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Radiation reaction and Gravitational waveform from EMRI

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Yasushi Mino

The Center for Gravitational Wave Astronomy,
University of Texas at Brownsville
E-mail : mino@phys.utb.edu

Extreme Mass Ratio Binary: Radiation reaction and gravitational waveform

1: Introduction: Problem solved?

By a recent effort, we come to have the self-force induced by a linear perturbation.

$$\frac{D}{d\tau}V^{\alpha} = F^{\alpha}$$

So, what? Is it really available in constructing LISA templates? We must know the validity of this calculation.

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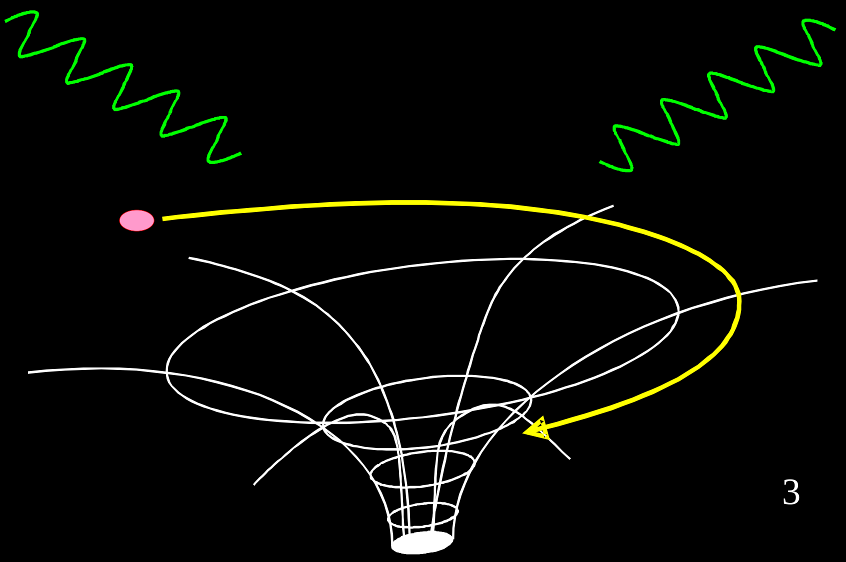
For mass-ratio $\mu \approx 10^{-5}$

dynamical time: $T_{dynamical} \approx 10 - 100\,\mathrm{sec}$

de-phasing time: $T_{de-phase} \approx v^{-2.5}\mu^{-0.5}T_{dynamical} \approx 1 day - 1 week$

radiation reaction time: $T_{reaction} \approx v^{-5}\mu^{-1}T_{dynamical} \approx 1 - 10\, years$

We need the orbital evolution of the radiation reaction time.



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Orbit Energetics:

Suppose we calculate the energy loss by a linear perturbation, the 2nd order perturbation acts as an error.

$$\dot{E} = \dot{E}^{(1)} + E^{(2)} + \ldots$$

$$\left(E\right)_{Error} \approx \dot{E}^{(2)} t$$

The linear approximation is valid enough.

$$T_{error} \approx v^{-7} \mu^{-2} T_{dynamical} \approx 10^{7}\ years$$

This type of estimate was widely done in PN formalism.
This estimate does not apply in the self-force calculation in general.

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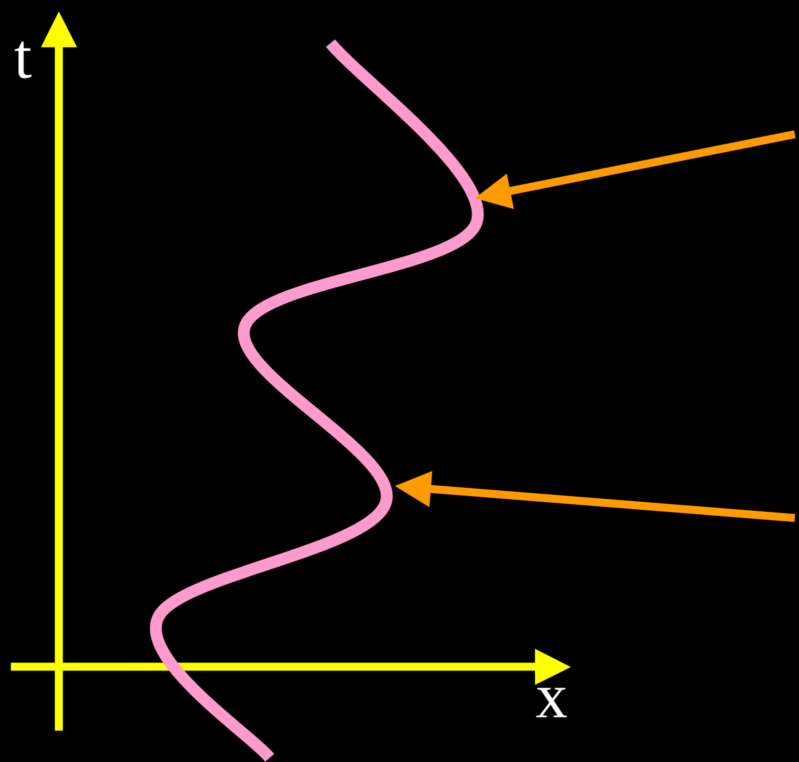
2: Orbital Evolution & Gauge

We consider an orbital evolution by the adiabatic picture and discuss the gauge dependence of the resulting orbit.

$$\frac{D}{d\tau}V^{\alpha} = F^{\alpha}$$



a geodesic

$$(E_1, L_1, C_1, \lambda_1, \bar{t}_1, \bar{\phi}_1)$$

These are different b/c the self-force.

a geodesic

$$(E_2, L_2, C_2, \lambda_2, \bar{t}_2, \bar{\phi}_2)$$

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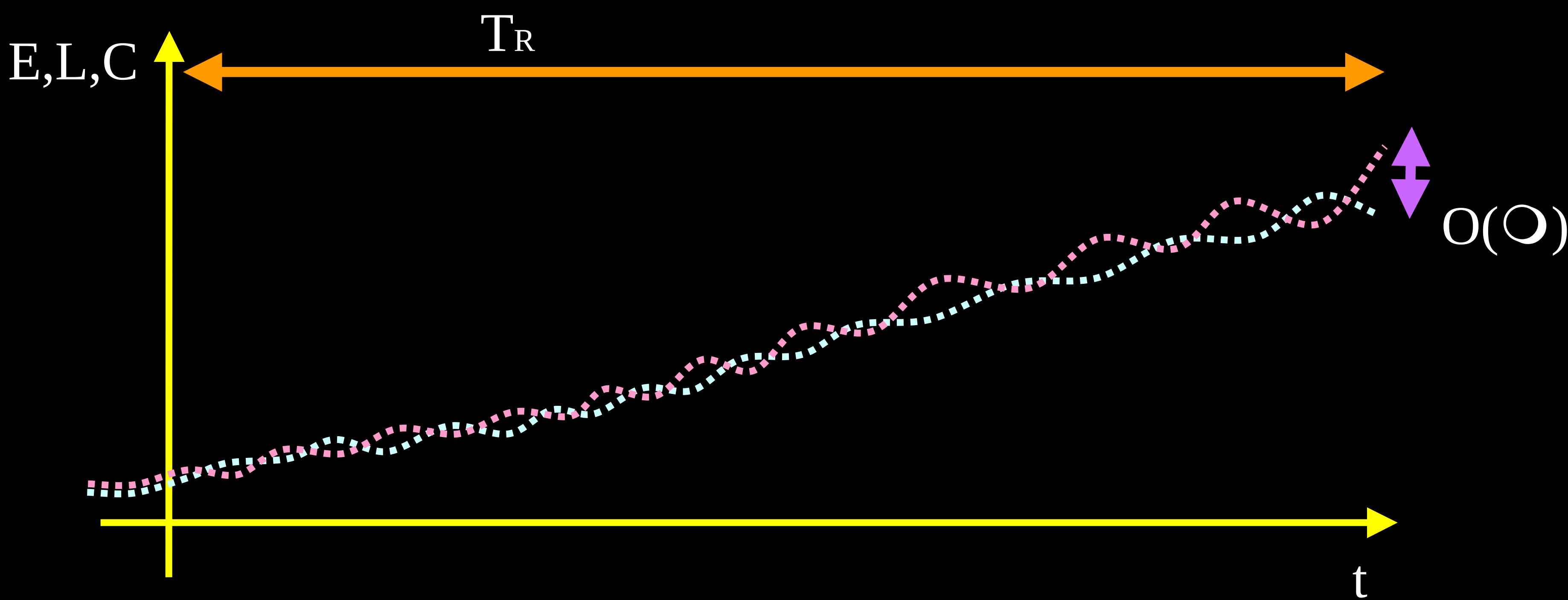
We compare the orbital evolution by the self-force calculated by different gauges.



We have a same prediction of the orbital evolution.

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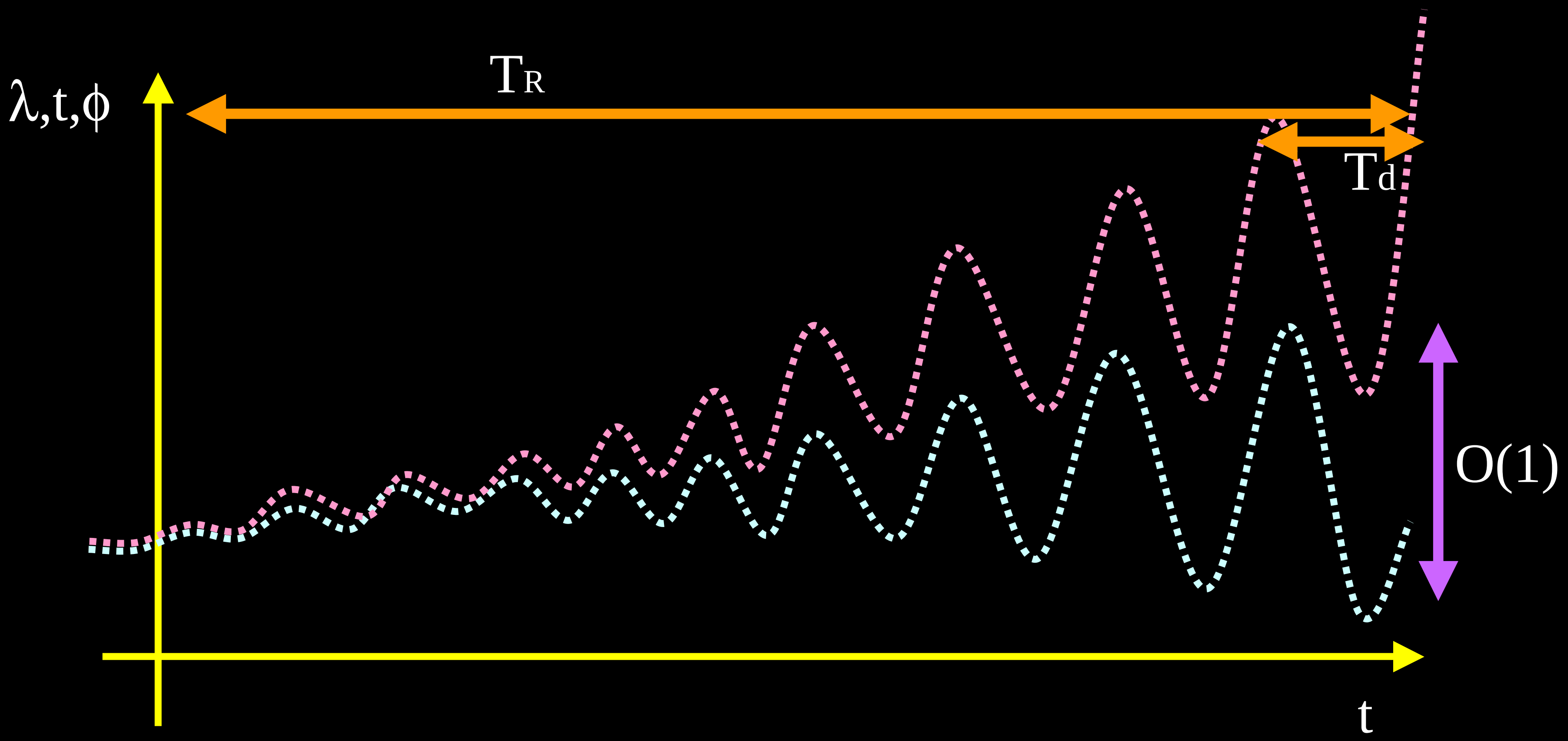

We have different predictions on the orbital evolution.
The adiabatic approximation is broken down.

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Different results of different gauge conditions suggest that we may not have a correct prediction at the radiation reaction time scale by the self-force calculation.

What is going on?

The gauge is growing!

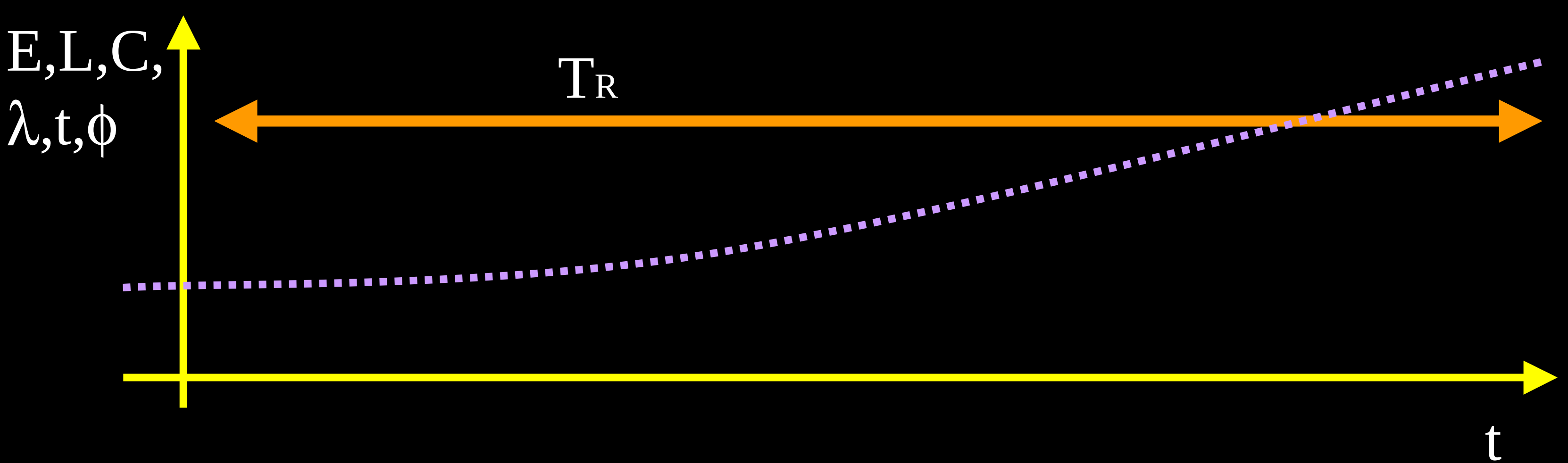
$$g_{\alpha\beta} = g_{\alpha\beta}^{BH} + h_{\alpha\beta}^{(1)} + h_{\alpha\beta}^{(2)} + \dots$$

$$h_{\alpha\beta}^{(1)} \approx \mu \times t \qquad h_{\alpha\beta}^{(2)} \approx \mu^2 \times t^2$$

The system becomes non-perturbative at the radiation reaction time scale. We need to study 2^{nd} order perturbation to find an optimal gauge.

3: Radiation Reaction Gauge

We consider the self-force problem in a non-perturbative manner, and we find a special gauge condition (Adiabatic gauge). In this gauge, the orbital equation is written in an gauge invariant manner. (Improved Radiation Reaction Formula)



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Summary of this formula;

1) The estimate of the orbital energetics applies, hence, we can predict the orbital evolution beyond the radiation reaction time scale.
2) This is a non-trivial extension of PN radiation reaction formula used for a quasi-circular orbit, which was successfully applied to Nobel-awarded observation of the Hulse-Taylor binary.
3) This fully describes the orbital motion, including the orbital phasing.
4) The required computation is minimal. The regularization of the self-force is naturally done.

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$$\frac{d}{d\tau} E^i = F^i(E)$$

$$E^i = E^i(z, v)$$

4: Waveform calculation

The self-force calculation:

10^2(10^6)	the self-force for a given geodesic
x	
10^4	evolution for 1 year
x	
10^11	number of templates
x	
10^-6?	extrapolation(?)
sec	

We need 100 years to generate templates!

Apology: I almost finished my coding, however, at the end stage of coding, I found the problem on the validity of the self-force, and, I concentrated on this instead of finishing up the program.

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Improved Radiation Reaction Formula:

1 Radiation reaction for a given geodesic

x

10 evolution for 1 year

x

10^11 number of templates

x

10^-6? extrapolation(?)

We need 1 month to generate templates

5: Conclusion

We found a class of gauges with which we can predict an orbital evolution necessary for LISA project.

Making an efficient program to generate templates is in progress.