(Super)massive Black Holes in Galactic Nuclei and LISA

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LISA sources

- Cosmological backgrounds (e.g. from electroweak phase transition)
- Burst sources (e.g. from cosmic string cusps and kinks)
- Galactic white dwarf binaries (double degenerates and AM CVn)
- Supermassive and intermediate mass black hole binaries merging.
- Compact objects spiralling into supermassive black holes in galactic nuclei.
Merging supermassive black holes

3C75 in Abell 400
R=7kpc, 1/173 3CRR

Also LBQS 0103-2753
R=2kpc, $\tau_{df}=10^7$y, 1/500QSO
Merging supermassive black holes

Post-Newtonian. Consider $10^6+10^6$ Msun at $z=1$, drawn to scale:

- 0.00002Hz, SNR=5, $t=2\text{yr}$, 700 orbits, 20 L-S precessions to go
- 0.00003Hz, $t=1\text{yr}$, SNR=16
- 0.0001Hz, $t=13\text{d}$, SNR=65
- 0.0003Hz, $t=1\text{d}$, SNR=120
Merging supermassive black holes

Hard! cf. Bernard Schutz talk. Dynamical strong gravity. Nonlinear numerical relativity in 4 space-time dimensions
Merging supermassive black holes

Quasi-normal modes of black hole spacetime. Amplitudes unknown, but \( \text{Re}(\omega_i) \) and \( \text{Im}(\omega_i) \) known as functions of \( M, a/M \).

Find simultaneous solutions for all observed \( i \) -> determine \( a/M, M \).

Find out how much pre-merger \( L \) went into final \( S=aM \). Can this make rapid spins?
Merging supermassive black holes

Points: 1yr 1 mo 1day (1 hour) ringdown

Extragalactic WDS (Farmer & Phinney)

Raw galactic WDs subtracted

Subtracted EMRI (raw)

LISA sensitivity

0.1/y

0.5/y

<20/y

<100/y
Local mass density of black holes dominated by $>10^8$ Msun. AGN light implies 50-100% of their growth by gas accretion: no GWs!

LISA is insensitive to these.

Sensitive mainly to $<3 \times 10^6$ Msun holes:

Only 8 reliably known, and unmeasured outside local group except in Low L AGN (cf Barth) where occupation fraction undetermined.

We know the present space density, and dark halo merger history. But not black hole growth (by accretion, at least 50%) and merger history: LISA rate model-dependent.
Merger Rate(s) of Supermassive Black holes

Given CDM halo merger tree: depends on

* Occupation density of seeds in halos
* Redshift at which they grow to $>10^5$ Msun
* Retention in small halos (GW recoil, 3body)
* Relative growth via gas accretion vs merger.
Do the black holes merge?

1. BHs orbit stellar potential
2-3. BHs orbit each other, eject stars
4. BHs orbit each other, GR inspiral

1. Dynamical friction
2. Stars ejected
3. Loss cone depleted
4. Gravitational radiation

\( (1-e)^{3/2} \)
Even for spherical galactic nuclei, $M<4\times10^6\,M_{\odot}$ black hole pairs merge: only ones LISA can see anyway!

$$\frac{GM_{\text{hole}}}{2a} = \sigma^2$$

GALAXY MERGER
Dynamical friction

Eject loss cone

Eject returning stars

2-body refilling cone

Even for spherical galactic nuclei, $M<4\times10^6\,M_{\odot}$ black hole pairs merge: only ones LISA can see anyway!
Rates of merger of supermassive black holes

• Every respectable galaxy with a bulge contains a central black hole. CDM: every galaxy has merged more than once.
  ✓ Milky Way and M31 in 5,000,000,000AD,
  ✓ 1/100 local galaxies AGN. ~1/100 AGN have second incoming black hole.
• ~$10^{10}$ galaxies in z<2.
• If black holes find each other, expect ~1 SMBH/SMBH merger per year.
• Could be much higher: each galaxy today product of ~1000 mergers of subunits. If each subunit had MMBH, up to 1000 mergers per year!
Merger Rate(s) of Supermassive Black holes

Mainly gas accretion. $>10^6$ Msun seeds in large low z fragments to avoid Supereddington, recoil:
LISA merger rate $\sim 1/y$, $z<5$, $10^6$-$10^7$ Msun.

Haehnelt 1997
Kauffman & Haehnelt 2000

Mainly mergers. $<10^4$ Msun seeds in small high-z fragments
LISA merger rate $\sim 300/y$, $z\sim 20$, $10^4$-$10^5$ Msun.
A Nuisance!

Wyithe & Loeb 2003
Volonteri et al 2003
Menou & Haiman 2004
17 dimensional parameter space of black hole binary waveforms:

- 3 center of mass coords rel to ☀, e.g. $D_L$, $\lambda$, $\beta$ (ecliptic coords).
- 2 angles times 2 BHs’ spin directions.
- 2 masses, and 2 dimensionless spin parameters for BHs.
- 6 initial phase space coordinates of the relative motion. Conveniently, 3 secularly varying actions (or functions thereof, e.g. $a$, $e$, $i$), and 3 rapidly varying phases.

Note: if one $M_2 \ll M_1$, can drop $M_2$’s spin and spin direction, reducing to 14 parameters.

NB: Strong covariance of some parameters in waveform templates (e.g. in Newtonian limit, only one phase, a frequency $\sqrt{M/a^3}$, and $\lambda$ and $\beta$ matter).
Accuracy of Parameter determination


Shown: $10^6+10^6$ at $z=1$
Spin-orbit coupling removes degeneracy between position and inclination, increases accuracy of $D,\text{pos} \times 5$

$a/M= 0.9 \quad 0.5 \quad 0$
Merging SMBH - what can we learn from gravitational waves?

- What merging black holes (M,z) can LISA see? [Compare with pulsar timing - large M]
- What are possible/plausible today rates?
- M,z distributions?
- What parameters can LISA measure?
  - z not direct, but can do indirectly!
  - spins, mass, mass ratio, spin-orbit inclination.
  - Tests of strong-field time-dependent gravity, cosmic censorship... (similar to LIGO/Virgo... but more precise).
Are “black holes” black holes?

Einstein 1939 [Ann of Math numerical paper]:
"The essential result of this investigation is a clear understanding as to why 'Schwarzschild singularities' do not exist in physical reality."

Oppenheimer and Snyder 1939:
Maximum neutron star mass. Stars more massive than this are unstable and the collapsing star "tends to close itself off from any communication with a distant observer; only its gravitational field persists."
Are “black holes” black holes?

Chandrasekhar [Ryerson lecture 1975, reprinted in Truth & Beauty 1987]:

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity, discovered by the New Zealand mathematician, Roy Kerr, provides the absolutely exact representation of untold numbers of massive black holes that populate the universe."

Touching faith! Is it true?
Are “black holes” black holes?

1. Mass from dynamics, or luminosity: radiation pressure limits mass $M > L\sigma_T/(4\pi GMm_p c)$.

2. Rapid large amplitude variability implies small size $R \sim c\Delta t$ So $GM/R \sim c^2$.

3. Lifetimes $t$ so $Lt/Mc^2 > 0.01 \rightarrow$ accretion, not nuclear.

4. Jets with relativistic speeds.

5. Some low luminosity sources seem to have $L \ll 0.01Mc^2$. No hard surface $\rightarrow$ horizon?

6. X-ray Fe K-shell lines—redshifts and Doppler boosts imply orbits at $v \sim 0.5c$. 
Are “black holes” black holes?

Feynman's remark: If you had one good reason, you wouldn't have to give six.

These arguments depend on hydrodynamics, plasma physics, cooling, radiation transport, magnetic fields, particle acceleration....

Black holes are defined by their vacuum space time structure and geodesics.

Can we diagnose black holes using vacuum space time dynamics alone?
LISA as a Dark Energy Probe

Notice $D_L$ determined to 0.4% at $z=0.5-3$, better than with Supernovae. Limited by weak lensing changes to effective $D_L$. If EM signal also gives $z$, get $D_L(z)$ and $w$, $w'$. Otherwise if cosmology known, can invert $D_L(z)$ to get $z$ and hence rest-frame black hole mass ($\text{GW signal invariant}$ $(1+z)M$).

At lowest PN level:

$$\frac{1}{f} \frac{df}{dt} = \frac{96}{5} M_z^{5/3} (\pi f)^{8/3}$$

$$h_{\cdot \times} = \frac{M_z^{5/3} f^{2/3}}{D_L}$$

times functions of inclination and orientation angles.

Here $f(t)$ is the frequency of grav waves at earth, $M = M_1 + M_2$, $0 < \eta = M_1 M_2 / M^2 < 1/4$, $M = \eta^{3/5} M$, $M_z (1+z)M$ is redshifted chirp mass.

Observing $f$ and $\dot{f}$ gives $M_z$, and the observed $h$ and fitted angles gives $D_L$.

$\delta D_L / D_L \sim 0.02$ for $a/M = 0$, 0.004 for $a/M = 0.9$ (so precession breaks a degeneracy in inclination/sky position) or if an optical counterpart determines the sky position.
LISA as a Dark Energy Probe

\( D_L^2 \) times the gravitational wave flux (\( \sim f^2 h^2 \)) is (up to angular factors) the rate of change of orbital energy \( \dot{E} \), and \( \dot{E} \) causes observable secular change in the gravitational waveform, we can use this to determine \( E' \), and from the measured \( f \), \( h \) and fitted angular factors, determine \( D_L \). At lowest PN level:

\[
\frac{1}{f_r} \frac{df_r}{dt_r} = \frac{96}{5} M_0^{5/3} (\pi f_r)^{8/3}
\]

\[
\frac{1}{f} \frac{df}{dt} = \frac{96}{5} M_0^{8/3} (\pi f)^{8/3}
\]

\[
h_+ \propto \frac{M_0^{5/3} f_r^{2/3}}{D_M} \frac{M_0^{5/3} f^{2/3}}{D_L}
\]

times functions of inclination and orientation angles.

Here \( f \) is the frequency of grav waves at earth, \( f_r = (1 + z)f \) is the emission frequency at source at proper motion distance \( D_M \), \( D_L = (1 + z)D_M \) is luminosity distance, \( M = M_1 + M_2 \), \( 0 < \eta = M_1 M_2/M^2 < 1/4 \), \( M = \eta^{1/5} M \), \( M_0 = (1 - z)M \).

Observing \( f \) and \( \dot{f} \) gives \( M_0 \), and the observed \( h \) and fitted angles gives \( D_L \).

Typical fractional errors in \( D_L \) are 0.02 for \( a/M = 0 \), 0.004 if \( a/M = 0.9 \) (so precession breaks a degeneracy in inclination/sky position) or if an optical counterpart determines the sky position.

If cosmological parameters perfectly known, get \( 1 + z \) to 1–2 times frac precision of \( D_L \).

If not, better than supernovae, clustering, etc. limited only by knowledge of weak lensing contribution to \( D_L \) (\( \sim \) few %).
Science return from SMBH

• **Merger rates** of $10^4$ - $10^6$Msun black holes at $z=1$-100.
• Measure black hole spins, $M(1+z)$ distribution.
• High S/N: precision match of inspiral parameters ($P^n N$) with ringdown: final black hole.
• vs numerical GR: test of **strong-field dynamical GR**. SNR high - so templates not needed for determining merger signal!
• **Precision** (<1%: limited by weak lensing!) **distance measurements**: self calibrating standard candles. If optical display gives independent $z$, also: **cosmography**, $w$, $w'$.
Stars in the nuclear cusp of the Milky Way

Sgr A* black hole, $3 \times 10^6 M_\odot$

What about even more eccentric orbits!? 

Plunge into black hole (or tidal disruption if extended)

Genzel et al 2003

Ghez et al 2003

Extreme Mass Ratio Inspiral
Extreme Mass Ratio Inspiral

2-body encounter scatters wd/ns/bh to orbit with peri $\sim 5-15M_{\odot}$

Gravitational radiation at pericenter: shrinks apocenter so fast no more 2-body.
Diffusion into the EMRI Loss Cone

EMRI capture loss cone defined by $\frac{d\ln(a)}{dt}$ due to gravitational radiation $<$ time to diffuse back out of loss cone.

$\frac{10}{10^6} z=0.1$

$a/M=0$, 0.998 pro

3/y Freitag rate,

Mass seg,

Kroupa IMF

$\frac{1}{10^6} z=0.25,$

$a/M=0$, 0.998 pro

3/y no mass seg,

WD only
EMRI rate/vol ingredients:
1) SMBH mass function (~obs)
2) Compact object mass function in galactic nuclei (? assume)
3) Density profiles of galactic nuclei (~obs)
4) Star formation histories of galactic nuclei (~obs)
5) Loss cone filling (asphericity..)
6) Accuracy of simulations?

Reasonable uncertainties: x100

EMRI LISA detection rate ingredients:
1) EMRI rate/vol
2) Black hole a/M, orbit inclination (sets upper freq)
3) Signal detection algorithm (effective SNR for detection)
Orbits and spiral-in of small bodies around spinning black holes

- Spiral-in and Circularization (GW energy and angular momentum Losses)
- Slow: ~ $M/m$ orbits

Orbit plane precession (spin – orbit; L-T)

Peribothron precession
a^* = 0.9, a = 100M, e = 0.05, i = 45

2x orbital freq

Peribothron precession freq

Integer combinations of 3 frequencies:

\[
l f_r + m f_\theta + n f_\phi
\]

L-T orbit plane precession freq

a^* = 0.9, a = 10M, e = 0.2, i = 45
Frequencies sweep and shift slowly as compact object spirals in, mapping space-time outside the horizon.  
cf. geodesy satellites mapping geopotential
$a/M=0.8$, $i=45$ prograde inspiral of 10 into $10^6$, 1 week before plunge, $e_p=0.4$

Comparison of slow motion quadrupole from exact orbit with full GR waveform From exact orbit. Same total power to 10%.
Tide induced on horizon dragged ahead by black hole rotation: orbiting body gains energy/angular momentum from hole, slowing inspiral ~10%!
Separating sources

\[ s(t) = \sum_{i} h_i(t) + n(t) \]

Signal = sources + noise

Wiener optimal filtering (optimal for Gaussian noise – ultimately others.)

\[ \langle h(t, \lambda), s(t) \rangle = \int_{-\infty}^{\infty} dt \ h(t, \lambda) s(t) \]

\[ = 4\Re \int_{0}^{\infty} df \ \frac{h^*(f, \lambda) \tilde{s}(f)}{Sh(f)} \]

Parameters:
- 2 trivial for `free’ t(0) (Fourier phase) and distance (amplitude of corr).
- White dwarfs additional 6: 4 extrinsic (source position, orbit \( L \) direction), 2 intrinsic: \( f, df/dt \).
- Inspiralling compact objects: additional 12: 5 extrinsic (source position, BH spin direction, initial phase of \( L \)), 7 intrinsic: \((M, m, S/M^2, L\cdot S, \text{initial } e, \text{initial phase in orbit and peribothron precession})\). SMBH add 3 more – spin \( s \) of second BH.

Must choose grid of templates fine enough that \( \frac{\langle h_i, h_j \rangle}{\langle h_i^2 \rangle} > M \sim 0.8 \), say

Note parameters correlated: optimal template grid is not square.
Merging supermassive black holes

Points: 1yr 1 mo 1 day (1 hour) ringdown

LISA sensitivity

extragalactic wds (Farmer & Phinney)

raw galactic WDs subtracted

subtracted EMRI (raw)
Computational requirements for EMRI detection

- Number of templates scales as $T^4$ for $T<4$ weeks. Faster for $T>4$ weeks (sky position variation affect minimum match).
- Each template requires $(0.1 T/s)$–point FFT.
- If have 50Tflops dedicated, can correlate templates up to 3 weeks. Then have to stack incoherently (lose factor of 2 in SNR compared to 5-year coherent search (would require $>10^{11}$Teraflops!)).
- Does not include computational cost of computing templates (likely to be similarly high).
Predicted number of EMRI sources to be observed by LISA with 5 year mission.

Contributions to SNR as function of frequency. For $10 \text{ into } 10^6 \text{ Msun}$, final $e_p=0.25$ from specified time intervals. 5-yr integrated SNR=331.
Precision of EMRI parameter determination

<table>
<thead>
<tr>
<th>$S/M^2$</th>
<th>$e_{LSC}$</th>
<th>0.1</th>
<th>0.1</th>
<th>0.5</th>
<th>0.5</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
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<tbody>
<tr>
<td>$\Delta(\ln M)$</td>
<td>3.6e-5</td>
<td>7.9e-5</td>
<td>4.5e-5</td>
<td>1e-4</td>
<td>6.3e-4</td>
<td>5.1e-5</td>
<td>2.6e-4</td>
<td>3.7e-4</td>
<td>2.6e-4</td>
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<tr>
<td>$\Delta(S/M^2)$</td>
<td>4.6e-5</td>
<td>1.5e-4</td>
<td>7.4e-5</td>
<td>6.8e-5</td>
<td>6.8e-5</td>
<td>1.0e-4</td>
<td>8.1e-5</td>
<td>9.1e-5</td>
<td>1.0e-3</td>
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<tr>
<td>$\Delta(e_0)$</td>
<td>6.3e-5</td>
<td>1.3e-4</td>
<td>2.9e-5</td>
<td>8.5e-5</td>
<td>2.8e-4</td>
<td>3.2e-5</td>
<td>1.2e-4</td>
<td>1.1e-4</td>
<td>1.6e-3</td>
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<td>$\Delta(\cos \lambda)$</td>
<td>6.0e-3</td>
<td>1.7e-2</td>
<td>1.3e-2</td>
<td>1.3e-3</td>
<td>5.8e-3</td>
<td>2.4e-4</td>
<td>5.5e-4</td>
<td>8.4e-4</td>
<td>4.7e-4</td>
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<td>$\Delta(\Omega_0)$</td>
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<td>1.6e-3</td>
<td>5.3e-4</td>
<td>1e-3</td>
<td>2.1e-3</td>
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<td>1.4e-3</td>
<td>8.3e-4</td>
<td>6.2e-4</td>
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<td>5.5e-2</td>
<td>4.7e-2</td>
<td>5.0e-2</td>
<td>5.2e-2</td>
<td>4.7e-2</td>
<td>5.5e-2</td>
<td>5.1e-2</td>
<td>4.8e-2</td>
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<td>$\Delta(\gamma_0)$</td>
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<td>6.3e-1</td>
<td>3.8e-1</td>
<td>1e+0</td>
<td>6.1e-1</td>
<td>3.9e-1</td>
<td>3.3e-1</td>
<td>3.4e-1</td>
<td>5.9e-1</td>
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<td>$\Delta(\Psi_0)$</td>
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<td>6.7e-1</td>
<td>2.2e-1</td>
<td>1e+0</td>
<td>7.5e-1</td>
<td>2.7e-1</td>
<td>1.8e-1</td>
<td>1.7e-1</td>
<td>3.3e-1</td>
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<td>$\Delta(\alpha_0)$</td>
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<td>5.4e-1</td>
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<td>$\Delta(\ln(\mu/D))$</td>
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<td>3.7e-2</td>
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<td>3.7e-2</td>
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<td>2.3e-1</td>
<td>1.3e-1</td>
<td>1.4e-2</td>
<td>2.5e-2</td>
<td>3.9e-2</td>
<td>5.5e-2</td>
</tr>
</tbody>
</table>

TABLE II. Parameter accuracy estimates for inspiral of a $10M_\odot$ CO onto a $10^6M_\odot$ MBH at: SNR=30 (based on data collected during the last year of inspiral). Shown are estimates for the accuracy in determining the various physical parameters, for various values of the MBH's spin magnitude $S$ and the final eccentricity $e_{LSC}$. The rest of the parameters are set as follows: $t_0 = (1/2)\tau_0$ (middle of integration); $\gamma_0 = 0$; $\Psi_0 = 0$; $\theta_0 = \pi/4$; $\phi_0 = 0$; $\lambda = \pi/6$; $\alpha_0 = 0$; $\theta_K = \pi/8$; $\phi_K = 0$.

Barak & Cutler 2004
Geodesy

Outside earth,
\[ \nabla^2 \Phi = 0 \]
\[ q(x) = -\nabla \Phi \]

Bothrodesy

Outside relativistic body (e.g. BH) metric can be expanded in
Mass multipoles
\[ M_l^m = \int d^3r \ r^l P_l(m\theta) \rho(x) \]

Current multipoles
\[ S_l^m = \int d^3r \ r^{l-1} (x \times \nabla \rho(x)) P_l(m\theta) \]

Axisymmetry: \( m = 0 \)
Equatorial reflection: \( M_{odd} = 0 \)
\[ \begin{cases} M_0 & \text{mass} \\ S_1 & \text{spin angular momentum} \\ M_2 & \text{mass quadrupole} \\ S_3 & \text{current octupole} \end{cases} \]

Diagnose interior structure by matching
\[ \nabla^2 \Phi = 4\pi \rho(x) \]
Science return from Bothrodesy

- Are astronomical `black holes’ really the Kerr solutions of GR? Or cubical solitons of exotic fields?
  - Precision test of no-hair
- Measure energy extraction from a black hole
  - 5-10% on inspiral rate
- Distribution of M, a/M
  - Important for astrophysics, formation history, accretion solutions, radio jet models…
- If see many (>100) events:
  - 2-10% could be white dwarfs, He stars: tidally disrupted in late stages giving electromagnetic signal, galaxy ID, cosmography.
  - 1% of inspirals in AGN, perturbed by accretion disk
<table>
<thead>
<tr>
<th>$M_\bullet$</th>
<th>$M_\odot$</th>
<th>space density</th>
<th>0.6$M_\odot$ WD</th>
<th>1.4$M_\odot$ MWD/NS</th>
<th>10$M_\odot$ BH</th>
<th>100$M_\odot$ PopIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^{6.5}±0.25</td>
<td>1.7</td>
<td>8.5</td>
<td>1.7</td>
<td>1.1</td>
<td>1.7</td>
<td>1.7×10^{-3}</td>
</tr>
<tr>
<td>10^{6.6}±0.25</td>
<td>1.7</td>
<td>6</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
<td>10^{-3}</td>
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<tr>
<td>10^{5.5}±0.25</td>
<td>1.7</td>
<td>3.5</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>7×10^{-4}</td>
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</table>

Table 1: Rates of EMRI merger for three ranges of supermassive black hole mass and four types of compact objects, based on Aller and Richstone 2002 E+S0+Sa/Sb black hole space densities, Freitag 2001 merger rates for 10^{6.5}$M_\odot$ black hole, and rate scaling with black hole mass from equation 2.

Considering observed central cusp properties, relaxation and dynamical friction leads to simple prediction of EMRI rate as function of cosmic time $t$:

\[
\frac{1}{2} \frac{f M_{\odot}}{mt} \simeq \left(\frac{f M_\bullet}{2mt}\right) \tilde{M}^{3/8} \langle m \rangle^{1/2} \simeq 10^{-4} f \tilde{M}^{3/8} (m_*/M_\odot)^{-1/2} t^{-1}, \tag{2}
\]

where we used $\langle m \rangle = 1M_\odot$. For $M = M_\bullet = 5 \times 10^4 M_\odot$, the EMRI rates predicted by this simple equation agree with Freitag's rates (perfectly for 10$M_\odot$, factor of 4 overest for WDs). We have used this functional form to extrapolate Freitag's rates for the model Milky Way to other $M_\bullet$. 

EMRI rates per year detectable by LISA and incoherent summation.

<table>
<thead>
<tr>
<th>$M_*$</th>
<th>$m$</th>
<th>LISA Optimistic</th>
<th>LISA Pessimistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{5.5} \pm 0.25$</td>
<td>0.6</td>
<td>8</td>
<td>0.7</td>
</tr>
<tr>
<td>$10^{5.5} \pm 0.25$</td>
<td>10</td>
<td>739</td>
<td>89</td>
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<tr>
<td>$10^{5.5} \pm 0.25$</td>
<td>100</td>
<td>1*</td>
<td>1*</td>
</tr>
<tr>
<td>$10^{5.0} \pm 0.25$</td>
<td>0.6</td>
<td>94</td>
<td>9</td>
</tr>
<tr>
<td>$10^{5.0} \pm 0.25$</td>
<td>10</td>
<td>1000*</td>
<td>800</td>
</tr>
<tr>
<td>$10^{5.0} \pm 0.25$</td>
<td>100</td>
<td>1*</td>
<td>1*</td>
</tr>
<tr>
<td>$10^{5.5} \pm 0.25$</td>
<td>0.6</td>
<td>67</td>
<td>2</td>
</tr>
<tr>
<td>$10^{5.5} \pm 0.25$</td>
<td>10</td>
<td>1700*</td>
<td>134</td>
</tr>
<tr>
<td>$10^{5.5} \pm 0.25$</td>
<td>100</td>
<td>2*</td>
<td>1*</td>
</tr>
</tbody>
</table>

Table 2: Conservative rates would divide WD rates ($0.6 M_\odot$) by 100 (Sigurdsson & Rees and Bender & Hils rates instead of Freitag). $10 M_\odot$ black hole capture so efficient that reduction of 10 is conservative. Sheth et al SMBH density (probably wrong) would reduce numbers by $\sim 20$. 
Geodesy: Use orbiting satellites to measure (vacuum) external gravitational potential of earth. General solution to Laplace equation is:

$$\Phi = -\frac{GM}{r} + \frac{GM}{R} \sum_{lm} \left(\frac{R}{r}\right)^{l+1} B_{lm} Y_{lm}(\theta, \phi)$$

External multipoles $B_{lm}$ are determined by matching to internal structure at surface: so provide info on interior!

GRACE mission: doing this for earth with high precision.
Bothrodesy: geodesy for black holes

[Bothros (βοθροσ): ancient Greek for sacrificial pit.]

Use properties of satellite orbits to “map” the spacetime of massive “Black Hole [?]. Powerful test of black hole hypothesis: black holes have very special multipole moment structure. Axisymmetric, so $B_{lm} = B_1$ and external spacetime can be built from “mass moments” $M_l$ and “current moments” $S_l$:

$$M_l + iS_l = M(i\alpha)^l$$

Only TWO moments are independent!!

Once we measure two of them, we have enough information to falsify black hole hypothesis.
“It is well known that the Kerr solution ... provides the unique solution for stationary black holes ... in the universe. But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the foreseeable future.”

Subrahmanyan Chandrasekhar, The Karl Schwarzschild Lecture, 18 Sept 1986
Bothrodesy: pros and cons

+ Numerical GR probably not needed: small expansion parameter ($m/M<10^{-5}$). ODEs!
+ Solved for circular orbits in Kerr, general orbits in Schwarzschild.
~ Only `solved in principle’ (?) for general eccentric inclined orbits in Kerr (Mino 2003, Barack & Ori 2003)
- Best current estimates: can do coherent brute-force template-matching only for weeks-month. Will have to stack/hierarchical search segments.
Is sum of 6 sources: 2 chirps and 4 fixed freq
Separation of more complicated sources

- Number of independent waveforms LISA could measure with power S/N given by Shannon’s theorem
- \( N_{\text{ind}} = 2^{(f T \log_2(1+S/N))} \) in bandwidth f.
- LISA in limit S/N<<1, so coherent detection amplitude \( \text{SNR} = (fT*S/N)^{1/2} \)
- \( N_{\text{ind}} = 10^{(0.59 \ \text{SNR}^2)} = 10^{531} \) for S/N=30 detection threshold!
- Much larger than \( 10^{20} \) templates actually searched! So templates sparse in signal space.
\( \theta(r) = \sqrt{2r_{\text{min}}/3r} \),

\[
\frac{t_{GW}}{t_{\text{orb}}} = \frac{24\sqrt{2}}{85\pi} \left( \frac{3}{2} \right)^{7/2} \frac{M_h}{m_*} \left( \frac{r}{r_s} \right)^{5/2} \theta^7.
\]

Consider two regimes; where the scattering angle is small compared to \( \theta \), which we refer to as "diffusion", and scattering where the scattering angle is large compared to \( \theta \), which we refer to as "kicks". The scattering in the respective regimes to gravitational radiation, we require \( t_{\text{scat}} \geq t_{GW} \), or that \( \theta \leq \theta_{\text{crit}} \). Hence for "kicks", we find

\[
\theta_{\text{crit}} = \sqrt{3} \left( \frac{85\pi}{24\sqrt{2}} \right)^{1/7} \left( \frac{M_h}{m_* N_*(r)} \right)^{1/7} \left( \frac{r}{r_s} \right)^{-5/14}, \tag{13}
\]

and for diffusion

\[
\theta_{\text{crit}} = \sqrt{3} \left( \frac{85\pi}{24\sqrt{2}} \right)^{1/5} \left( \frac{M_h}{m_* N_*(r)} \right)^{1/5} \left( \frac{r}{r_s} \right)^{-1/2}. \tag{14}
\]
Swan songs

• Inspiral of compact objects into supermassive black holes from $r=4M$ to last stable orbit.

• Is 13 Msun inspiraling into $1.3 \times 10^5$ Msun, sped up by $10^4$ to get audio frequency (i.e. 0.013 into 130Msun).

\[
\frac{a}{M} = 0.359, \text{ circular, } i=20, \text{ view 60 deg from pole: }
\]

\[
\frac{a}{M} = 0.998, \text{ circular, } i=20, \text{ view 60 deg from pole: }
\]

\[
\frac{a}{M} = 0.95, \ e=0.95, \ i=25, \ \text{semilatus Rectum=} a(1-\ e^2)=5M
\]

\[
\]