μ} *and* a^{μ} *for P in* $(M, g_{\mu\nu})$ *and* $\mathbf{P}\,(\mathbf{M}, \mathbf{g}_{\mu\nu})$ *via Riemann normal coordinates*

$$f^{\mu} - \boldsymbol{f}^{\mu} = \lim_{r \to 0} \{ [(\tfrac{1}{2} \nabla^{\mu} h_{\alpha\beta} - \nabla_{\beta} h^{\mu}{}_{\alpha}) - [(\tfrac{1}{2} \boldsymbol{\nabla}^{\mu} \boldsymbol{h}_{\alpha\beta} - \boldsymbol{\nabla}_{\beta} \boldsymbol{h}^{\mu}{}_{\alpha})] \}$$

2. Flat spacetime axiom:
If $(M, g_{\mu\nu})$ *is Minkowski spacetime the perturbed metric* $h_{\alpha\beta}$, *is half advanced and half retarded solution*

$$h_{\alpha\beta} = \quad [h^{+}{}_{\alpha\beta} \tfrac{1}{2}+ h^{-}{}_{\alpha\beta}] \quad \textit{then} \quad m\, u^{\nu} \nabla^{(0)}{}_{\nu} (u^{\mu}) = f^{\mu} = 0\,.$$

The Solution

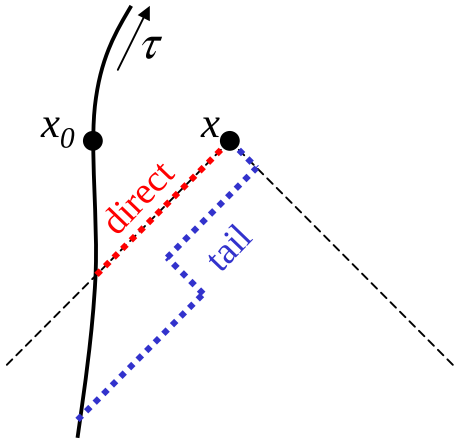
MiSaTaQuWa - Self -Force

$$F^{\alpha}_{\text{full}}(x)[= m\nabla^{\alpha\beta\gamma}h_{\beta\gamma}(x)] \;=\; \underbrace{F^{\alpha}_{\text{direct}}(x)}_{} \;+\; \underbrace{F^{\alpha}_{\text{tail}}(x)}_{}$$

from propagation *along* light cone

from Scattering *inside* light-cone

$$F^{\alpha}_{self} = F^{\alpha}_{\text{tail}}(x \to x_0)$$



$$F^{\mu}_{\text{tail}} = m\, u^{\alpha} u^{\beta} \int_{-\infty}^{\tau^-} \left[\tfrac{1}{2} \nabla^{\mu} G^{-}_{\alpha\beta\gamma\delta} - \nabla_{\beta} G^{-}{}_{\alpha}{}^{\mu}{}_{\gamma\delta} - \tfrac{1}{2} u^{\mu} u^{\lambda} \nabla_{\lambda} G^{-}_{\alpha\beta\gamma\delta}\right] u^{\gamma} u^{\delta} d\tau'$$

The Problems of the solution

- How to implement this regularization?
- How to compute the full and singular metric perturbations?
- All in the Harmonic Gauge

Mode Sum Regularization scheme (Barack & Ori, Burko, Lousto, Mino …)

$$F = \lim_{x \to z(0)} \sum_l \left(F^l_{\text{full}}(x) - F^l_{\text{direct}}(x) \right)$$

Multipole contributions finite at the particle, $F^l_{\text{full}}, F^l_{\text{direct}} (l \to \infty) \propto l$

$$\Downarrow$$

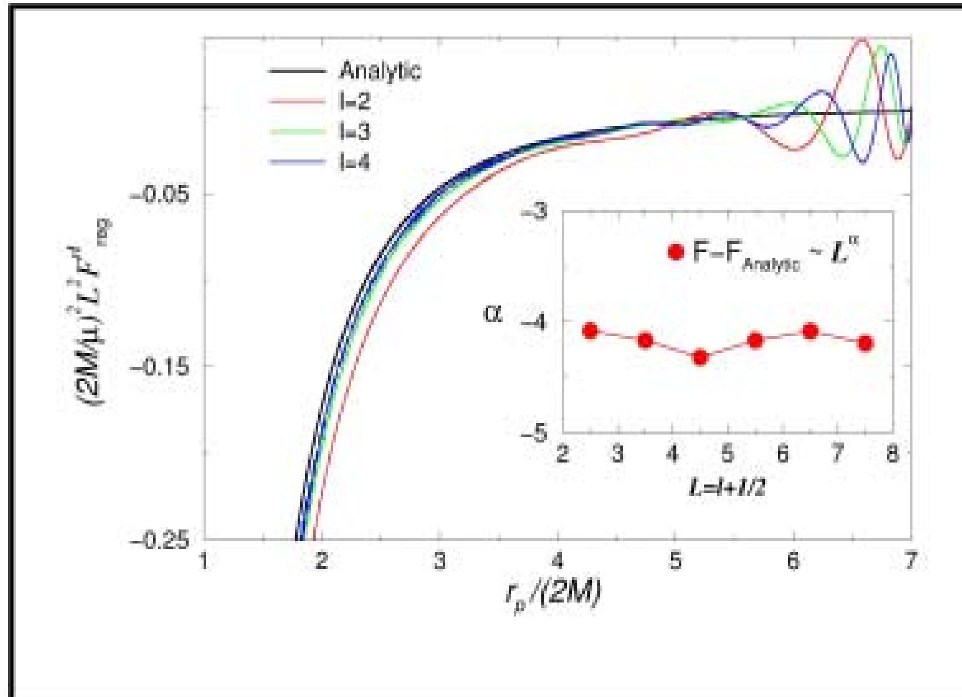
$$F = \sum_l \left(F^l_{\text{full}}(0) - A \cdot l - B - C/l \right) - \underbrace{\sum_l \left(F^l_{\text{direct}}(0) - A \cdot l - B - C/l \right)}_{D}$$

•"Regularization parameters" A^μ, B^μ, C^μ, D^μ derived analytically by local analysis (Kerr background)

UTB TSC

The applications: Headon

- For Schwarzshild background we can decompose into (l,m) multipoles
- The metric perturbations are finite for each (l,m) and can be computed in the RW gauge
- The sum diverges: Use mode regularization scheme
- Analytic form of (l=0,1) multipoles



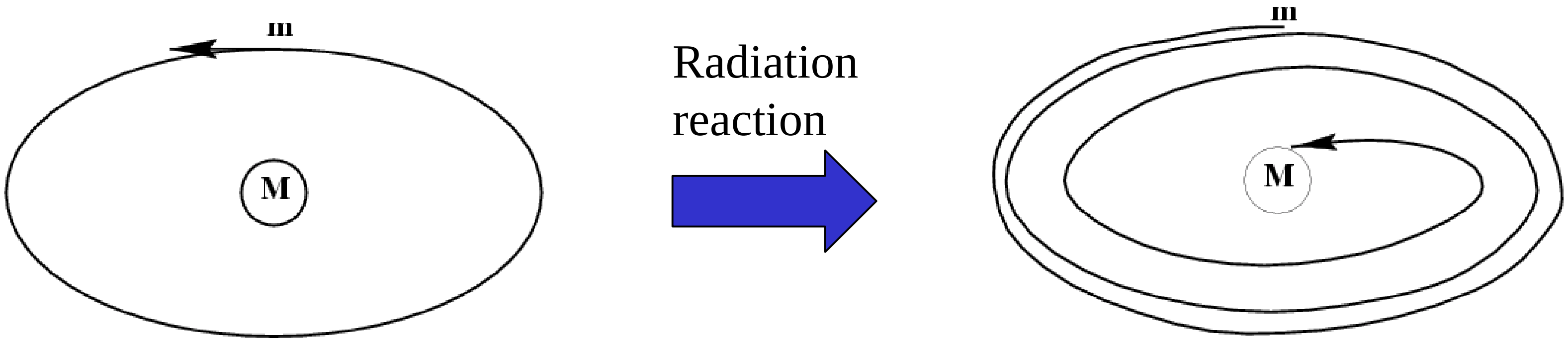
❖ Local analysis can be pushed to higher order in $1/l$, giving analytic approximation for the self force. For example:

$$F^{rl} = -\frac{15}{16} m^2 \frac{E^2}{r^2}\left(E^2 + 4M/r - 1\right)(l+1/2)^{-2} + O(l^{-4})$$

(Barack & Lousto 2002)

The applications: Circular orbits

- Multipole decomposition
- Compute metric perturbations in the RW gauge
- Transform the 'force' into the Harmonic gauge
- Compute the ISCO



It didn't work for us: This is the 'so called' gauge problem…

New Results:
The solution to the problems of the solution

Solve the field equations of GR directly in the Harmonic gauge! (Barack)

Linearize Einstein's equations in perturbation $h_{\alpha\beta}(x)$ about BH background $g_{\alpha\beta}$. Take source to be a point particle moving on a geodesic $x = x_p(\tau)$ of $g_{\alpha\beta}$. Get

$$\Box \bar{h}_{\alpha\beta} + 2R^{\mu}{}_{\alpha}{}^{\nu}{}_{\beta}\bar{h}_{\mu\nu} + g_{\alpha\beta}\bar{h}^{\mu\nu}{}_{;\mu\nu} - 2g^{\mu\nu}\bar{h}_{\mu(\alpha;\nu\beta)}$$
$$= -16\pi\mu \int_{-\infty}^{\infty} (-g)^{-1/2}\, \delta^4[x^\mu - x_p^\mu(\tau)] u_\alpha u_\beta \, d\tau \equiv S_{\alpha\beta},$$

where

$$\bar{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} h.$$

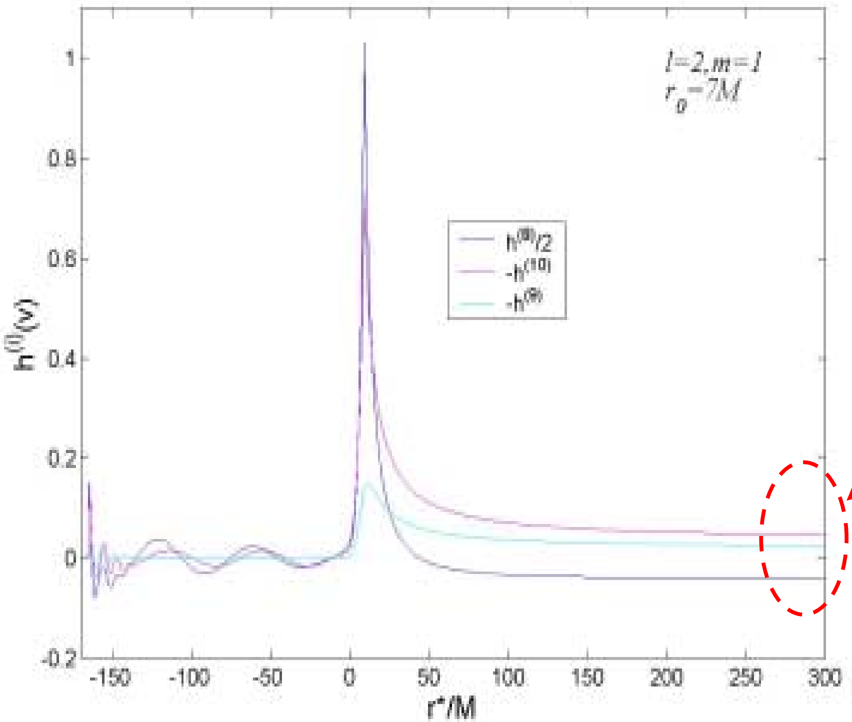
Impose Harmonic gauge condition,

$$g^{\beta\gamma}\bar{h}_{\alpha\beta;\gamma} = 0.$$

Get

$$\Box \bar{h}_{\alpha\beta} + 2R^{\mu}{}_{\alpha}{}^{\nu}{}_{\beta}\bar{h}_{\mu\nu} = S_{\alpha\beta}$$

New Results:
The solution to the problems of the solution



Energy flux (Circular orbit at r_0=7.9456M)

l	m	Ours: t-domain, from $h_{\alpha\beta}$	Poisson's: f-domain, from ψ	Martel's: t-domain, from ψ
2	1	8.1636e−07	8.1633e−07 [0.00%]	8.1623e−07 [0.02%]
3	2	2.5246e−07	2.5199e−07 [0.19%]	2.5164e−07 [0.36%]
4	1	8.3825e−13	8.3956e−13 [0.16%]	8.3507e−13 [0.38%]
	3	5.7828e−08	5.7751e−08 [0.13%]	5.7464e−08 [0.63%]
5	2	2.7897e−12	2.7896e−12 [0.00%]	2.7587e−12 [1.12%]
	4	1.2296e−08	1.2324e−08 [0.23%]	1.2193e−08 [0.84%]

Benefits:

•Work directly with the variables to compute the force (only first derivatives needed)

•Field equations have less singular source terms (only Dirac's deltas)

•Regularization parameters already computed (no gauge problem)

New Problems: Second order perturbations

- Consistency requires to compute second order perturbations
- Harmonic first order gauge is AF: Convenient to use in the second order Teukolsky Equation to compute waveforms
- The problem still needs regularization

Rosenthal & Ori

Distance from the world line

$D[h^{(1)}]=S^{(1)}[\delta(x-x_p)]$; $h^{(1)}_{\text{Harmonic}} \sim \varepsilon^{-1} + O(\varepsilon^{0})$;

$D[h^{(2)}]=S^{(2)}[\nabla h^{(1)} \cdot \nabla h^{(1)}, h^{(1)} \cdot \nabla^2 h^{(1)}] \sim a\,\varepsilon^{-4} + b\,\varepsilon^{-3} + O(\varepsilon^{-2})$

Non integrable terms

Solution for the scalar field: $\phi^{(2)} = \psi + \phi_{SH}$

$$\psi = \frac{1}{2}\left(\int_{-\infty}^{\infty} G^{ret}(x \mid z(\tau))d\tau\right)^2 \; ; \qquad \phi_{SH} = -\int_{-\infty}^{\infty} \phi^{(1)R}(\tau) G^{ret}(x \mid z(\tau))d\tau$$

New Results: 18PN! (Hikida et al.)

§ 5 Simple case (Schwarzschild + Scalar + Circular)

Comparison with previous results

The regularized self-force obtained by Detweiler et al. is

$$F_r^R = 1.37844828(2) \times 10^{-5} \quad (r_0 = 10M).$$

The most accurate self-force in our calculation is

$$F_r^R = 1.378448203 \times 10^{-5} \quad (r_0 = 10M).$$

coincidence at the accuracy 10^{-8}!!

- Table: the r-component of the self-force

PN order	$F_r^R(r_0 = 6M)$	$F_r^R(r_0 = 10M)$	$F_r^R(r_0 = 20M)$
4	$-3.698897009 \times 10^{-4}$	$5.438965544 \times 10^{-8}$	$4.009204942 \times 10^{-7}$
6	$3.900997486 \times 10^{-5}$	$1.215734502 \times 10^{-5}$	$4.900744665 \times 10^{-7}$
8	$1.469034988 \times 10^{-4}$	$1.370724270 \times 10^{-5}$	$4.937547086 \times 10^{-7}$
10	$1.634644402 \times 10^{-4}$	$1.377874928 \times 10^{-5}$	$4.937898906 \times 10^{-7}$
12	$1.665705633 \times 10^{-4}$	$1.378392510 \times 10^{-5}$	$4.937905702 \times 10^{-7}$
14	$1.674516681 \times 10^{-4}$	$1.378443247 \times 10^{-5}$	$4.937905862 \times 10^{-7}$
16	$1.676513985 \times 10^{-4}$	$1.378447488 \times 10^{-5}$	$4.937905865 \times 10^{-7}$
18	$1.677456783 \times 10^{-4}$	$1.378448203 \times 10^{-5}$	$4.937905865 \times 10^{-7}$