Evaluation of disturbances due to test mass charging for LISA and LISA Pathfinder

DNA Shaul,
HM Araujo, GK Rochester,
TJ Sumner, PJ Wass

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Electrostatic analyses (LTP GRS)
- FE model of entire GRS
- FE CM submodels
  + Comparison to // plate

Charging Disturbance Estimates
LISA & LISA PF Solar Max & Min

Method to Cope with Charging

COULOMB

LORENTZ
Why do TMs charge up?

- TMs can charge up due to:

  - Galactic cosmic rays

More in later talks
Why does charging matter?

Charging of TMs disturbs their geodesic motion

- **Unwanted forces from:**
  - **Coulomb interactions** with surrounding conductors of GRS

  **Charge-dependent Coulomb accn:**
  
  \[
  a_{Qk} = \frac{Q^2}{2C_T^2m} \frac{\partial C_T}{\partial k} + \frac{QV_T}{mC_T} \frac{\partial C_T}{\partial k} + \frac{Q}{mC_T} \sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k}
  \]

  \[\Rightarrow \text{Terms } \sim \Delta k\]
  \[\Rightarrow \text{Terms } \sim \Delta V \text{ & } V \Delta k\]

- **Lorentz interactions** as it moves through magnetic fields

  **Lorentz accn:**
  
  \[a_L = \left(\frac{Q}{m}\right)V \times B\]
Main charging Disturbances

1. Acceleration Noise (from fluctuations in: charge, TM position & velocity, voltages, magnetic field)
2. Modification of stiffness (from position dependence)
3. Coherent Fourier components (as $Q(t) \sim t$)

Parasitic coupling

Stray forces

Courtesy: S. Vitale

Target LISA Strain Noise

$\sqrt{S_h(1/\sqrt{Hz})}$

$1. \times 10^{-16}$

$1. \times 10^{-17}$

$1. \times 10^{-18}$

$1. \times 10^{-19}$

$1. \times 10^{-20}$

$1. \times 10^{-21}$

$0.0001$ $0.001$ $0.01$ $0.1$ $1$ $f(\text{Hz})$
Electrostatic FE model of LTP GRS

Coulomb accn is capacitance and hence geometry dependent

\[ a_{Qk} = \frac{Q^2}{2C_T^2m} \frac{\partial C_T}{\partial k} + \frac{QV_T}{mC_T} \frac{\partial C_T}{\partial k} - \frac{Q}{mC_T} \sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k} \]

Determine energy in system for given geometry & voltage distribution using FE analysis (ANSYS software)

- Change V distribution => capacitances between specific conductors
- Change TM position => capacitance derivatives

Move in 1 DOF, Sym => \( C_T(k) \approx C_T(k = 0) + \frac{1}{2} \frac{\partial^2 C_T}{\partial k^2} k^2 \)

"YZ-injection" sensor design* (LISA PF LTP GRS EM design)

46mm side TM

Gaps:
X face: 4 mm sens
Y face: 2.9 mm sens, 4 mm inj
Z face: 3.5 mm sens, 4 mm inj

* B Weber et al. '03 SPIE

<table>
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<tr>
<th>ANSYS (1σ errors#~ 1%)</th>
<th>% &gt; than // plate</th>
</tr>
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<td>( C_T(k = 0) = 33.7 \pm 0.2 \text{pF} )</td>
<td>30%</td>
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<tr>
<td>( \frac{\partial^2 C_T}{\partial x^2} = 1.15 \pm 0.01 \text{μF/m}^2 )</td>
<td>21%</td>
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<td>( \frac{\partial^2 C_T}{\partial y^2} = 2.50 \pm 0.03 \text{μF/m}^2 )</td>
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<tr>
<td>( \frac{\partial^2 C_T}{\partial z^2} = 1.62 \pm 0.03 \text{μF/m}^2 )</td>
<td>27%</td>
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#Shaul & Sumner '04 CNME
LTP TM CM: 1 Plunger, 4 stoppers & corresponding TM recesses, per z face. (Plunger tip level with recessed, injection electrodes during science mode)

(1) Pyramidal plunger & baseball-glove-shaped stoppers (ht 1.25mm)*

OR (2) Pyramidal plunger & alternative, elongated stoppers (l'gth 12mm, ht 1mm) *

• For either set of features, $C_T$ & $\frac{\partial^2 C_T}{\partial z^2}$ unchanged to within $0.4\sigma$ & $1.2\sigma$ respectively

* D Smart, S Tobin et al.
Submodel CM => estimate CM influence more accurately

- Positions of borders chosen as compromise between model size & distance from RoI => minimise effect on couplings of interest.
- BCs were not derived from the full model => avoid confounding models' accuracies

<table>
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<th>Plunger</th>
<th>Baseball glove stopper</th>
<th>Alternative, elongated stopper</th>
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<td>[Image 1]</td>
<td>[Image 2]</td>
<td>[Image 3]</td>
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Reduced model complexity (effects results only at the ~1% level)

Sign depends on which side of the sensor the feature is located

\[
C_{CTM,CM} = \begin{cases} 
(0.031411 \pm 0.000004)z + (12.10 \pm 0.01)z^2 + (3334 \pm 58)z^3 \\
(0.104081 \pm 0.000004)z + (17.87 \pm 0.01)z^2 + (2660 \pm 62)z^3 \\
(0.087078 \pm 0.000009)z + (29.31 \pm 0.03)z^2 + (8279 \pm 137)z^3
\end{cases}
\]

- Figures represent the properties of CM, NOT increase in capacitance when CM added as not compared “no CM” case
- Other capacitances will be affected by the inclusion of CM.
- These figures indicate CM level of influence.
Comparison of CM to full sensor

- Total $C_{TM,CM} < 3\%$ $C_T$ for both types of stopper
- Total $\frac{\partial^2 C_{TM,CM}}{\partial z^2} < 4\%$ & $9\%$ for CM with bbg stoppers & elongated stoppers
  - Only $0.7\%$ from plungers
  - In $dC_T/dk$, this gets multiplied by TM offset from the centre of opposing features

\[
a_{Qk} = \frac{Q^2}{2C_{Tm}^2} \frac{\partial C_T}{\partial k} + \frac{QV_T}{mC_T} \frac{\partial C_T}{\partial k} - \frac{Q}{mC_T} \sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k}
\]

$\Rightarrow$ Terms $\sim \frac{\partial^2 C_T}{\partial z^2} \Delta z$ $\Rightarrow$ requirements of the retracted CM

(E.g. TM offset of $50\mu m$ from the centre of the plungers $\Rightarrow$ contribution of $\sim3\%$ of that when the TM is offset $10\mu m$ wrt entire sensor)

- $\frac{\partial C_{TM,CM}}{\partial z} < 9\%, 6\%$ & $14\%$ of that of a single $z$-face, sensing electrode for a pyramidal plunger, a bbg stopper & an elongated stopper

\[
a_{Qk} = ... - \frac{Q}{mC_T} \sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k}
\]

$\Rightarrow$ Terms $\sim \frac{\partial C_i}{\partial z} \Delta V_i$

$\Rightarrow$ Need to maintain a high level of uniformity in the CM surfaces, to minimise work function differences, as patch effects could multiply these gradients into significant effects

- Similarly, to minimise the CM contribution to stiffness, the mean voltage of opposing features should also be minimised.
Magnitude of Charging Disturbances
(using ANSYS results for sensitive x-axis)
## Charging Rates

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<tr>
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<th>LISA, solar min</th>
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<td>Net charging rate (GEANT GCR) (+e/s)</td>
<td>100</td>
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<td>&quot;Worst case&quot; net charging rate assumed (+e/s)</td>
<td>191</td>
<td>99</td>
<td>176</td>
<td>93</td>
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<tr>
<td>Effective charging rate, $R_{eff}$ (+e/s)</td>
<td>708</td>
<td>462</td>
<td>746</td>
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- Assume "worst case" charging conditions:
  - $+ 30\%$ on the GEANT GCR charging rates
  - $60 + e/s$ for kinetic low-energy secondary electron emission
    (may $\sim$ cancel in the actual sensor)

- Rate of SEP events expected to be low enough that data acquisition could be suspended for their duration, but needs to be verified

- $[\delta Q (CHz^{-0.5}) = (2R_{eff})^{0.5}e/2\pi f ]$

Coherent Charging Signals

For $f \sim 10^{-4}$Hz $\Leftrightarrow$ estimates largest ($\tau = 1$ yr; $T = 1$ day)

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<td>$f_x(t)$ (S/N)</td>
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<td>201.38</td>
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<td>$l_x(t)$ (S/N)</td>
<td>0.57</td>
<td>0.30</td>
<td>0.53</td>
<td>0.28</td>
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S/N is the ratio of the charging signal to the acceleration noise budget for LISA

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Coulomb signals dependent on:
- $\Delta k$ (10$\mu$m), $\Delta V$ (1mV), $V$ (100mV), $V_T$ (100mV)
- Lorentz signal dependent on mean $B_{IP}$
  - ($\sim 0.25nT \rightarrow \sim 40nT$; median $\sim 6nT$).
- Even estimates @ $40nT <$ accn noise) ($\eta = 0.01$)
- Exact spectral shape depends on discharging scheme & variability of e.g. mean charging rate
  - (Shaul et al, CQG '04)

Magnitudes plotted at primary peaks of sinc functions
**Stiffness:**

<table>
<thead>
<tr>
<th>Stiffness, $s_{Qx} (s^{-2})$</th>
<th>LISA, solar min</th>
<th>LISA, solar max</th>
<th>LISA PF, solar min</th>
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<td>-5.27E-09</td>
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$S_{Qk} = -m \frac{\partial a_{Qk}}{\partial k}$

Terms that dominate are $\sim$ independent of $\Delta k$

$\sim V$ (100mV)

Independent of voltage $\sim V_T$ (100mV)

**GRS Requirement:** $-2 \times 10^{-8} \rightarrow 8 \times 10^{-8} \text{ s}^{-2}$

Within Limit? √
## Coulomb Noise

For $f \sim 10^{-4}$Hz  
(Charge noise $\sim 1/f$)

<table>
<thead>
<tr>
<th>Noise Type</th>
<th>LISA, solar min</th>
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<td><strong>Displacement noise</strong> ($ms^{-2}Hz^{-0.5}$)</td>
<td>1.70E-17</td>
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\[
\delta a_{Q_k}^2 = \left( \frac{\partial a_{Q_k}}{\partial k} \right)^2 \delta k^2 + \sum_{i=1}^{N-1} \left( \frac{\partial a_{Q_k}}{\partial V_i} \right)^2 \delta V_i^2 + \left( \frac{\partial a_{Q_k}}{\partial Q} \right)^2 \delta Q^2
\]

\[
\frac{\partial a_{Q_k}}{\partial k} = -\frac{1}{m} s_{Q_k}
\]

\[
\delta x = 2.5 \text{nmHz}^{-0.5}
\]

\[
\frac{\partial a_{Q_k}}{\partial Q} = \frac{Q}{C_T^2 m} \frac{\partial C_T}{\partial k} + \frac{V_T}{mC_T} \frac{\partial C_T}{\partial k} - \frac{1}{mC_T} \sum_{i=1}^{N-1} \frac{\partial C_{i,N}}{\partial k}
\]

\[
\frac{\partial a_{Q_k}}{\partial V_p} = \frac{Q}{mC_T^2} \frac{\partial C_T}{\partial k} C_{p,N} - \frac{Q}{mC_T} \frac{\partial C_{p,N}}{\partial k}
\]

\[
\delta V = 10 / \sqrt{2} \mu \text{VHz}^{-0.5}
\]

Dominant term

\[\delta a_{Q_k} \sim 1/f\]
**Lorentz Noise**

For $f \sim 10^{-4}$Hz
(Lorentz noise $\sim 1/f^{1.6}$)

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\[
\overline{B}_{IP} = 3 \times 10^{-8} \text{T} \\
\delta B_{IP} = 3 \times 10^{-7} \text{THz}^{-0.5} \\
\overline{B}_{SC} = 8 \times 10^{-7} \text{T} \\
\delta B_{SC} = 1 \times 10^{-7} \text{THz}^{-0.5}
\]

Lorentz accn:

\[
a_L \approx \eta \overline{Q} \overline{V} \times \overline{B}_{IP} + \eta \overline{Q} \overline{V} \times \delta B_{IP} + \eta \overline{Q} \delta \overline{V} \times \overline{B}_{IP} + \eta \overline{Q} \delta \overline{V} \times \delta B_{IP} + \eta \delta \overline{Q} \overline{V} \times \overline{B}_{IP} / m
\]

Fluctuations in $B_{IP}$
Fluctuations in TM velocity
Charging “shot noise”

Accn Noise

 Dominant term by $> 3$ orders of magnitude so $\sim$ insensitive to range of $B_{IP}$

Note: The image contains a table with data and a complex equation for Lorentz noise, along with a diagram illustrating the components of the noise and their magnitudes.
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**Noise Allocation =20% of LISA noise budget = \(0.6 \times 10^{-15} \left[1 + \left(f/3\text{mHz}\right)^2\right] \text{ms}^{-2}\text{Hz}^{-0.5}\)**

- Reduce T by ~2 hrs => noise within spec
- If no contribution of secondaries -> net charging rate, total noise ↓ ~10-20%
- If V=1mV, the \(e(t)\) ↓ ~55%, stiffness ↓ ~90%, total noise ↓ ~20-30%
Comparison to //plate

- // plate approx overestimates
  - Coulomb noise, stiffness, $e_x(t)$ by ~10%
  - $f_x(t)$ by ~40%

- BUT LEVEL OF AGREEMENT IS DEPENDENT ON SENSOR GEOMETRY
- E.g. For similar “Trento torsion” sensor design, FE results (validated in expt: Carbone et al 2003 Physical Review Letters 91 151101) & //plate approx give similar levels of agreement
  - But if exclude guard rings for this design, agreement ↓:
    - $C_T(\text{FE}) = C_T(//) \times 1.56$ (from 1.30)
    - agreement of 2nd derivatives wrt $x$, $y$ & $z$ ~same (~20-30%)
    - //plate approx overestimates:
      - noise, stiffness, $e_x(t)$ by ~30-40%
      - $f_x(t)$ by ~110%
      - guard rings minimise fringing fields
      - exact level of agreement between derived quantities also dependent on other parameters e.g. $V$

=> USE CAUTION WHEN EMPLOY // PLATE APPROX
Management of disturbances

- UV light => discharge the TMs via pe effect.
- Nominally, T ~ 1 day => accn noise & stiffness within budget
- But coherent charging signals above noise target
- To remove charging signals:
  - spectral analysis?
  - continuous discharging of the TMs at a rate exceeding the charging rate?
  Disadvantage = increased noise

\[
\begin{align*}
\text{Acceleration noise (ms}^{-2}\text{Hz}^{-0.5}) \\
\text{Discharging rate (-e/s)}
\end{align*}
\]

Variation of total noise for LISA solar min, with discharging rate, assuming Q=0

⇒ Discharging rate = ~6 \times \text{Charging rate}, to reach nominal noise budget
⇒ Variation in charging rate between solar min & max ~ 50% ⇒ plausible
  that discharging rate could be set high enough to cope with variations in the GCR flux, without exceeding budgets

• Still need to quantify e.g. impact of restoration of equilibrium of charging and discharging currents following changes in the mean charging rate

Thank-you