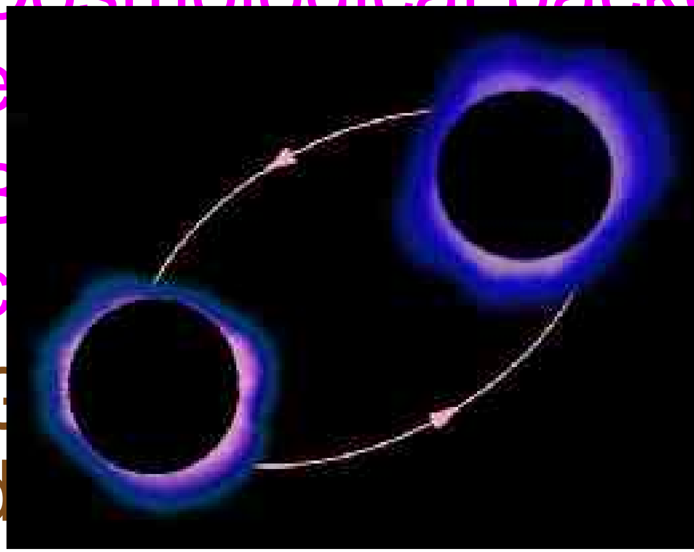
(Super)massive Black Holes in Galactic Nuclei and LISA

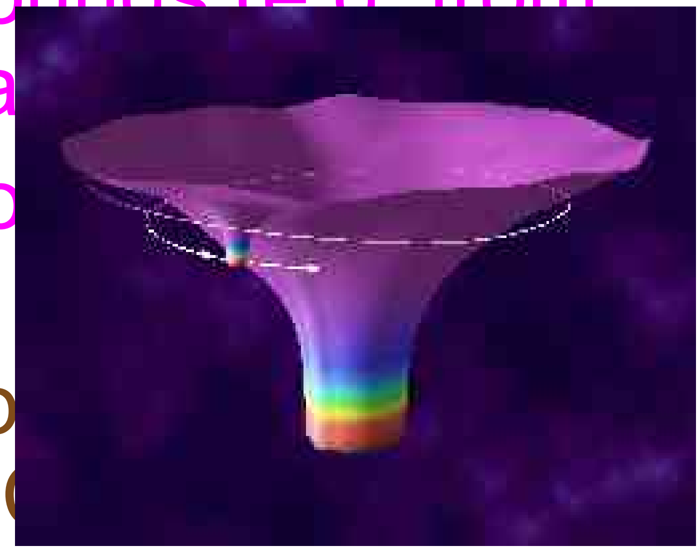
Sterl Phinney
Caltech

14 July 2004: 5th LISA symposium

LISA sources

- Cosmological backgrounds (e.g. from e … tra
- B … fro c
- G … rf b d … M (





- Supermassive and intermediate mass black hole binaries merging.
- Compact objects spiralling into supermassive black holes in galactic nuclei.

Merging supermassive black holes

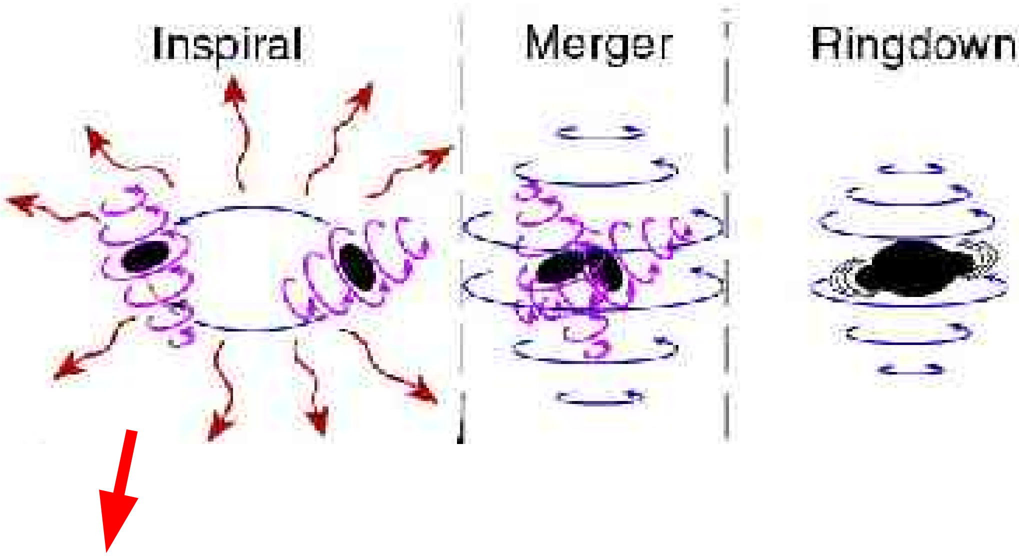
R=1kpc, 1/50 ULIRG

3C75 in Abell 400

R=7kpc, 1/173 3CRR

Also LBQS 0103-2753
Junkkarinen, Shields et al 2001.
R=2kpc, $\tau_{df}=10^7$y, 1/500QSO

Merging supermassive black holes



Post-Newtonian. Consider 10^6+10^6 Msun at z=1, drawn to scale:

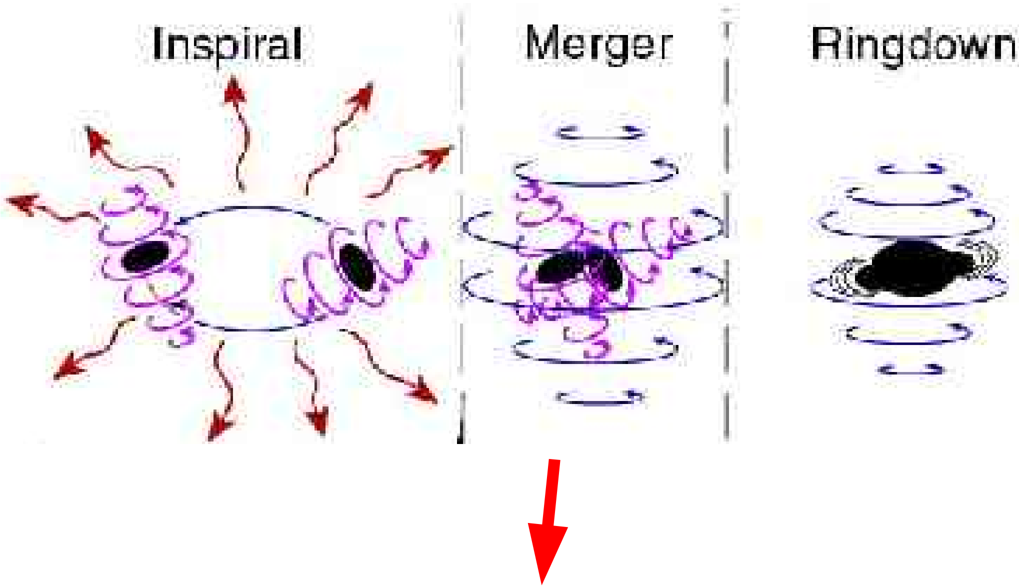
0.00002Hz, SNR=5, t=2yr, 700 orbits, 20 L-S precessions to go

0.00003Hz, t=1yr, SNR=16

0.0001Hz, t=13d, SNR=65

0.0003Hz, t=1 d, SNR=120

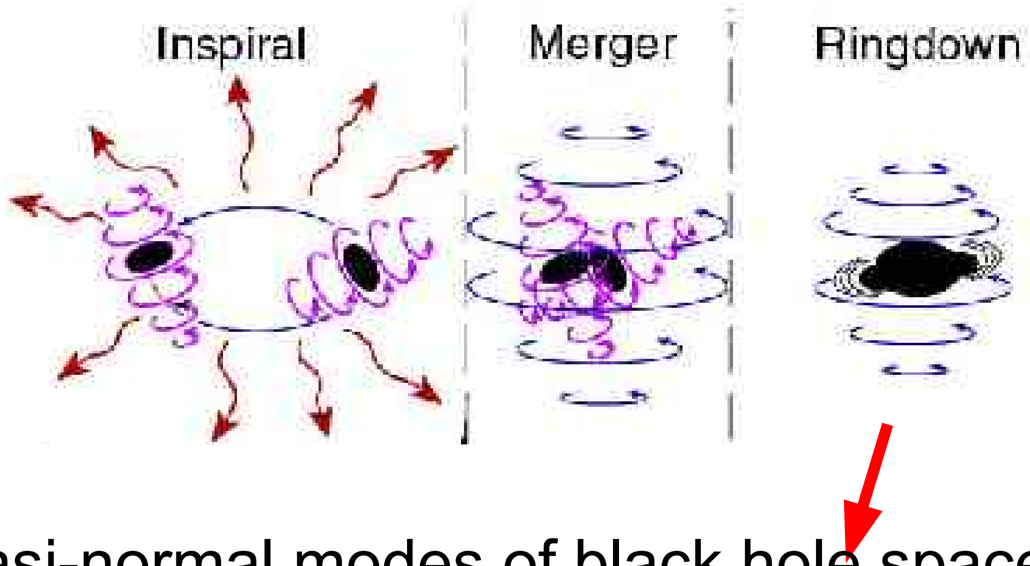
Merging supermassive black holes



???

Hard! cf. Bernard Schutz talk. Dynamical strong gravity.
Nonlinear numerical relativity in 4 space-time dimensions

Merging supermassive black holes



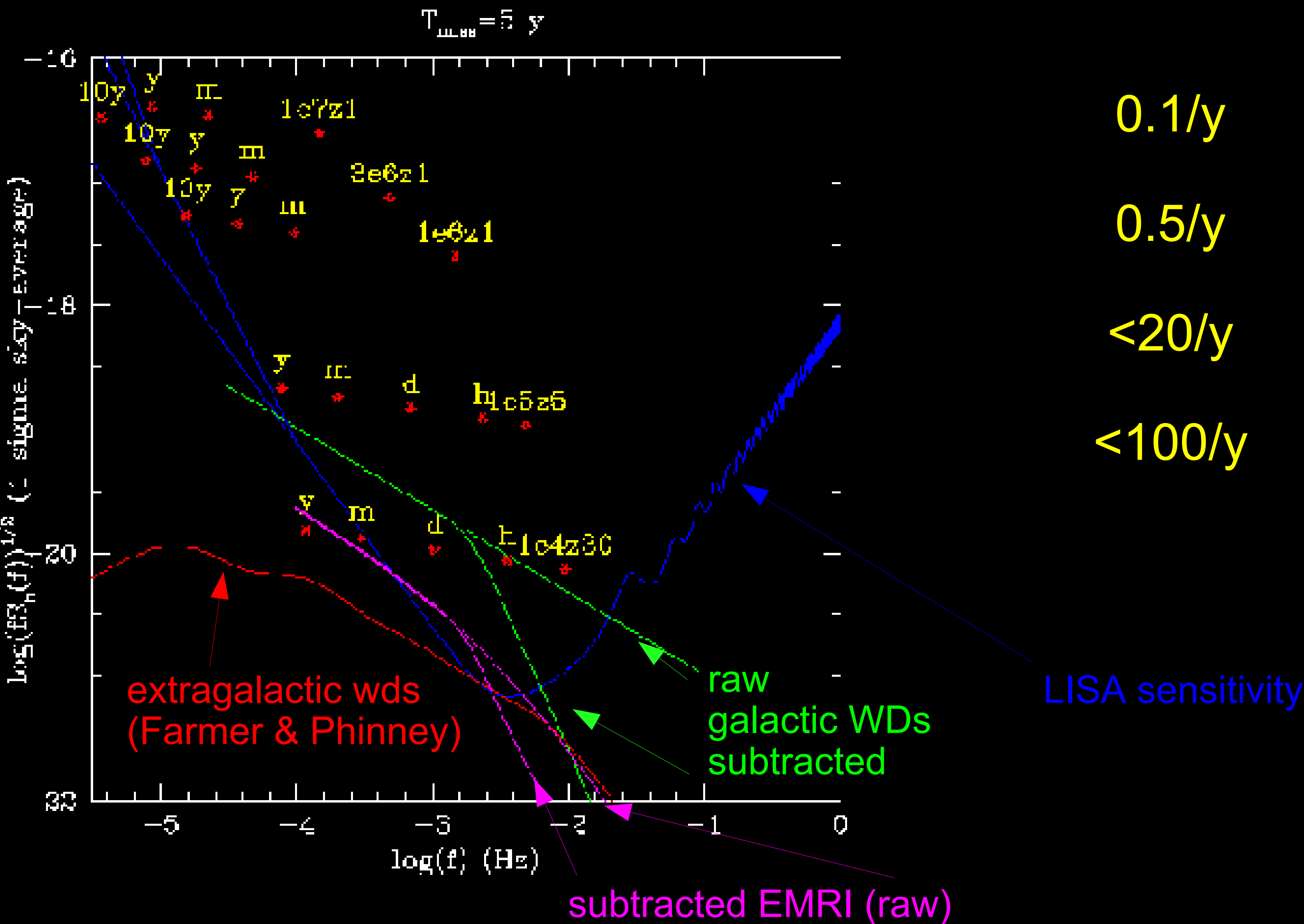
Quasi-normal modes of black hole spacetime.
Amplitudes unknown, but Re(ω_i) and Im(ω_i) known as functions of M, a/M.

Find simultaneous solutions for all observed i -> determine a/M, M.

Find out how much pre-merger L went into final S=aM. Can this make rapid spins?

Merging supermassive black holes

Points: 1yr 1 mo 1day (1 hour) ringdown

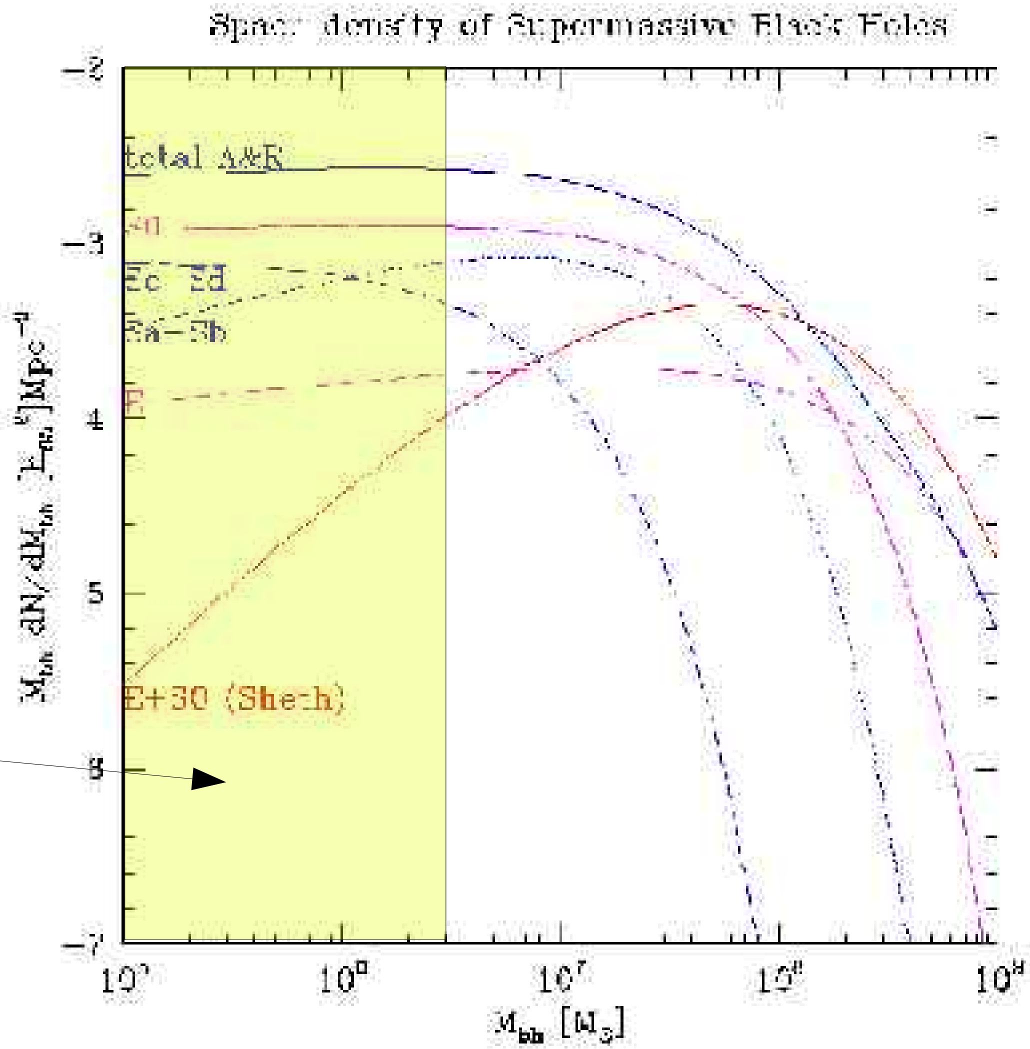


Local mass density of black holes dominated by $>10^8$ Msun. AGN light implies 50-100% of their growth by gas accretion: no GWs!

LISA is insensitive to these.

Sensitive mainly to $<3\times10^6$ Msun holes:

Only 8 reliably known, and unmeasured outside local group except in Low L AGN (cf Barth) where occupation fraction undetermined.



We know the present space density, and dark halo merger history. But not black hole growth (by accretion, at least 50%) and merger history: LISA rate model-dependent.

Merger Rate(s) of Supermassive Black holes

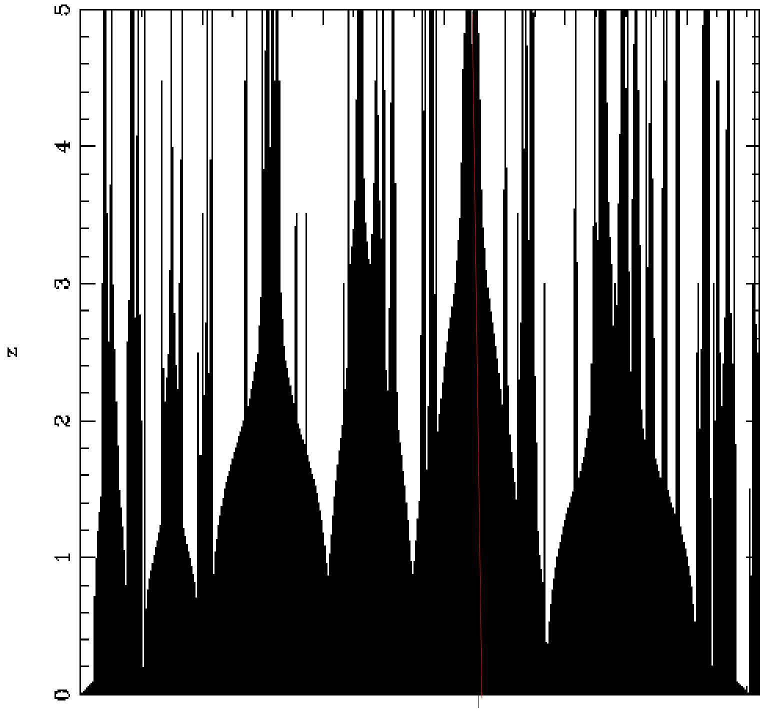


Figure 1. Merger tree for an elliptical galaxy of mass ~ 2.3 × 10^{11} $M_{\odot}$, beginning at redshift 5, and proceeding up to the

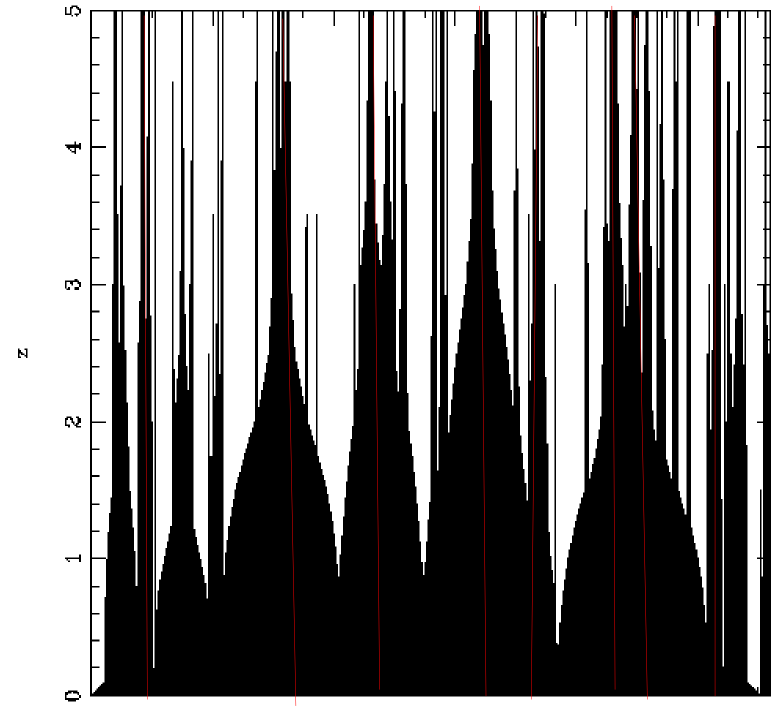


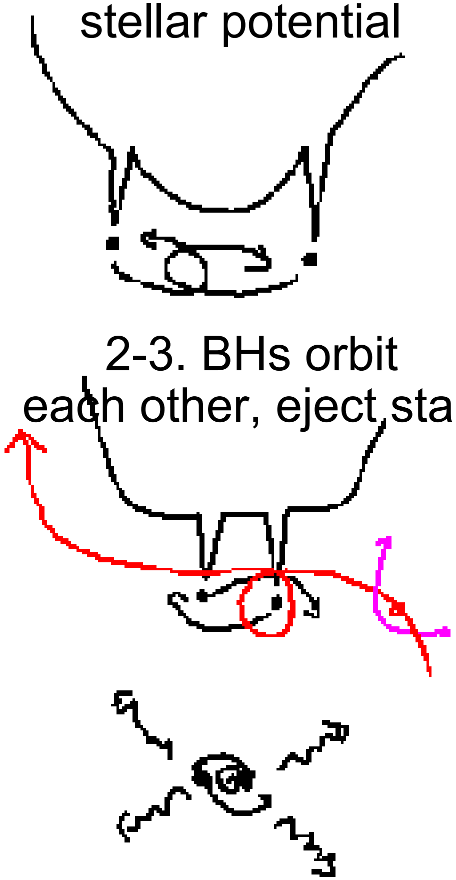
Figure 1. Merger tree for an elliptical galaxy of mass ~ 2.3 × 10^{11} $M_{\odot}$, beginning at redshift 5, and proceeding up to the

Given CDM halo merger tree: depends on
*Occupation density of seeds in halos
*Redshift at which they grow to >10^5 Msun
*Retention in small halos (GW recoil, 3body)
*Relative growth via gas accretion vs merger.

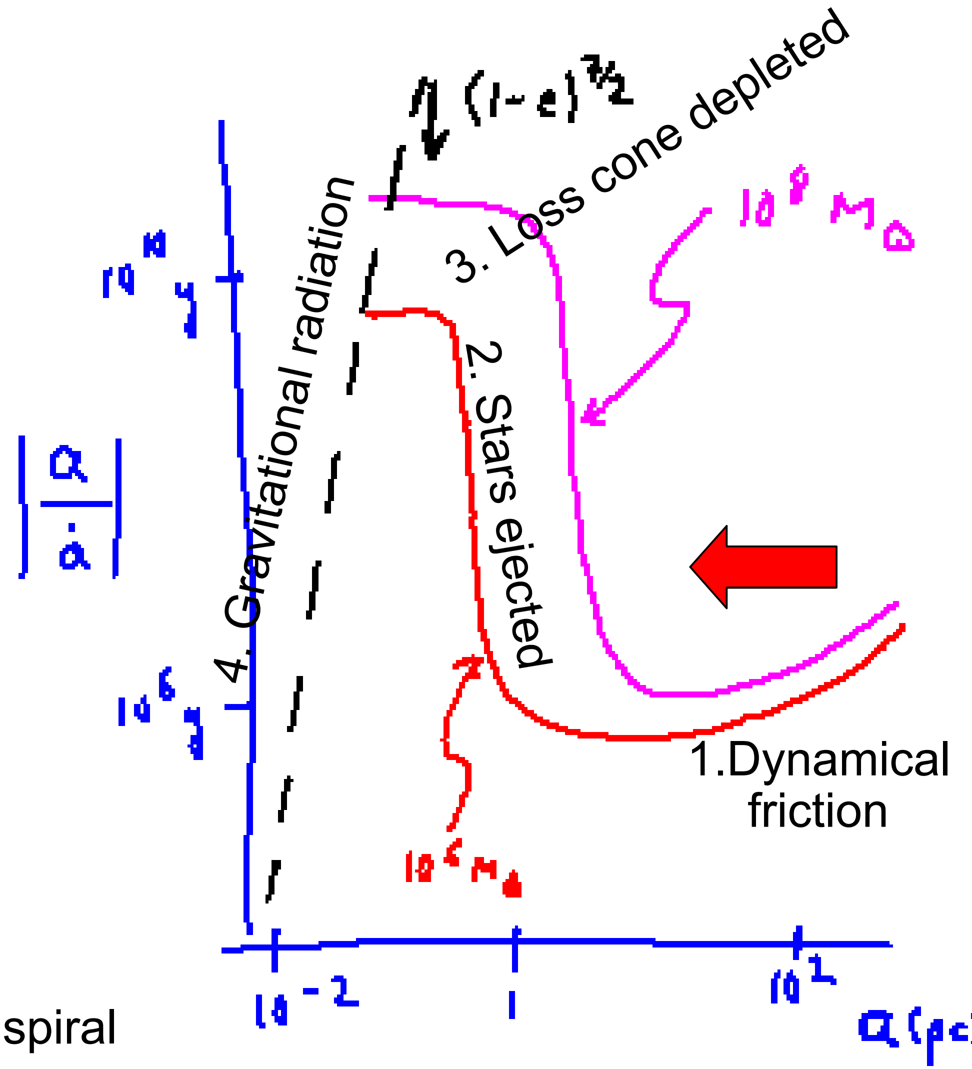
Do the black holes merge?

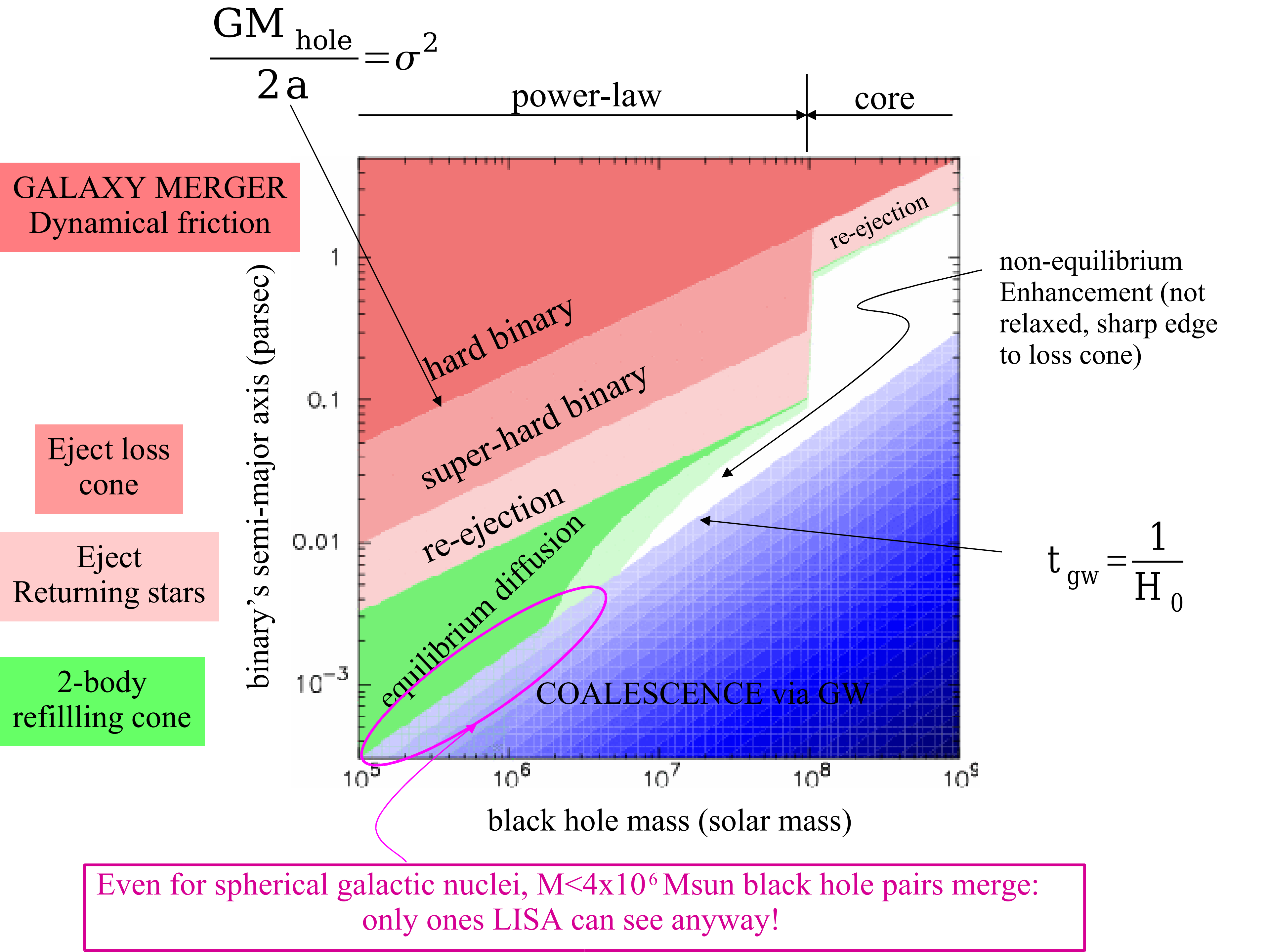
1. BHs orbit stellar potential

2-3. BHs orbit each other, eject stars



4. BHs orbit each other, GR inspiral




$\frac{GM_{hole}}{2a} = \sigma^2$
power-law
core
GALAXY MERGER
Dynamical friction
Eject loss cone
Eject Returning stars
2-body refillling cone
hard binary
super-hard binary
re-ejection
re-ejection
equilibrium diffusion
COALESCENCE via GW
non-equilibrium Enhancement (not relaxed, sharp edge to loss cone)
$t_{gw} = \frac{1}{H_0}$
binary's semi-major axis (parsec)
1
0.1
0.01
10^{-3}
10^5
10^6
10^7
10^8
10^9
black hole mass (solar mass)
Even for spherical galactic nuclei, M<4x10^6 Msun black hole pairs merge:
only ones LISA can see anyway!

Rates of merger of supermassive black holes

- Every respectable galaxy with a bulge contains a central black hole. CDM: every galaxy has merged more than once.

✓ Milky Way and M31 in 5,000,000,000AD,

✓ Toomre 1977, Carlberg et al 2000 ApJ 532 L1: merger rate per galaxy 1 per 10^{10}y

✓ 1/100 local galaxies AGN. ~1/100 AGN have second incoming black hole.

- ~10^{10} galaxies in z<2.
- If black holes find each other, expect ~1 SMBH/SMBH merger per year.
- Could be much higher: each galaxy today product of ~1000 mergers of subunits. If each subunit had MMBH, up to 1000 mergers per year!

Merger Rate(s) of Supermassive Black holes

Mainly gas accretion. >10^6 Msun seeds in large low z fragments to avoid Supereddington, recoil:
LISA merger rate ~1/y, z<5, 10^6-10^7 Msun.

Haehnelt 1997
Kauffman & Haehnelt 2000

Mainly mergers. <10^4 Msun seeds in small high-z fragments
LISA merger rate ~300/y, z~20, 10^4-10^5 Msun.
A Nuisance!

Wyithe & Loeb 2003
Volonteri et al 2003
Menou & Haiman 2004

17 dimensional parameter space of black hole binary waveforms:

- 3 center of mass coords rel to $\odot$, e.g. D_L, λ, β (ecliptic coords).
- 2 angles times 2 BHs' spin directions.
- 2 masses, and 2 dimensionless spin parameters for BHs.
- 6 initial phase space coordinates of the relative motion. Conveniently, 3 secularly varying actions (or functions thereof, e.g a, e, i), and 3 rapidly varying phases.

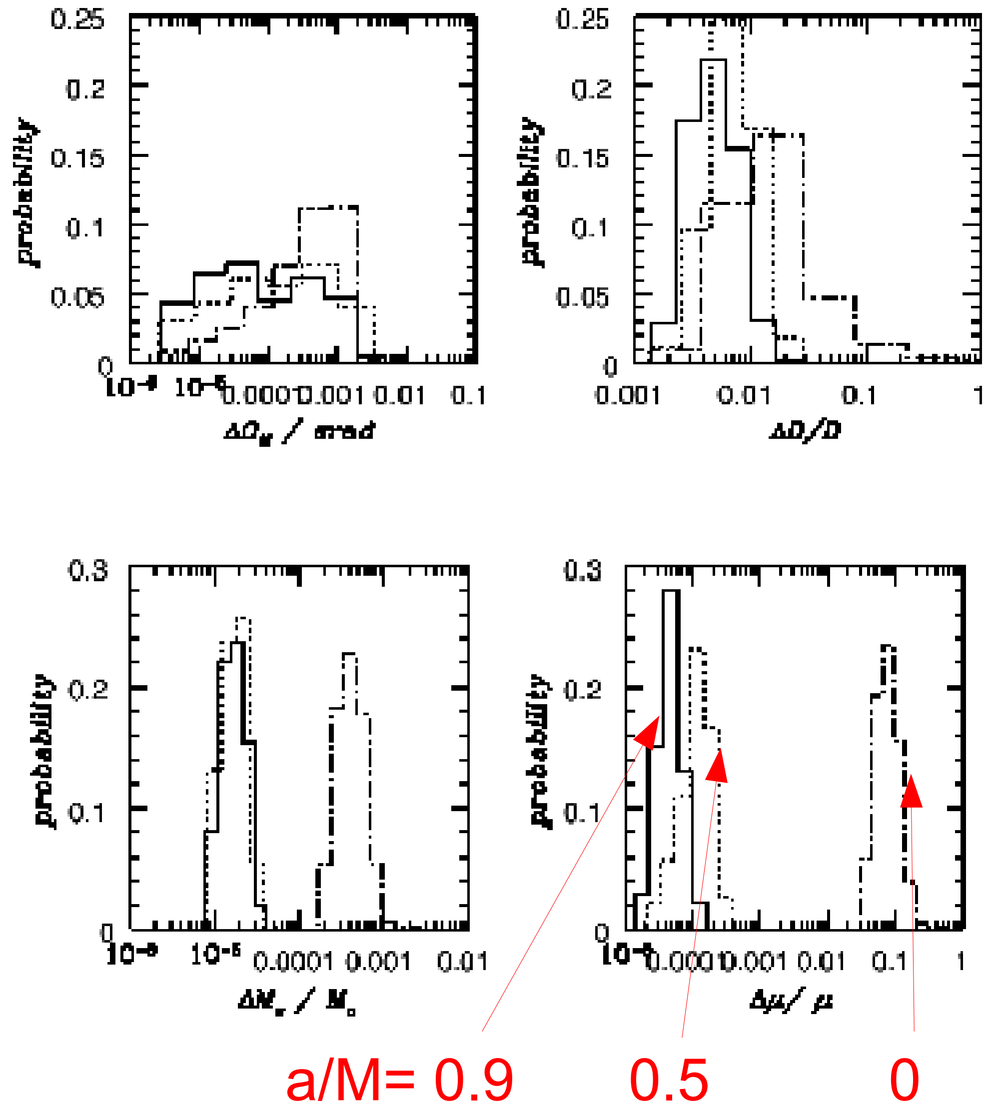
Note: if one $M_2 \ll M_1$, can drop M_2's spin and spin direction, reducing to 14 parameters.

NB: Strong covariance of some parameters in waveform templates (e.g. in Newtonian limit, only one phase, a frequency $\sqrt{M/a^3}$, and λ and β matter).

Accuracy of Parameter determination

- C. Cutler, Phys. Rev. D 57, 7089 (1998). [phase only]
- S. Hughes, MNRAS 331, 805 (2002). [phase only]
- A. Vecchio, astro-ph/0304051 (2004). [phase and amplitude]

Shown: 10^6+10^6 at z=1
Spin-orbit coupling removes degeneracy between position and inclination, increases accuracy of D,pos x5



Merging SMBH -what can we learn from gravitational waves?

- What merging black holes (M,z) can LISA see? [Compare with pulsar timing -large M]
- What are possible/plausible today rates?
- M,z distributions?
- What parameters can LISA measure?
  - z not direct, but can do indirectly!
  - spins, mass, mass ratio, spin-orbit inclination.
  - Tests of strong-field time-dependent gravity, cosmic censorship... (similar to LIGO/Virgo... but more precise).

Are “black holes” black holes?

Einstein 1939 [*Ann of Math* numerical paper]:

"The essential result of this investigation is a clear understanding as to why 'Schwarzschild singularities' do not exist in physical reality."

Oppenheimer and Snyder 1939:

Maximum neutron star mass. Stars more massive than this are unstable and the collapsing star

"tends to close itself off from any communication with a distant observer; only its gravitational field persists."

Are “black holes” black holes?

Chandrasekhar [*Ryerson lecture* 1975, reprinted in *Truth & Beauty* 1987]:

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity, discovered by the New Zealand mathematician, Roy Kerr, provides the absolutely exact representation of untold numbers of massive black holes that populate the universe."

Touching faith! Is it true?

Are “black holes” black holes?

1. Mass from dynamics, or luminosity: radiation pressure limits mass $M > L\sigma_T/(4\pi G M m_p c)$.

2. Rapid large amplitude variability implies small size $R \sim c\Delta t$ So $GM/R \sim c^2$.

3. Lifetimes t so $Lt/Mc^2 > 0.01 \rightarrow$ accretion, not nuclear.

4. Jets with relativistic speeds.

5. Some low luminosity sources seem to have $L \ll 0.01\dot{M}c^2$. No hard surface $\rightarrow$ horizon?

6. X-ray Fe K-shell lines —redshifts and Doppler boosts imply orbits at $v \sim 0.5c$.

Are “black holes” black holes?

Feynman's remark: If you had one *good* reason, you wouldn't have to give six.

These arguments depend on hydrodynamics, plasma physics, cooling, radiation transport, magnetic fields, particle acceleration....

Black holes are defined by their vacuum space time structure and geodesics.

Can we diagnose black holes using vacuum space time dynamics alone?

LISA as a Dark Energy Probe

Notice D_L determined to 0.4% at z=0.5-3, better than with Supernovae. Limited by weak lensing changes to effective D_L. If EM signal also gives z, get $D_L(z)$ and w, w'. Otherwise if cosmology known, can invert $D_L(z)$ to get z and hence rest-frame black hole mass (GW signal invariant $(1+z)M$).

At lowest PN level:

$$\frac{1}{f}\frac{df}{dt} = \frac{96}{5}\mathcal{M}_z{}^{5/3}(\pi f)^{8/3}$$

$$h_{+,\times} = \frac{\mathcal{M}_z^{5/3} f^{2/3}}{D_L}$$

times functions of inclination and orientation angles.

Here $f(t)$ is the frequency of grav waves at earth, $M = M_1 + M_2$, $0 < \eta = M_1 M_2/M^2 < 1/4$, $\mathcal{M} = \eta^{3/5}M$, $\mathcal{M}_z = (1+z)\mathcal{M}$ is redshifted chirp mass.

Observing f and $\dot{f}$ gives $\mathcal{M}_z$, and the observed h and fitted angles gives D_L.

$\delta D_L/D_L \sim 0.02$ for $a/M = 0$, 0.004 for $a/M = 0.9$ (so precession breaks a degeneracy in inclination/sky position) or if an optical counterpart determines the sky position.

LISA as a Dark Energy Probe

D_L^2 times the gravitational wave flux ($\sim f^2h^2$) is (up to angular factors) the rate of change of orbital energy $\dot{E}$, and $\dot{E}$ causes observable secular change in the gravitational waveform, we can use this to determine $\dot{E}$, and from the measured f, h and fitted angular factors, determine D_L. At lowest PN level:

$$\frac{1}{f_r}\frac{df_r}{dt_r} = \frac{96}{5}\mathcal{M}^{5/3}(\pi f_r)^{8/3}$$

$$\frac{1}{f}\frac{df}{dt} = \frac{96}{5}\mathcal{M}_z{}^{5/3}(\pi f)^{8/3}$$

$$h_{+,\times} \sim \frac{\mathcal{M}^{5/3} f_r^{2/3}}{D_M} = \frac{\mathcal{M}_z^{5/3} f^{2/3}}{D_L}$$

times functions of inclination and orientation angles.

Here f is the frequency of grav waves at earth, $f_r = (1+z)f$ is the emission frequency at source at proper motion distance D_M, $D_L = (1+z)D_M$ is luminosity distance, $M \equiv M_1 + M_2$, $0 < \eta \equiv M_1M_2/M^2 < 1/4$, $\mathcal{M} = \eta^{3/5}M$, $\mathcal{M}_z = (1+z)\mathcal{M}$

Observing f and $\dot{f}$ gives $\mathcal{M}_z$, and the observed h and fitted angles gives D_L.

Typical fractional errors in D_L are 0.02 for $a/M = 0$, 0.004 if $a/M = 0.9$ (so precession breaks a degeneracy in inclination/sky position) or if an optical counterpart determines the sky position.

If cosmological parameters perfectly known, get $1+z$ to 1–2 times frac precision of D_L.

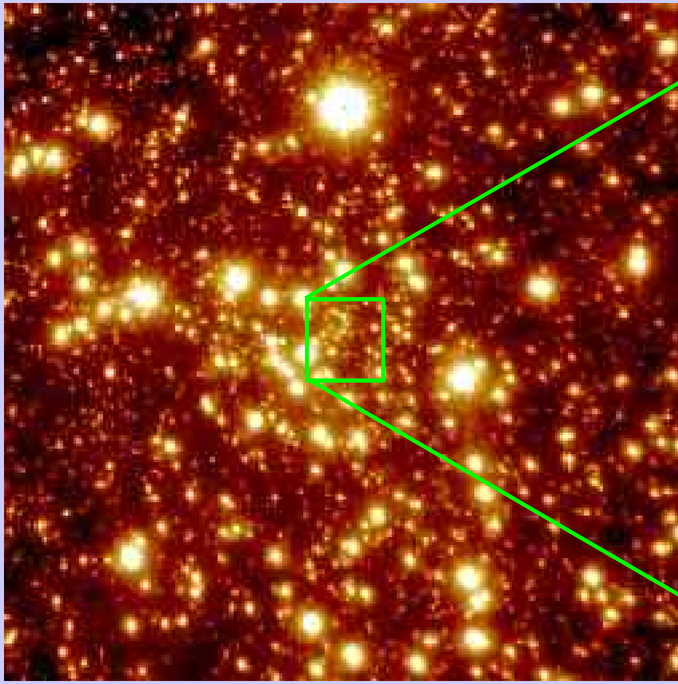
If not, *better than supernovae, clustering, etc* — limited only by knowledge of weak lensing contribution to D_L ($\sim$ few %).

Science return from SMBH

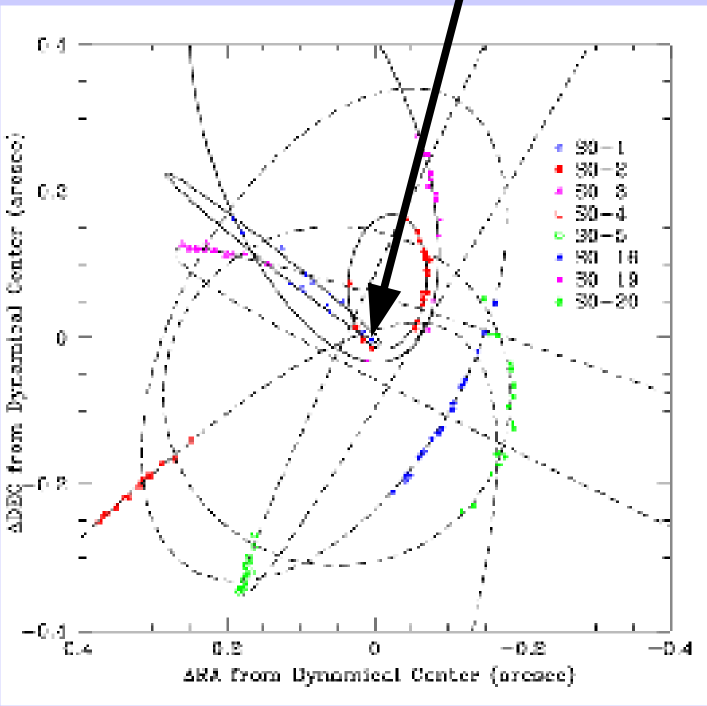
- Merger rates of 10^4 - 10^6Msun black holes at z=1-100.
- Measure black hole spins, $M(1+z)$ distribution.
- High S/N: precision match of inspiral parameters (P^nN) with ringdown: final black hole.
- vs numerical GR: test of strong-field dynamical GR. SNR high -so templates not needed for determining merger signal!
- Precision (<1%: limited by weak lensing!) distance measurements: self calibrating standard candles. If optical display gives independent z, also: cosmography, w, w'.

Stars in the nuclear cusp of the Milky Way

Sgr A* black hole, $3x10^6 M_o$



Genzel et al 2003



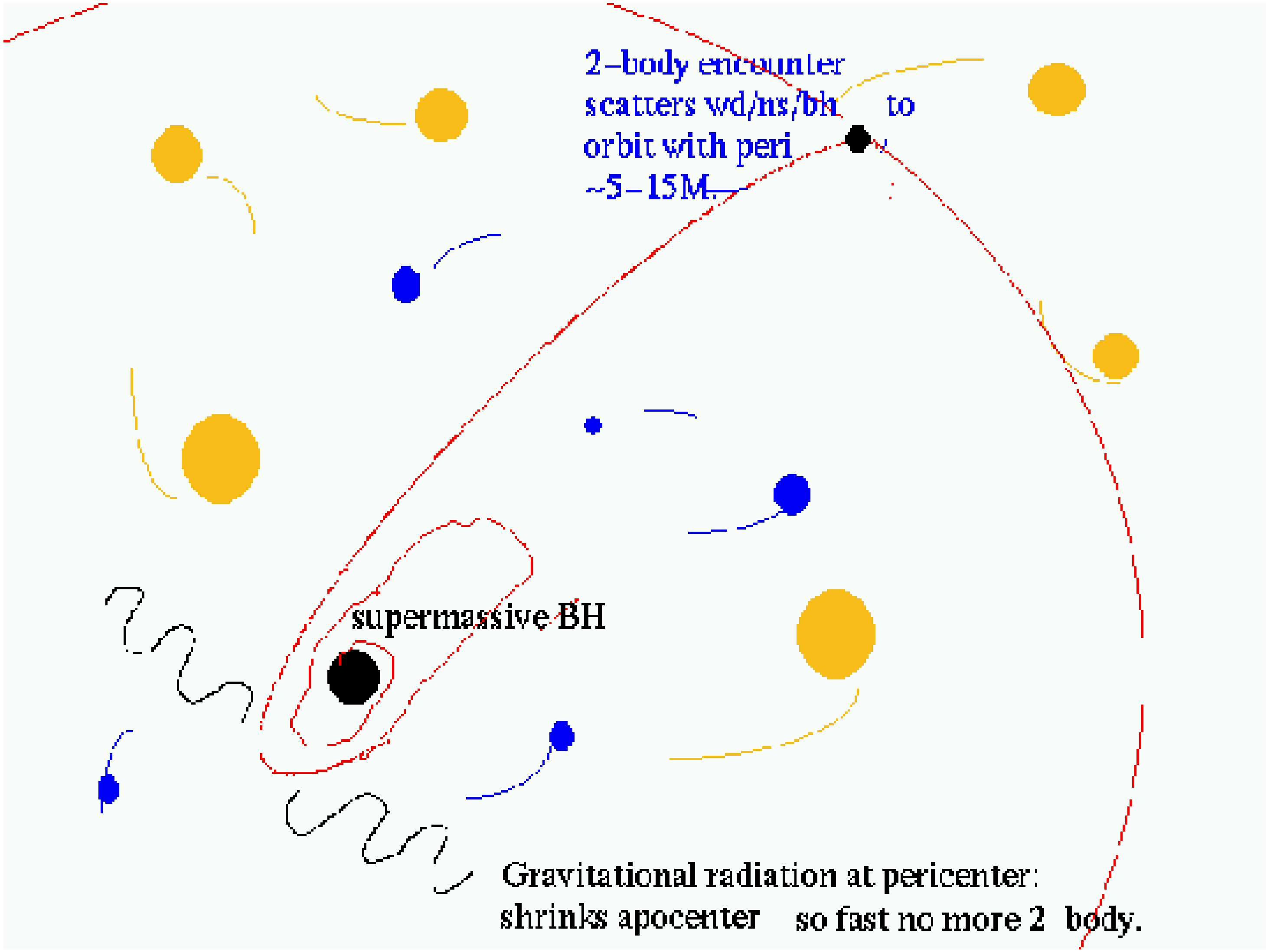
Ghez et al 2003

What about even more eccentric orbits!?

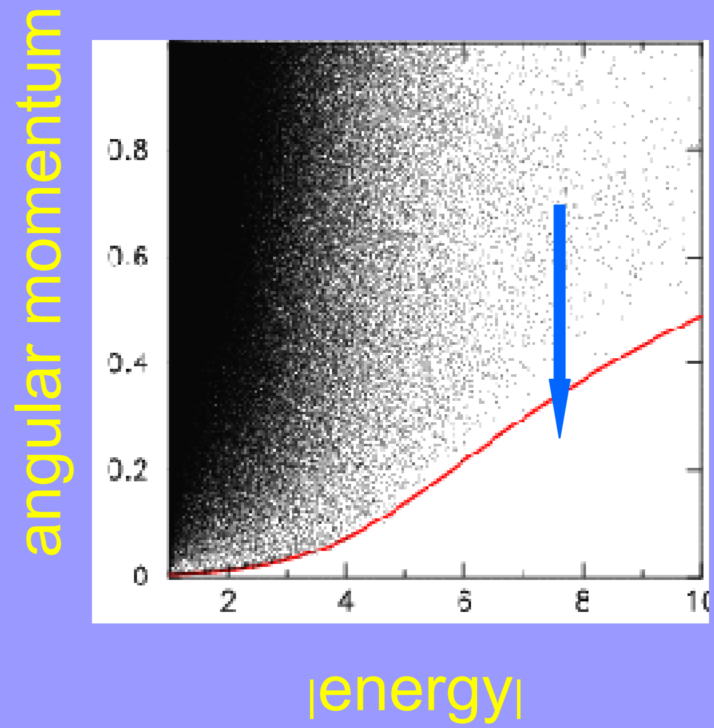
Plunge into black hole
(or tidal disruption if extended)

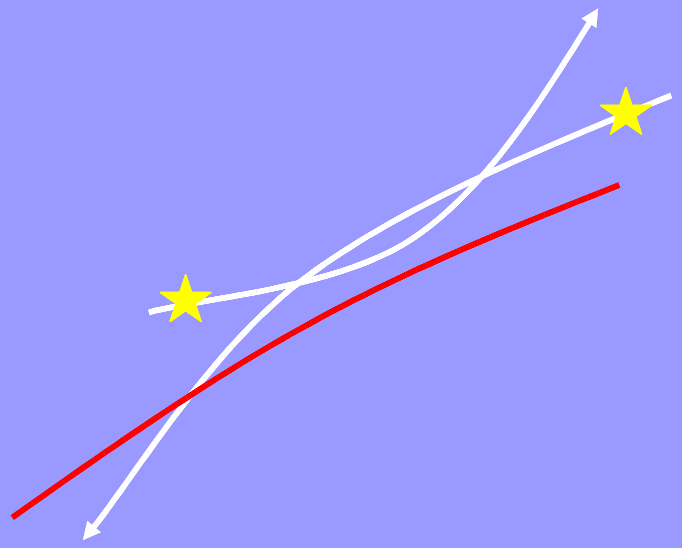
Extreme Mass Ratio Inspiral

Extreme Mass Ratio Inspiral



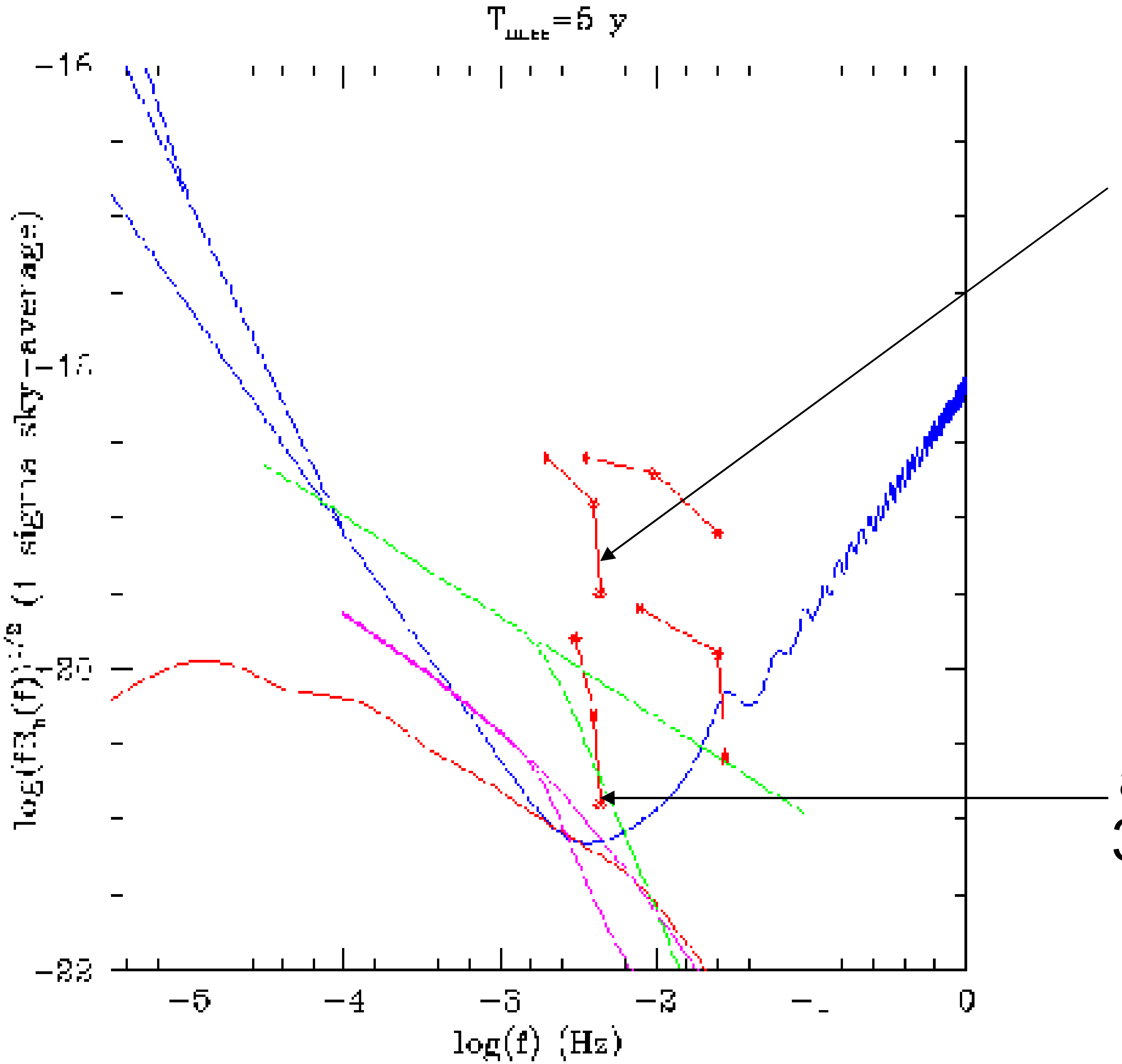
Diffusion into the EMRI Loss Cone





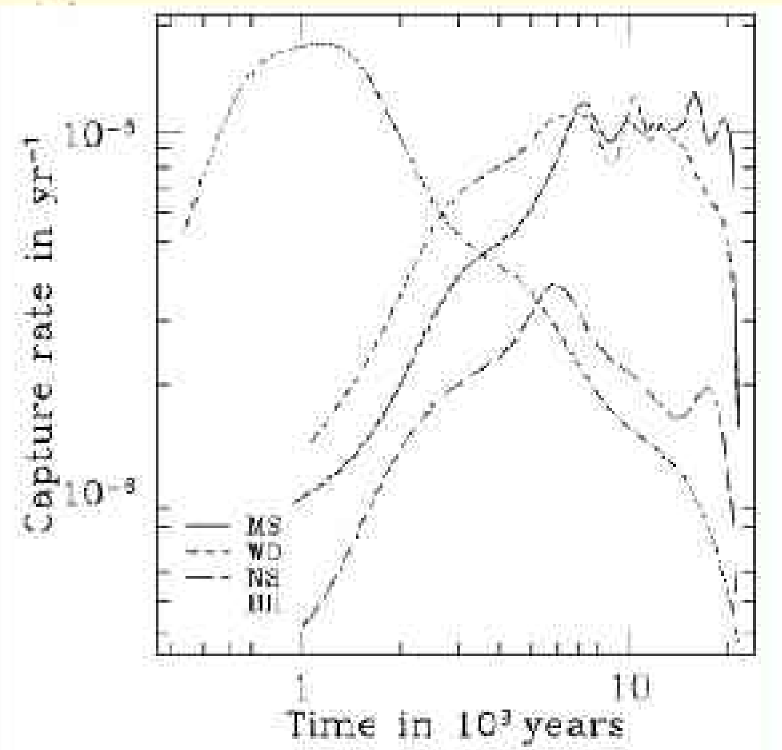
EMRI capture loss cone defined by dln(a)/dt due to gravitational radiation < time to diffuse back out of loss cone.

Hils & Bender 1995, Sigurdsson & Rees 1997, Miralda-Escude & Gould 2000, Freitag 2001,2003



$10/10^6$ z=0.1
a/M=0, 0.998 pro
3/y Freitag rate,
Mass seg,
Kroupa IMF

$1/10^6$ z=0.25,
a/M=0, 0.998 pro
3/y no mass seg,
WD only



Freitag's Milky Way simulation of EMRI capture rates in a model Milky Way ($M_\bullet$(now) = $4 \times 10^6 M_\odot$), as function of cosmic time. WD, white dwarfs, mean mass $0.7 M_\odot$, BH black holes $9 M_\odot$.

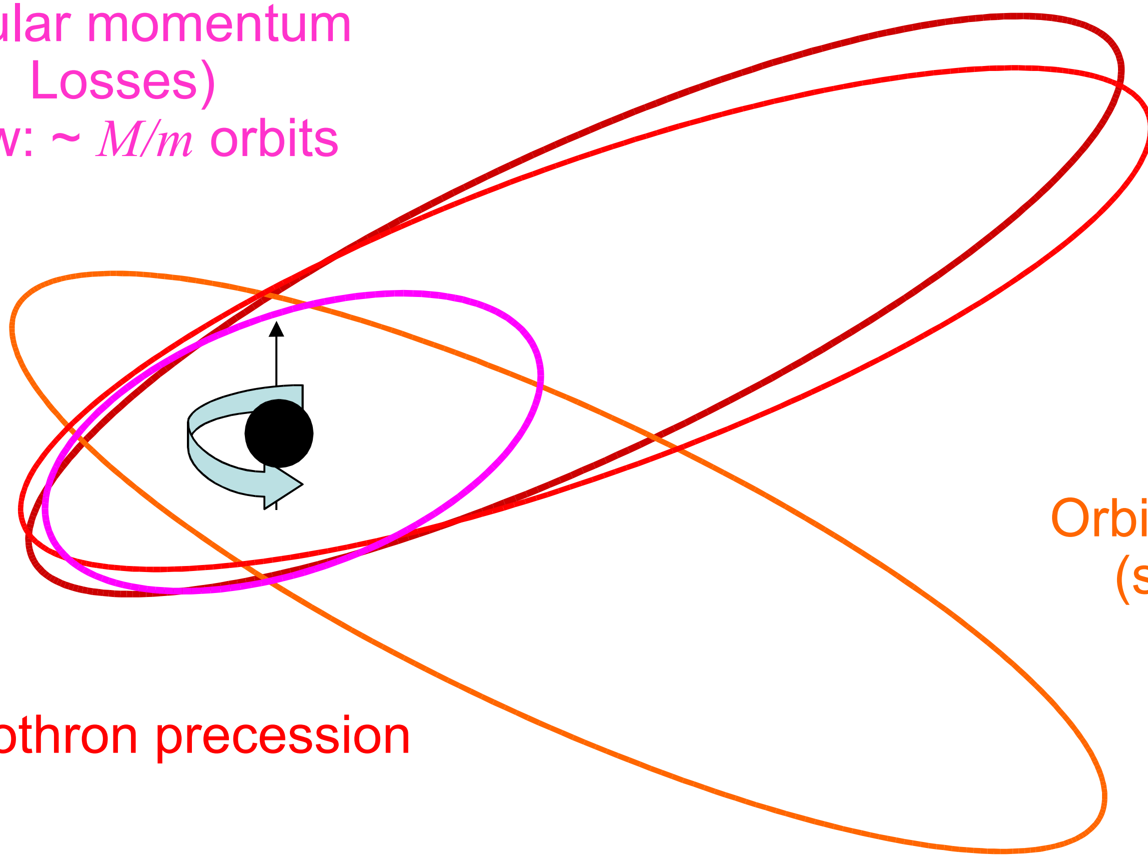
EMRI rate/vol ingredients:
1) SMBH mass function (~obs)
2) Compact object mass function in galactic nuclei (? assume)
3) Density profiles of galactic nuclei (~obs)
4) Star formation histories of galactic nuclei (~obs)
5) Loss cone filling (asphericity..)
6) Accuracy of simulations ?

Reasonable uncertainties: x100

EMRI LISA detection rate ingredients:
1) EMRI rate/vol
2) Black hole a/M, orbit inclination (sets upper freq)
3) Signal detection algorithm (effective SNR for detection)

Orbits and spiral-in of small bodies around spinning black holes

Spiral-in and
Circularization
(GW energy and
angular momentum
Losses)
Slow: ~ M/m orbits



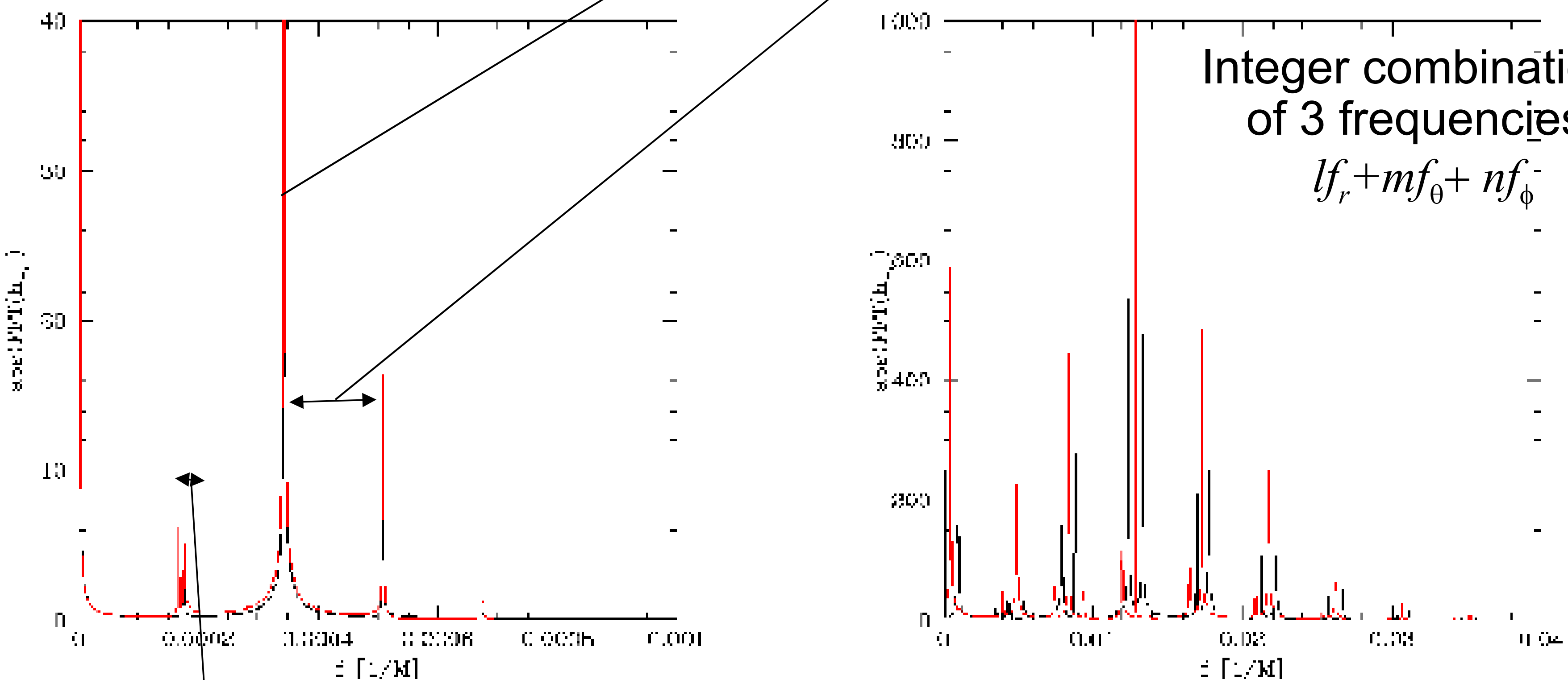
Orbit plane precession
(spin – orbit; L-T)

Peribothron precession

2x orbital freq

a*=0.9, a=100M, e=0.05, i=45

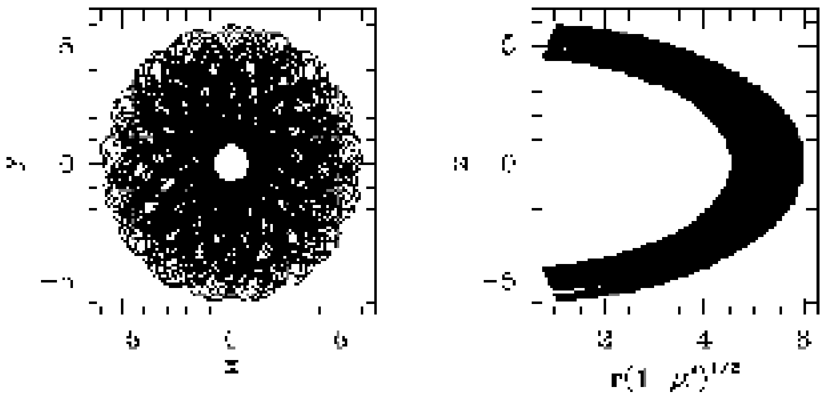
Peribothron precession freq

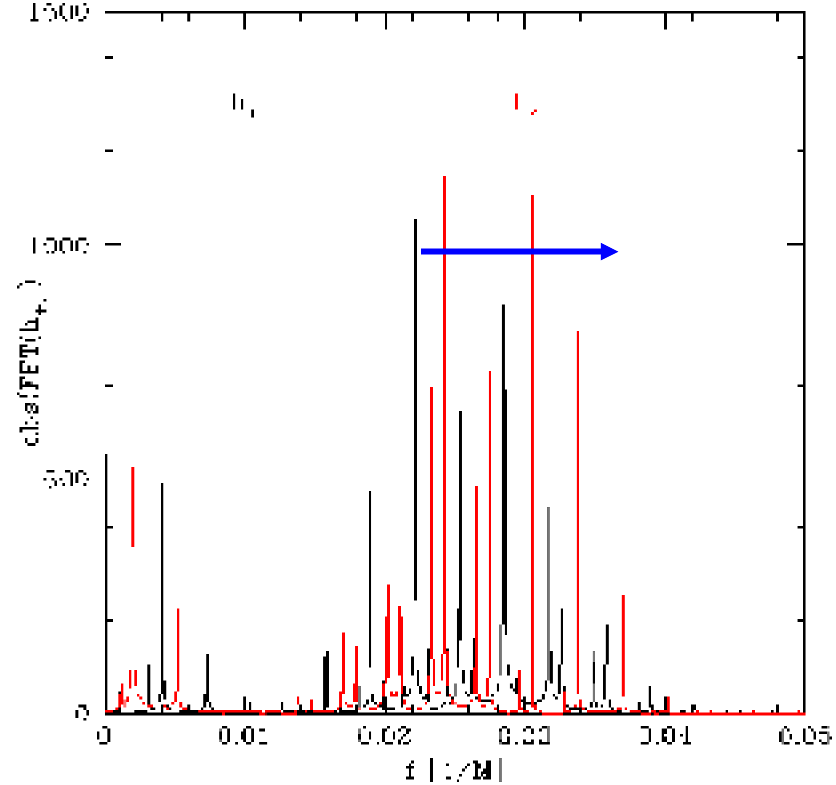


L-T orbit plane precession freq

a*=0.9, a=10M, e=0.2, i=45

a*=0.95, a=6M, e=0.2, i=80

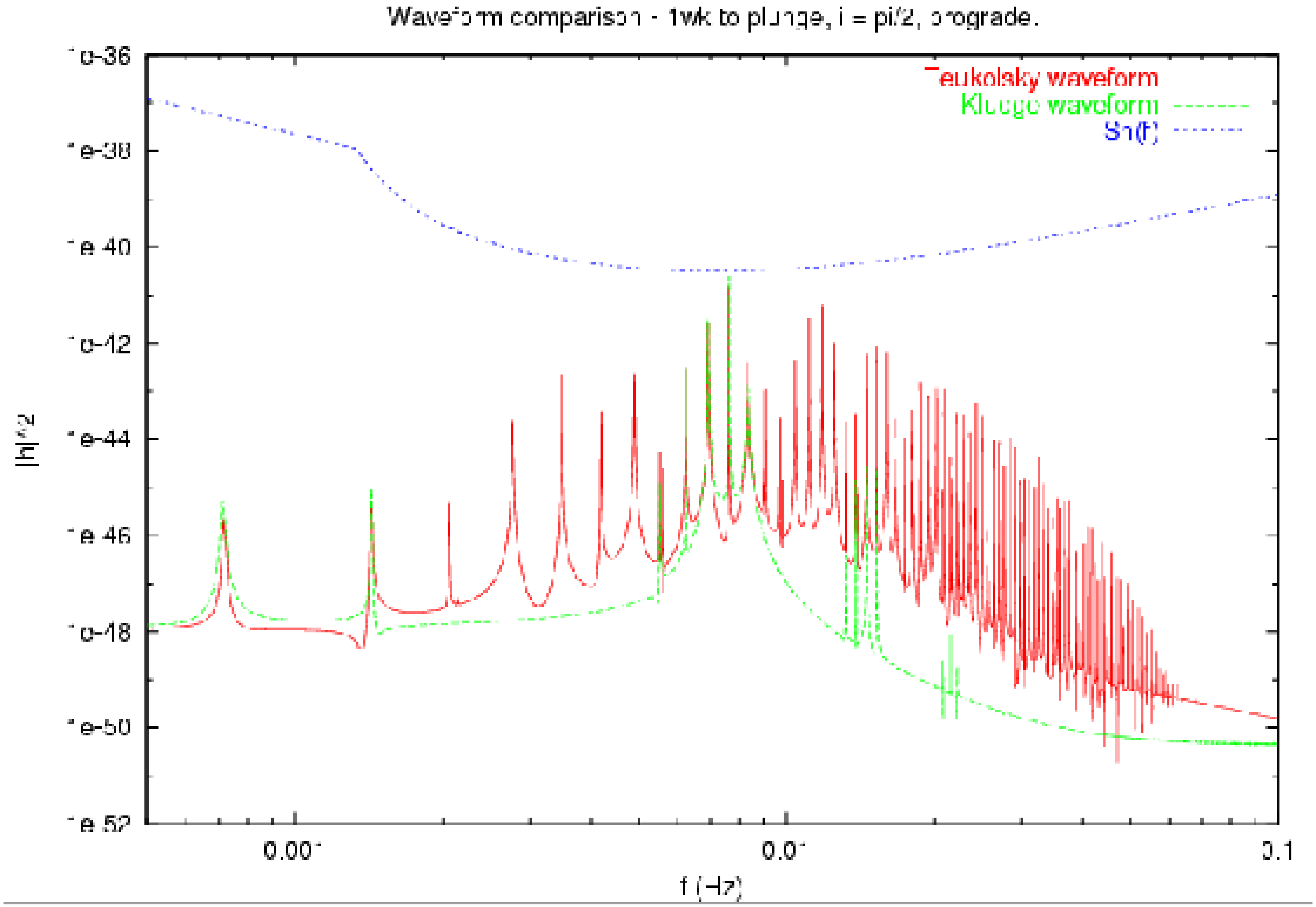


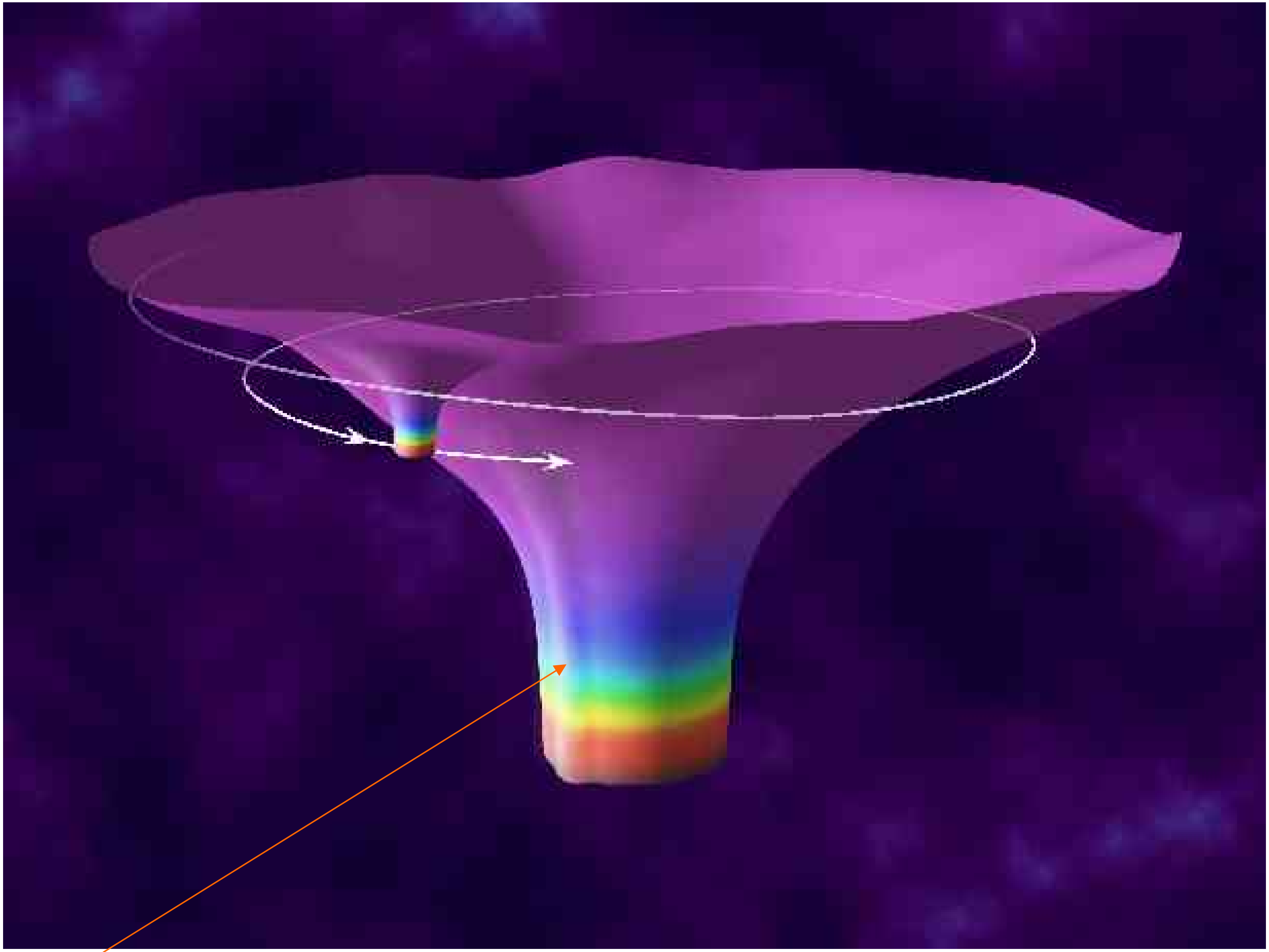


Frequencies sweep
and shift slowly as
compact object spirals in,
mapping space-time outside
the horizon.

cf. geodesy satellites mapping
geopotential

a/M=0.8, i=45 prograde inspiral of 10 into 10^6, 1 week before plunge, e_p=0.4
Comparison of slow motion quadrupole from exact orbit with full GR waveform
From exact orbit. Same total power to 10%.





Tide induced on horizon dragged ahead by black hole rotation: orbiting body gains energy/angular momentum from hole, slowing inspiral ~10% !

Separating sources

$$s(t)=\sum_i h_i(t)+n(t$$

Signal = sources + noise

Wiener optimal filtering (optimal for Gaussian noise –ultimately others.)

$$\langle h(t,\vec{\lambda}), s(t)\rangle = \int_{-\infty}^{\infty} dt\, h(t,\vec{\lambda})s(t)$$

$$= 4\mathrm{Re}\int_0^{\infty} df\, \frac{\tilde{h}^*(f,\vec{\lambda})\tilde{s}(f)}{S_h(f)}$$

template

parameters

$$\tilde{h}(f) = \int_{\infty}^{\infty} dt \exp(i2\pi f t) h(t)$$

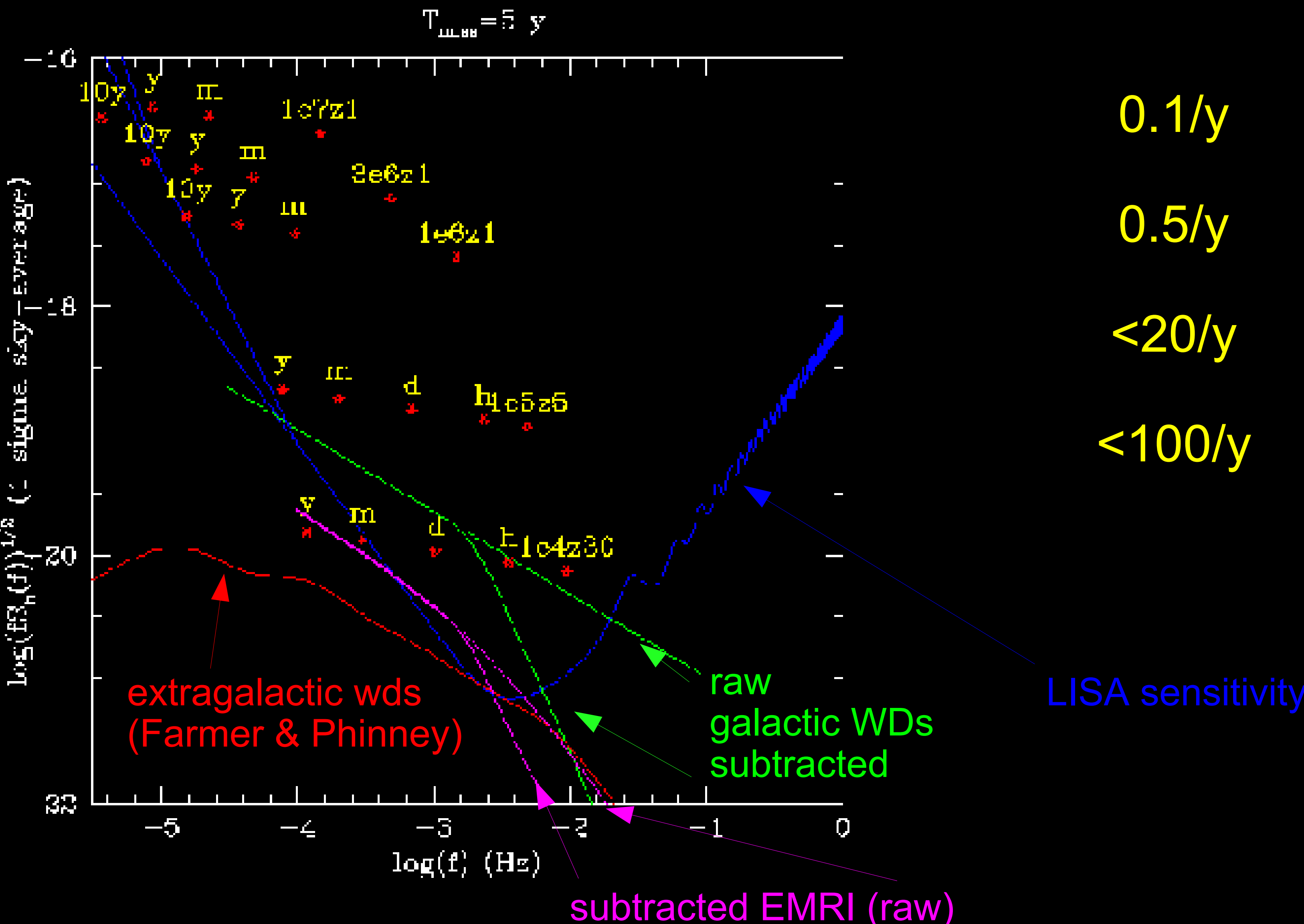
Parameters: 2 trivial for `free' t(0) (Fourier phase) and distance (amplitude of corr).
white dwarfs additional 6: 4 extrinsic (source position, orbit ***L*** direction), 2 intrinsic: *f, df/dt.*
inspiralling compact objects: additional 12: 5 extrinsic (source position, BH spin direction, initial phase of ***L***), 7 intrinsic: (*M,m,S/M*^{2},***L.S***,initial e,initial phase in orbit and peribothron precession). **SMBH** add 3 more –spin ***s*** of second BH.

Must choose grid of templates fine enough that $\frac{\langle h_i, h_j\rangle}{\langle h_i, h_i\rangle}$ > M ~ 0.8, say

Note parameters correlated: optimal template grid is not square.

Merging supermassive black holes

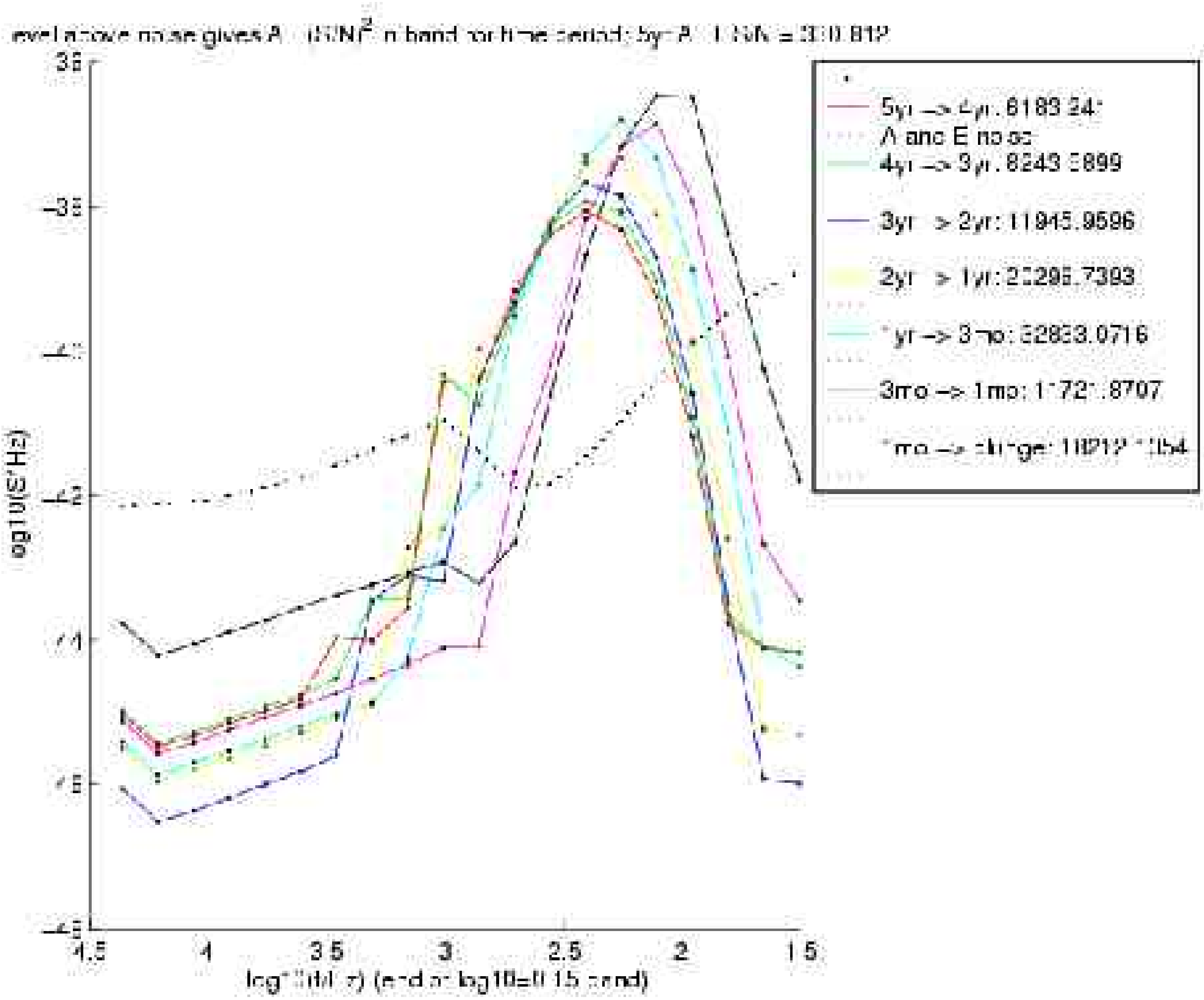
Points: 1yr 1 mo 1day (1 hour) ringdown



Computational requirements for EMRI detection

- Number of templates scales as T^4 for $T<4$ weeks. Faster for $T>4$ weeks (sky position variation affect minimum match).
- Each template requires ($0.1\,T$/s)–point FFT.
- If have 50Tflops dedicated, can correlate templates up to 3 weeks. Then have to stack incoherently (lose factor of 2 in SNR compared to 5-year coherent search (would require $>10^{11}$Teraflops!).
- Does not include computational cost of computing templates (likely to be similarly high).

Contributions to SNR as function of frequency
For 10 into 10^6 Msun, final e_p=0.25 from specified
Time intervals. 5-yr integrated SNR=331.



Predicted number of EMRI sources to be Observed by LISA with 5 year mission.

$M_\bullet$	m	LISA Optimistic	LISA Pessimistic
300 000	0.6	8	0.7
300 000	10	739	89
300 000	100	1*	1*
1 000 000	0.6	94	9
1 000 000	10	1000*	800
1 000 000	100	1*	1*
3 000 000	0.6	67	2
3 000 000	10	1700*	134
3 000 000	100	2*	1*

Precision of EMRI parameter determination

S/M^2	0.1	0.1	0.1	0.5	0.5	0.5	1	1	1
e_{LSC}	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
$\Delta(\ln M)$	2.6e−4	5.6e−4	5.3e−5	2.7e−4	9.2e−4	7.7e−5	2.8e−4	2.5e−4	1.5e−4
$\Delta(S/M^2)$	3.6e−5	7.9e−5	4.5e−5	1.3e−4	6.3e−4	5.1e−5	2.6e−4	3.7e−4	2.6e−4
$\Delta(\ln \mu)$	6.8e−5	1.5e−4	7.4e−5	6.8e−5	9.2e−5	1.0e−4	5.1e−5	9.1e−5	1.0e−3
$\Delta(e_0)$	6.3e−5	1.3e−4	2.9e−5	8.5e−5	2.8e−4	3.2e−5	1.2e−4	1.1e−4	1.6e−4
$\Delta(\cos\lambda)$	6.0e−3	1.7e−2	1.3e−3	1.3e−3	5.8e−3	2.4e−4	5.5e−4	8.4e−4	4.7e−4
$\Delta(\Omega_S)$	1.4e−3	1.6e−3	5.3e−4	1.4e−3	2.1e−3	6.3e−4	1.4e−3	8.3e−4	6.2e−4
$\Delta(\Omega_K)$	5.6e−2	5.5e−2	4.7e−2	5.5e−2	5.2e−2	4.7e−2	5.5e−2	5.1e−2	4.8e−2
$\Delta(\tilde\gamma_0)$	4.0e−1	6.3e−1	3.8e−1	1.0e+0	6.1e−1	3.9e−1	9.3e−1	3.4e−1	3.9e−1
$\Delta(\Phi_0)$	2.6e−1	6.7e−1	2.2e−1	1.4e+0	7.5e−1	2.7e−1	1.5e−0	1.7e−1	3.3e−1
$\Delta(\alpha_0)$	6.2e−1	5.8e−1	5.5e−1	6.3e−1	5.9e−1	5.6e−1	5.4e−1	5.9e−1	5.9e−1
$\Delta[\ln(\mu/D)]$	8.7e−2	3.8e−2	3.7e−2	3.8e−2	3.7e−2	3.7e−2	3.8e−2	7.0e−2	3.7e−2
$\Delta(t_0)\nu_0$	4.5e−2	1.1e−1	3.3e−2	2.3e−1	1.3e−1	4.4e−2	2.5e−1	3.2e−2	5.5 − 2

TABLE II. Parameter accuracy estimates for inspiral of a $10 M_\odot$ CO onto a $10^6 M_\odot$ MBH at SNR=30 (based on data collected during the last year of inspiral). Shown are estimates for the accuracy in determining the various physical parameters, for various values of the MBH's spin magnitude S and the final eccentricity e_{LSC}. The rest of the parameters are set as follows: $t_0 = (1/2)$yr (middle of integration); $\tilde\gamma_0 = 0$; $\Phi_0 = 0$; $\theta_S = \pi/4$; $\phi_S = 0$; $\lambda = \pi/6$; $\alpha_0 = 0$; $\theta_K = \pi/8$; $\phi_K = 0$.

$$\frac{\delta \text{mass}}{\text{mass}} \simeq 10^{-4}\,, \qquad \frac{\delta \text{spin}}{\text{spin}} \simeq 10^{-4}$$

Barak & Cutler 2004

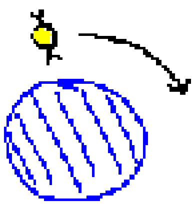
Geodesy

Outside earth,

$$\nabla^2 \phi = 0$$

$$g(\underline{r}) = -\nabla \phi$$

CHAMP, GRACE, GOCE



Measure

$$\frac{d^2 \underline{r}}{dt^2} = g(\underline{r})$$

$$\phi(\underline{r}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} M_{lm} Y^{lm}(\theta, \phi)\, r^{-(l+1)}$$

solve for M_{lm}

Diagnose interior structure by matching

$$\nabla^2 \phi = 4\pi \rho(\underline{r})$$

Bothrodesy

Outside relativistic body (e.g. BH) metric can be expanded in

Mass multipoles

$$M_l = \int d^3r\, r^l P_l(\cos\theta)\, \rho(\underline{r})$$

Current multipoles

$$S_l = \int d^3r\, r^{l-1} (\underline{r} \times \underline{v}\, \rho(\underline{r}))\, P_l(\cos\theta)$$

Axisymmetry : $m = 0$

Equatorial reflection $\begin{cases} M_{odd} = 0 \\ S_{even} = 0 \end{cases}$

M_0 mass

S_1 spin angular momentum

M_2 mass quadrupole

S_3 current octupole

Science return from Bothrodesy

- Are astronomical `black holes' really the Kerr solutions of GR? Or cubical solitons of exotic fields?
  - Precision test of no-hair
- Measure energy extraction from a black hole
  - 5-10% on inspiral rate
- Distribution of M, a/M
  - Important for astrophysics, formation history, accretion solutions, radio jet models…
- If see many (>100) events:
  - 2-10% could be white dwarfs, He stars: tidally disrupted in late stages giving electromagnetic signal, galaxy ID, cosmography.
  - 1% of inspirals in AGN, perturbed by accretion disk

No hair theorem:

Kerr M_0, S_1 specify all M_ℓ, S_ℓ:

$M_0 = M$, $S_1 = J \equiv aM$

$$M_\ell + i S_\ell = M (i a)^\ell$$

e.g. $M_2 = M a^2$, $M_3 = 0$ for Kerr hole

But for uniform density Newtonian rotating sphere

$$M_2 = \frac{25}{8}\left(\frac{R}{GM}\right) M a^2$$

and for earth

$M_3 \neq 0$ ('pear shape')

$M_\bullet$ $M_\odot$	space density $10^{-3}h_{65}^{2}\mathrm{Mpc}^{-3}$	Merger rate $\mathcal{R}$ $\mathrm{Gpc}^{-3}\mathrm{y}^{-1}$			
		$0.6M_\odot$ WD	$1.4M_\odot$ MWD/NS	$10M_\odot$ BH	$100M_\odot$ PopIII
$10^{6.5\pm0.25}$	1.7	8.5	1.7	1.7	1.7×10^{-3}
$10^{6.0\pm0.25}$	1.7	6	1.1	1.1	10^{-3}
$10^{5.5\pm0.25}$	1.7	3.5	0.7	0.7	7×10^{-4}

Table 1: Rates of EMRI merger for three ranges of supermassive black hole mass and four types of compact objects, based on Aller and Richstone 2002 E+S0+Sa/Sb black hole space densities, Freitag 2001 merger rates for $10^{6.5}M_\odot$ black hole, and rate scaling with black hole mass from equation 2.

Considering observed central cusp properties, relaxation and dynamical friction leads to simple prediction of EMRI rate as function of cosmic time t:

$$\frac{1}{2}\frac{fM_{dj}}{mt} \simeq (fM_{\bullet,*}/2mt)\tilde{M}^{3/8}(m/\langle m\rangle)^{1/2} \simeq 10^{-4}f\tilde{M}^{3/8}(m/M_\odot)^{-1/2}\mathrm{y}^{-1}\,, \qquad (2)$$

where we used $\langle m\rangle = 1M_\odot$. For $M = M_{\bullet,*} = 5\times10^{6}M_\odot$, the EMRI rates predicted by this simple equation agree with Freitag's rates (perfectly for $10M_\odot$, factor of 4 overest for WDs). We have used this functional form to extrapolate Freitag's rates for the model Milky Way to other $M_\bullet$.

EMRI rates per year detectable by LISA and incoherent summation.

$M_\bullet$	m	LISA	
		Optimistic	Pessimistic
$10^{5.5\pm0.25}$	0.6	8	0.7
$10^{5.5\pm0.25}$	10	759	89
$10^{5.5\pm0.25}$	100	1*	1*
$10^{6.0\pm0.25}$	0.6	94	9
$10^{6.0\pm0.25}$	10	1000*	800
$10^{6.0\pm0.25}$	100	1*	1*
$10^{6.5\pm0.25}$	0.6	67	2
$10^{6.5\pm0.25}$	10	1700*	134
$10^{6.5\pm0.25}$	100	2*	1*

Table 2: Conservative rates would divide WD rates ($0.6M_\odot$) by 100 (Sigurdsson & Rees and Bender & Hils rates instead of Freitag). $10M_\odot$ black hole capture so efficient that reduction of 10 is conservative. Sheth et al SMBH density (probably wrong) would reduce numbers by ~ 20.

Geodesy: Use orbiting satellites to measure (vacuum) external gravitational potential of earth. General solution to Laplace equation is:

$$\Phi = -\frac{GM}{r} + \frac{GM}{R}\sum_{lm}\left(\frac{R}{r}\right)^{l+1} B_{lm}Y_{lm}(\theta,\phi)$$

External multipoles B_{lm} are determined by matching to internal structure at surface: so provide info on interior!

GRACE mission: doing this for earth with high precision.

Bothrodesy: geodesy for black holes

[Bothros (βοθροσ): ancient Greek for sacrificial pit.]

Use properties of satellite orbits to "map" the spacetime of massive "Black Hole [?]. Powerful test of black hole hypothesis: black holes have *very* special multipole moment structure.
Axisymmetric, so $B_{lm}=B_l$ and external spacetime can be built from "mass moments" M_l and "current moments" S_l :

$$M_l + iS_l = M(ia)^l$$

Only TWO moments are independent!!

Once we measure two of them, we have enough information to falsify black hole hypothesis.

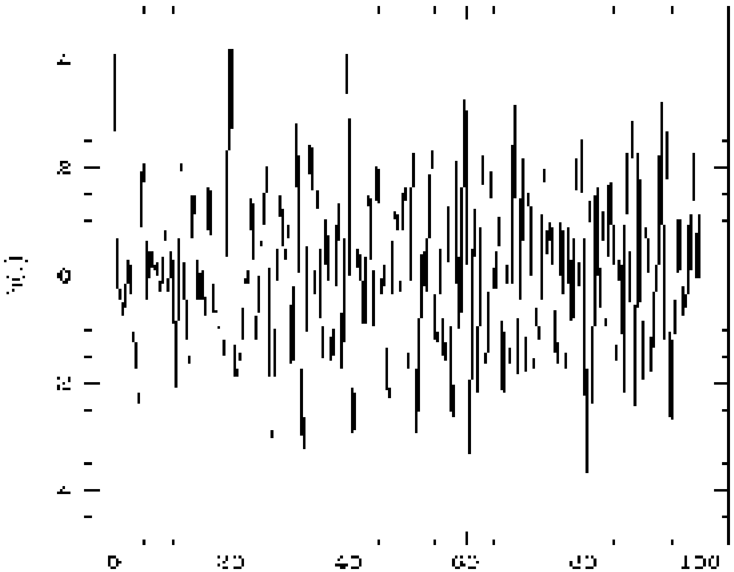
"It is well known that the Kerr solution ... provides the unique solution for stationary black holes ... in the universe. **But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the forseeable future."**

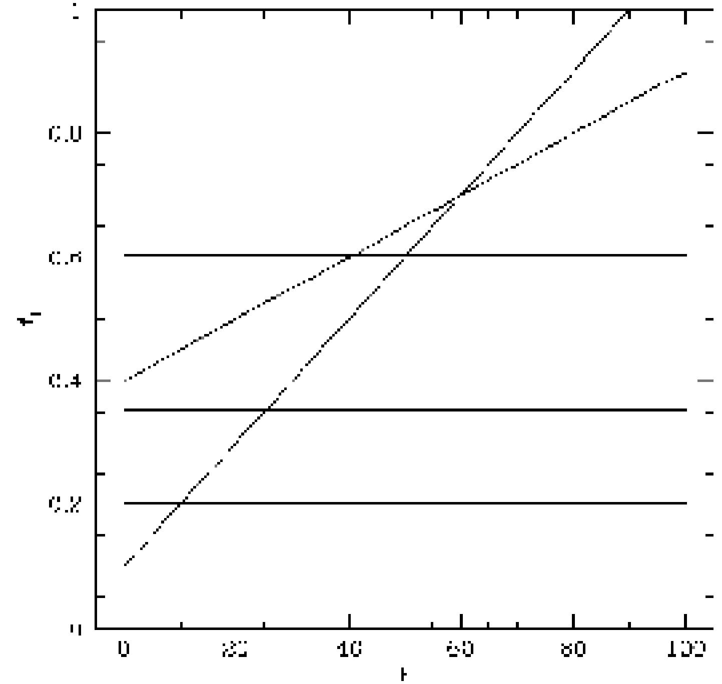
Subrahmanyan Chandrasekhar,
The Karl Schwarzschild Lecture, 18 Sept 1986

Unearthed by Scott Hughes

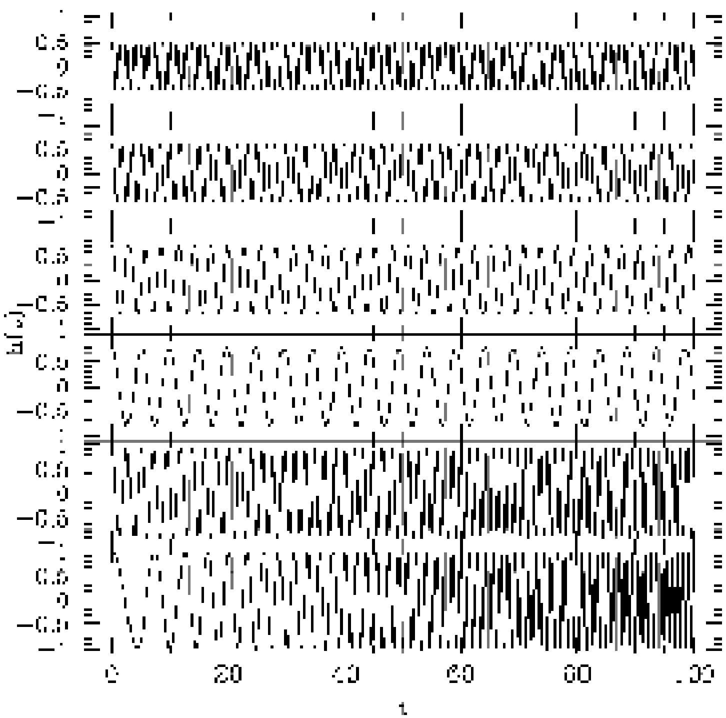
Bothrodesy: pros and cons

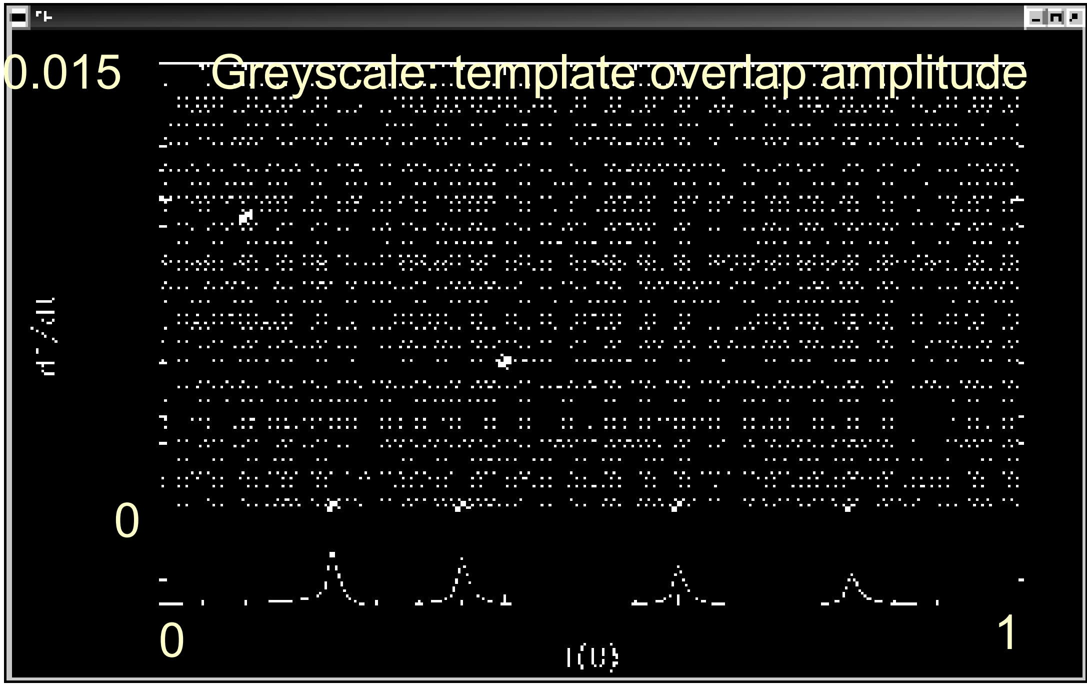
+ Numerical GR probably not needed: small expansion parameter ($m/M<10^{-5}$). ODEs!
+ Solved for circular orbits in Kerr, general orbits in Schwarzshild.
~ Only `solved in principle' (?) for general eccentric inclined orbits in Kerr (Mino 2003, Barack & Ori 2003)
- Many templates needed. Computing a problem for signal processing.
- Best current estimates: can do coherent brute-force template-matching only for weeks-month. Will have to stack/hierarchical search segments.





Is sum of 6 sources: 2 chirps and 4 fixed freq





Separation of more complicated sources

- Number of independent waveforms LISA could measure with power S/N given by Shannon's theorem
- $N_{ind}=2^{\wedge}(f\ T\ \log_2(1+S/N))$ in bandwidth f.
- LISA in limit S/N<<1, so coherent detection amplitude $SNR=(fT*S/N)^{1/2}$
- $N_{ind}=10^{\wedge}(0.59\ SNR^2)=10^{531}$ for S/N=30 detection threshold!
- Much larger than 10^{20} templates actually searched! So templates sparse in signal space.

"loss–conc", $\theta(r) = \sqrt{2r_{min}/3r}$,

$$\frac{t_{GW}}{t_{orb}} = \frac{24\sqrt{2}}{85\pi}\left(\frac{3}{2}\right)^{7/2}\frac{M_h}{m_*}\left(\frac{r}{r_S}\right)^{5/2}\theta^7.$$

sider two regimes; where the scattering angle is small compared to θ, which we refer to as "diffusion", and scattering where the scattering angle is large compared to θ, which we refer to as "kicks". The scattering in the respective regimes

to gravitational radiation, we require $t_{scat} \geq t_{GW}$, or that $\theta \leq \theta_{crit}$. Hence for "kicks", we find

$$\theta_{crit} = \sqrt{\frac{3}{2}}\left(\frac{85\pi}{24\sqrt{2}}\right)^{1/7}\left(\frac{M_h}{m_* N_*(r)}\right)^{1/7}\left(\frac{r}{r_S}\right)^{-5/14}, \quad (13)$$

and for diffusion

$$\theta_{crit} = \sqrt{\frac{3}{2}}\left(\frac{85\pi}{24\sqrt{2}}\right)^{1/5}\left(\frac{M_h}{m_* N_*(r)}\right)^{1/5}\left(\frac{r}{r_S}\right)^{-1/2}. \quad (14)$$

Swan songs

- Inspiral of compact objects into supermassive black holes from r=4M to last stable orbit.
- Is 13 Msun inspiraling into 1.3×10^5 Msun, sped up by 10^4 to get audio frequency (i.e. 0.013 into 130Msun).

a/M=0.359, circular, i=20, view 60 deg from pole:

a/M=0.998, circular, i=20, view 60 deg from pole:

a/M=0.95, e=0.95, i=25, semilatus Rectum=$a(1-e^2)$=5M