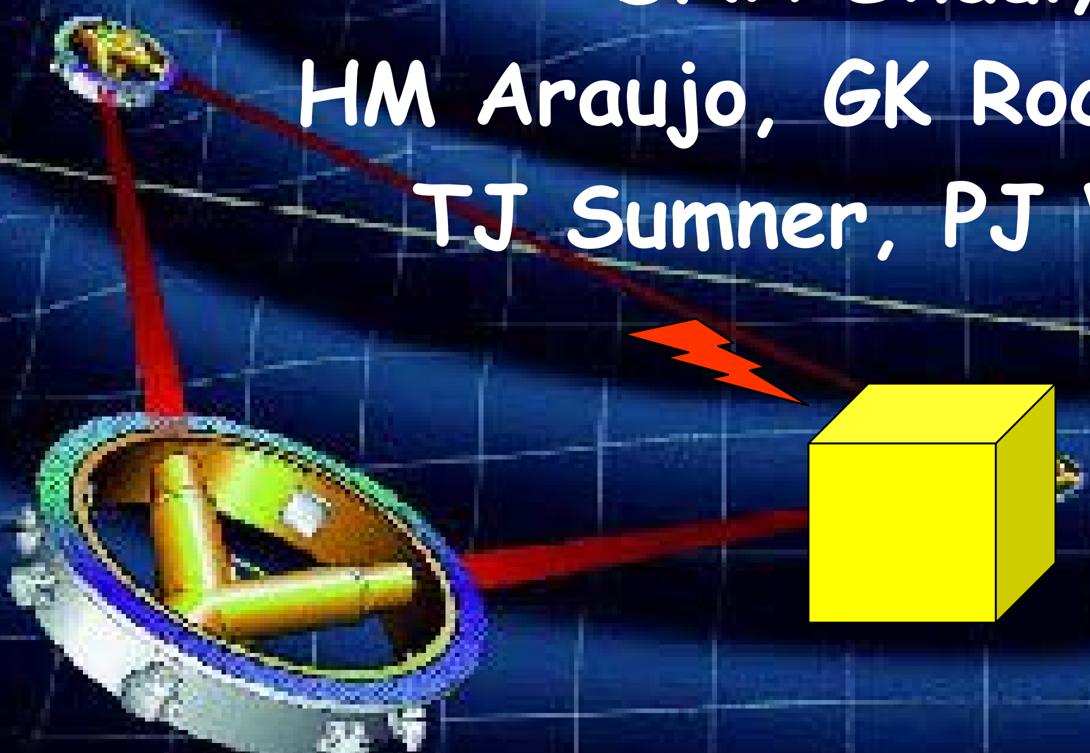
# Evaluation of disturbances due to test mass charging for LISA and LISA Pathfinder

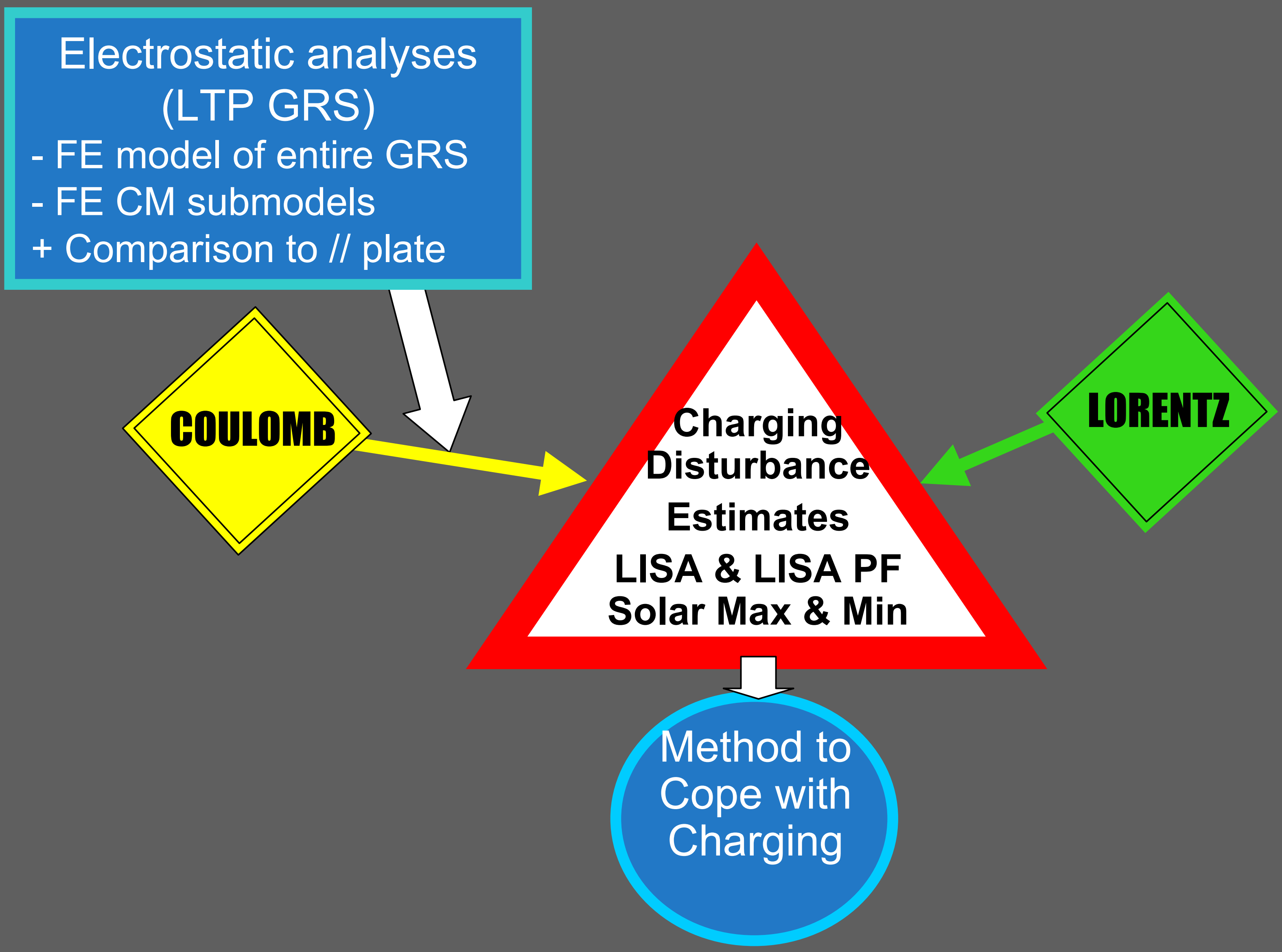
DNA Shaul,
HM Araujo, GK Rochester,
TJ Sumner, PJ Wass



5th International LISA Symposium, ESTEC, Wednesday 14th July 2004

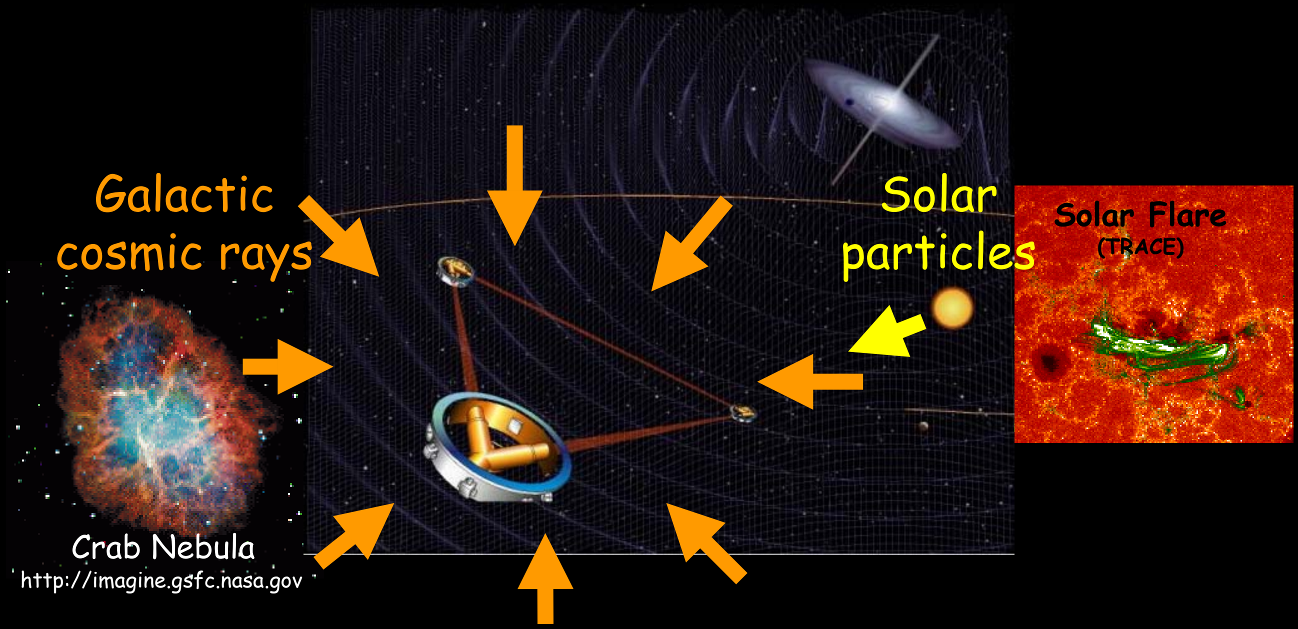
Imperial College London

# Talk Outline



# Why do TMs charge up?

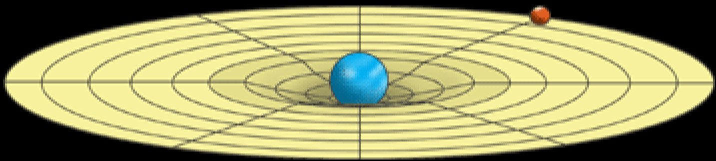
- TMs can charge up due to:



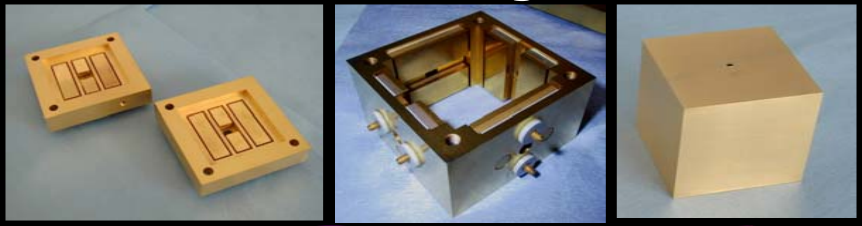

More in later talks

# Why does charging matter?

Charging of TMs disturbs
their geodesic motion



- Unwanted forces from:
  - Coulomb interactions with surrounding conductors of GRS



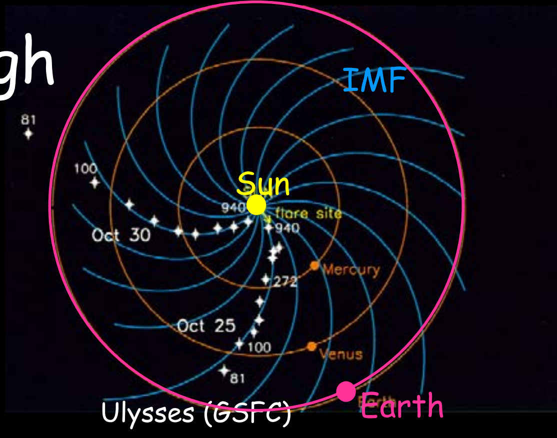
Charge-dependent Coulomb accn:

$$a_{Qk} = \frac{Q^2}{2C_T^2 m}\frac{\partial C_T}{\partial k} + \frac{QV_T}{mC_T}\frac{\partial C_T}{\partial k} - \frac{Q}{mC_T}\sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k}$$

=> Terms ~$\Delta k$ => Terms~$\Delta V$ & $V\Delta k$

  - Lorentz interactions as it moves through magnetic fields

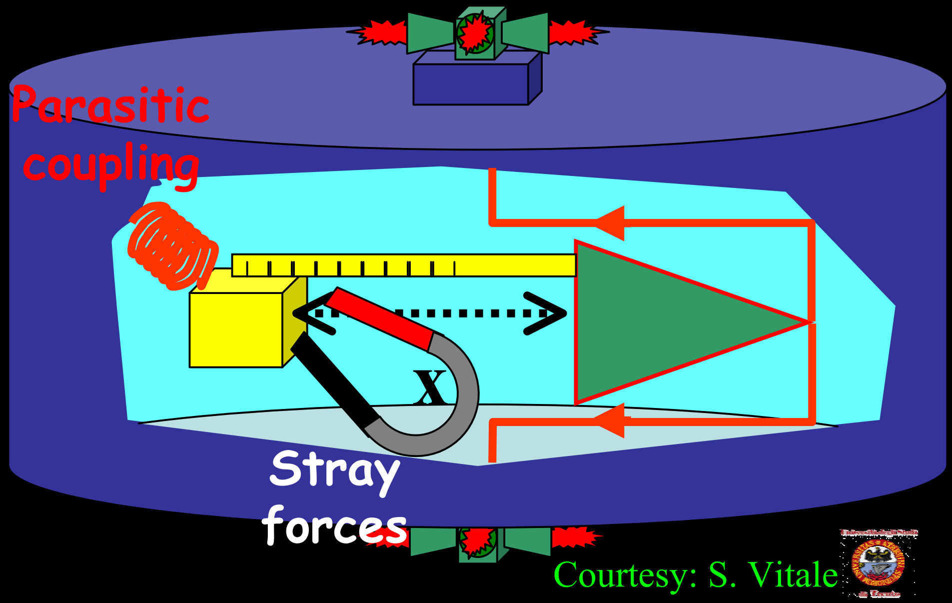
Lorentz accn: $a_L = (Q/m) V \times B$



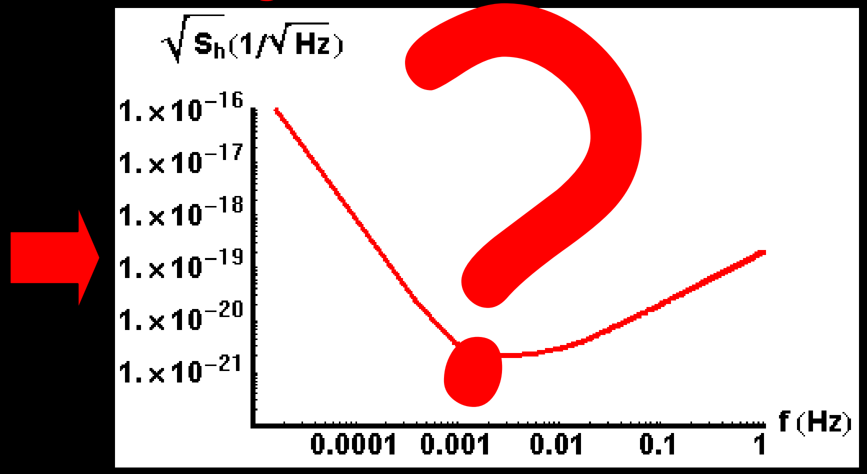
Ulysses (GSFC)

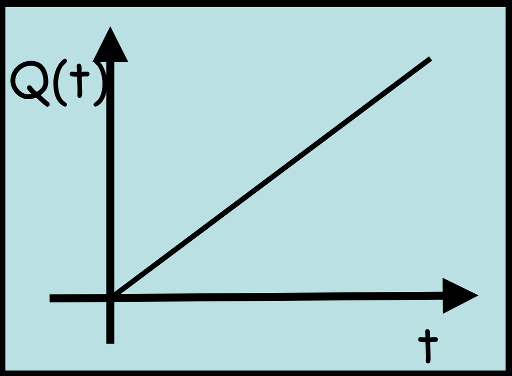
# Main charging Disturbances

1. Acceleration Noise (from fluctuations in: charge, TM position & velocity, voltages, magnetic field)
2. Modification of stiffness (from position dependence)
3. Coherent Fourier components (as Q(t) ~t)




Target LISA Strain Noise



# Electrostatic FE model of LTP GRS

Coulomb accn is capacitance and hence geometry dependent

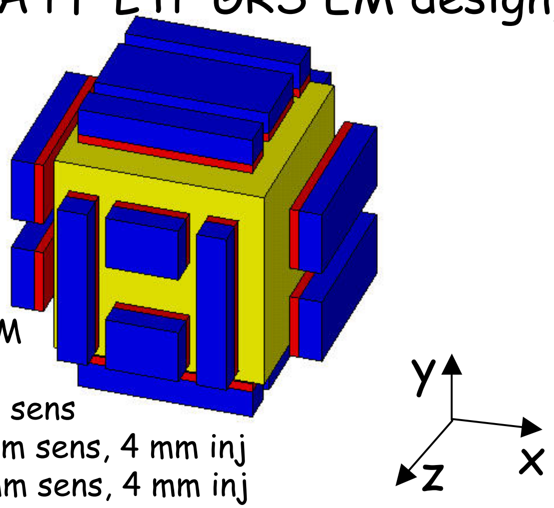
$$a_{Qk} = \frac{Q^2}{2C_T^2 m}\frac{\partial C_T}{\partial k} + \frac{QV_T}{mC_T}\frac{\partial C_T}{\partial k} - \frac{Q}{mC_T}\sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k}$$

Determine energy in system for given geometry & voltage distribution using FE analysis (ANSYS software)

- Change V distribution => capacitances between specific conductors
- Change TM position => capacitance derivatives

Move in 1 DOF, Sym => $C_T(k) \approx C_T(k=0) + \frac{1}{2}\frac{\partial^2 C_T}{\partial k^2}k^2$

"YZ-injection" sensor design*
(LISA PF LTP GRS EM design)




46mm side TM
Gaps:
X face: 4 mm sens
Y face: 2.9 mm sens, 4 mm inj
Z face: 3.5 mm sens, 4 mm inj

* B Weber et al. '03 SPIE

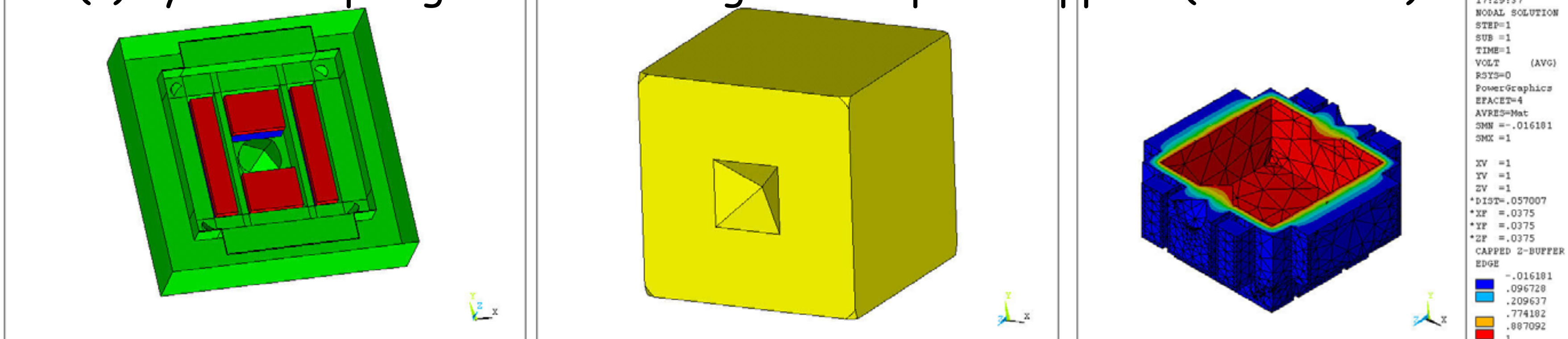
| ANSYS (1σ errors#~ 1%) | % > than // plate |
|---|---|
| $C_T(k=0) = 33.7 \pm 0.2\text{pF}$ | 30% |
| $\frac{\partial^2 C_T}{\partial x^2} = 1.15 \pm 0.01\mu\text{F}/\text{m}^2$ | 21% |
| $\frac{\partial^2 C_T}{\partial y^2} = 2.50 \pm 0.03\mu\text{F}/\text{m}^2$ | 21% |
| $\frac{\partial^2 C_T}{\partial z^2} = 1.62 \pm 0.03\mu\text{F}/\text{m}^2$ | 27% |

#Shaul & Sumner '04 CNME

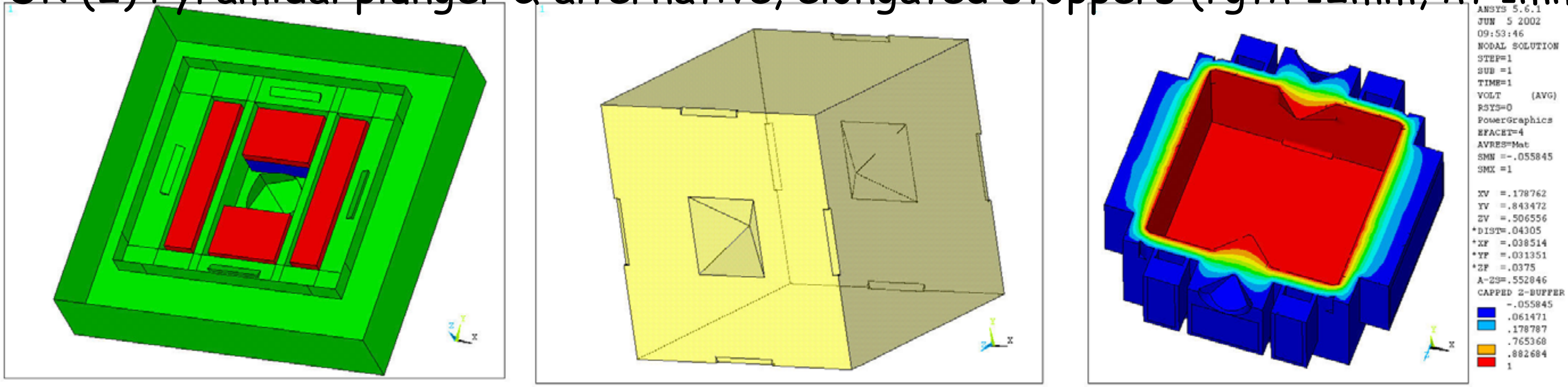
# LTP TM CM: 1 Plunger, 4 stoppers & corresponding TM recesses, per z face.

(Plunger tip level with recessed, injection electrodes during science mode)

(1) Pyramidal plunger & baseball-glove-shaped stoppers (ht 1.25mm)*



OR (2) Pyramidal plunger & alternative, elongated stoppers (l'gth 12mm, ht 1mm) *

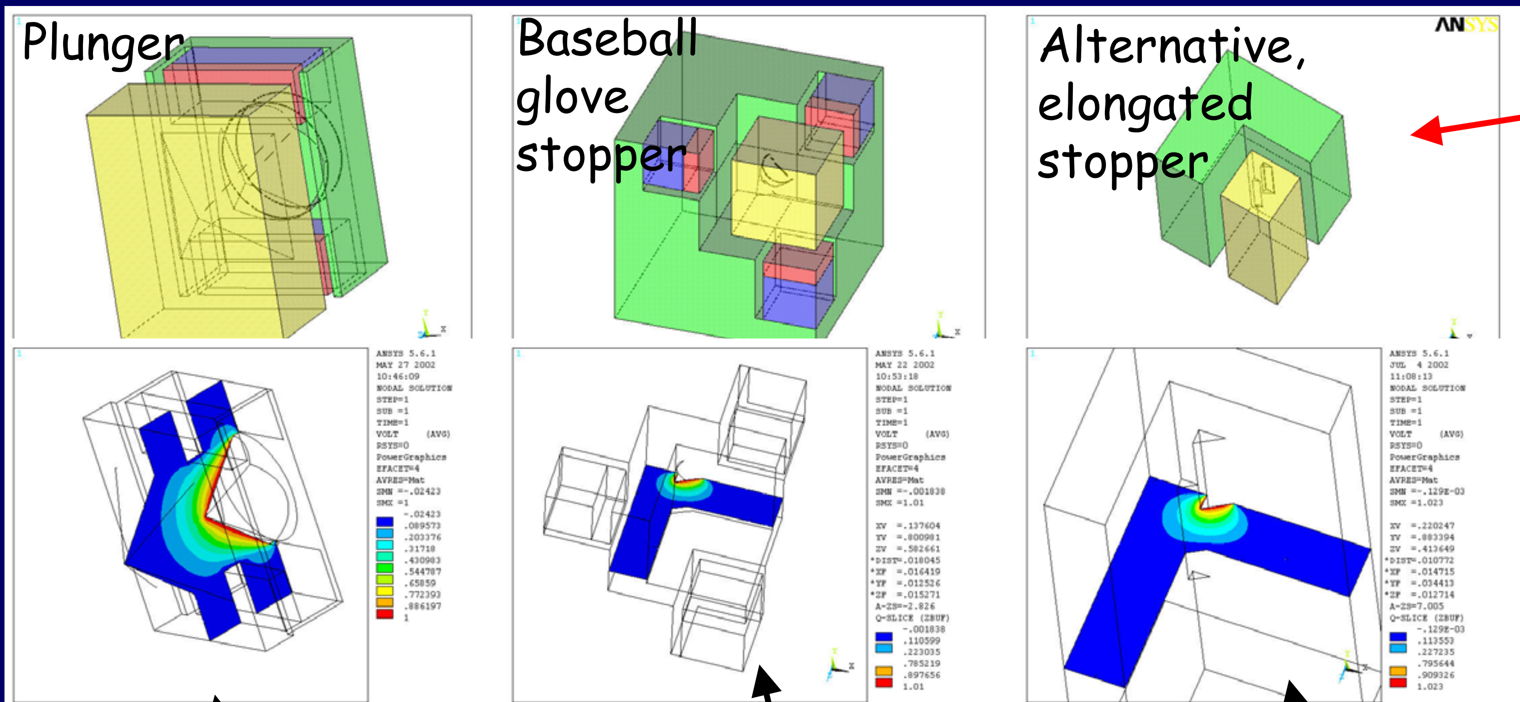


- For either set of features, $C_T$ & $\frac{\partial^2 C_T}{\partial z^2}$ unchanged to within 0.4σ & 1.2σ respectively

* D Smart, S Tobin et al.

# Submodel CM => estimate CM influence more accurately

- Positions of borders chosen as compromise between model size & distance from RoI => minimise effect on couplings of interest.
- BCs were not derived from the full model => avoid confounding models' accuracies




Reduced model complexity (effects results only at the ~1% level)

Sign depends on which side of the sensor the feature is located

$C_{TM,CM}$:

$$(0.104081 \pm 0.000004) \pm (17.87 \pm 0.01)z + (2660 \pm 62)z^2$$

$$(0.031411 \pm 0.000004) \pm (12.10 \pm 0.01)z + (3334 \pm 58)z^2$$

$$(0.087078 \pm 0.000009) \pm (29.31 \pm 0.03)z + (8279 \pm 137)z^2$$

- Figures represent the properties of CM, NOT increase in capacitance when CM added as not compared "no CM" case
- Other capacitances will be affected by the inclusion of CM.
- These figures indicate CM level of influence.

# Comparison of CM to full sensor

- Total $C_{TM,CM}$ < 3% $C_T$ for both types of stopper
- Total $\frac{\partial^2 C_{TM,CM}}{\partial z^2}$ < 4% & 9% $\frac{\partial^2 C_T}{\partial z^2}$ for CM with bbg stoppers & elongated stoppers
  - Only 0.7% from plungers
  - In $dC_T/dk$, this gets multiplied by TM offset from the centre of opposing features

$$a_{Qk} = \frac{Q^2}{2C_T^2 m}\frac{\partial C_T}{\partial k} + \frac{QV_T}{mC_T}\frac{\partial C_T}{\partial k} - \frac{Q}{mC_T}\sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k}$$

=> Terms ~ $\frac{\partial^2 C_T}{\partial z^2}\Delta z$ => ease the positioning requirements of the retracted CM

(E.g. TM offset of 50μm from the centre of the plungers => contribution of ~3% of that when the TM is offset 10μm wrt entire sensor)

- $\frac{\partial C_{TM,CM}}{\partial z}$ < 9%, 6% & 14% of that of a single *z*-face, sensing electrode for a pyramidal plunger, a bbg stopper & an elongated stopper

$$a_{Qk} = \ldots - \frac{Q}{mC_T}\sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k}$$

=> Terms ~ $\frac{\partial C_i}{\partial z}\Delta V_i$

=> Need to maintain a high level of **uniformity** in the CM surfaces, to minimise work function differences, as patch effects could multiply these gradients into significant effects

- Similarly, to minimise the CM contribution to stiffness, the mean voltage of opposing features should also be minimised.

# Magnitude of Charging Disturbances

## (using ANSYS results for sensitive x-axis)

# Charging Rates

| | LISA, solar min | LISA, solar max | LISA PF, solar min | LISA PF, solar max |
|---|---|---|---|---|
| *Net charging rate (GEANT GCR) (+e/s)* | *100* | *47* | *88* | *43* |
| *"Worst case" net charging rate assumed (+e/s)* | *191* | *99* | *176* | *93* |
| *Effective charging rate, $R_{eff}$ (+e/s)* | *708* | *462* | *746* | *469* |

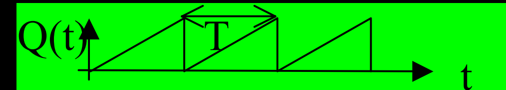
- Assume "worst case" charging conditions:
  - + error margin of **~30%** on the GEANT GCR charging rates
  - **+ 60 +e/s** for kinetic low-energy secondary electron emission (may ~cancel in the actual sensor)

- Rate of SEP events expected to be low enough that data acquisition could be suspended for their duration, but needs to be verified

- [$\delta Q$ ($CHz^{-0.5}$) = $(2R_{eff})^{0.5}e/2\pi f$]

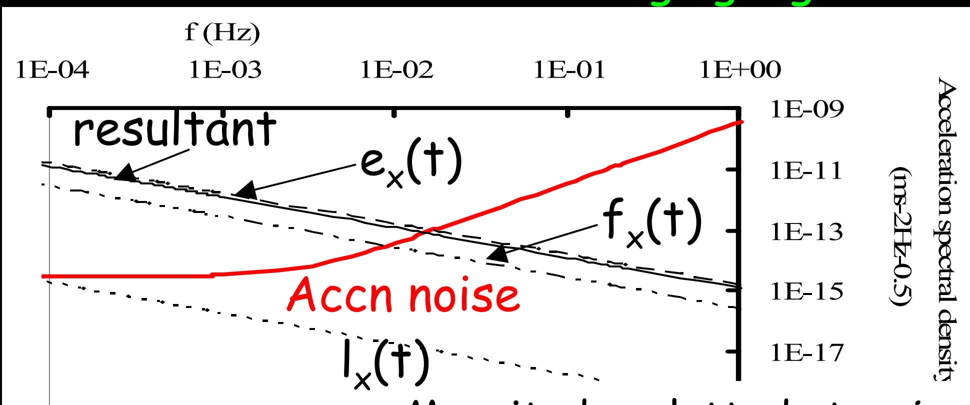
LISA: Araujo H *et al* submitted to Astroparticle Physics, arXiv:astro-ph/0405522.
LISA PF: Wass P *et al* in preparation.

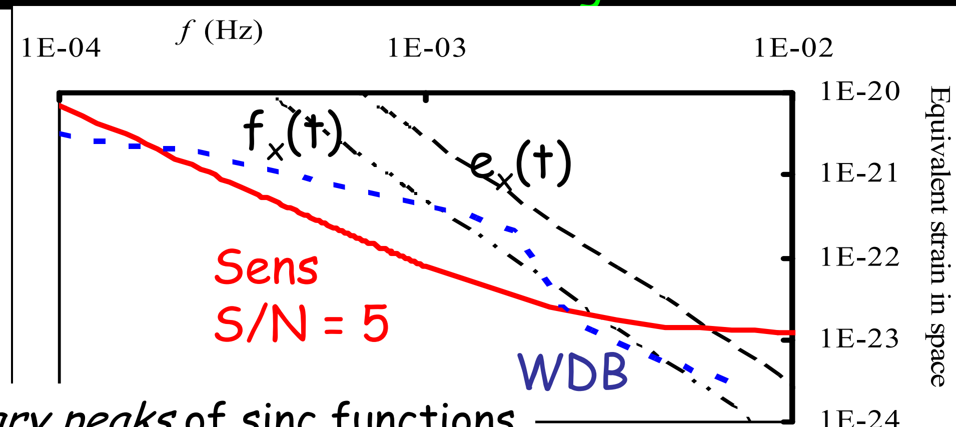
# Coherent Charging Signals




| For f ~ $10^{-4}$Hz ⇔ estimates largest ($\tau$ = 1 yr; T = 1day) | *LISA, solar min* | *LISA, solar max* | *LISA PF, solar min* | *LISA PF, solar max* |
|---|---|---|---|---|
| ***$e_x(t)$ (S/N)*** | 4,833.57 | 2,498.52 | 4,447.51 | 2,351.47 |
| ***$f_x(t)$ (S/N)*** | 850.87 | 227.35 | 720.38 | 201.38 |
| ***$l_x(t)$ (S/N)*** | 0.57 | 0.30 | 0.53 | 0.28 |

S/N is the ratio of the charging signal to the acceleration noise budget for LISA




Magnitudes plotted at *primary peaks* of sinc functions

$$e_k(t) \equiv \Theta_k t \equiv \frac{\bar{\dot{Q}} V_T t}{m C_T}\frac{\partial C_T}{\partial k} - \frac{\bar{\dot{Q}} t}{m C_T}\sum_{i=1}^{N-1} V_i \frac{\partial C_{iN}}{\partial k}$$

$$f_k(t) \equiv \Xi_k t^2 \equiv \frac{\bar{\dot{Q}}^2}{2 C_T^2 m}\frac{\partial C_T}{\partial k} t^2$$

$$l_x(t) \equiv \Phi_x t \equiv \left(\eta \bar{\dot{Q}} t / m\right)\left[\bar{V}_I \times \bar{B}_{IP}\right] \cdot \hat{x}$$

Coulomb signals dependent on:
$\Delta k$ (10μm), $\Delta V$ (1mV), V (100mV), $V_T$ (100mV)
Lorentz signal dependent on mean $B_{IP}$ (~ 0.25nT -> ~ 40nT; median ~6nT).
Even estimates @ 40nT < accn noise) ($\eta$ = 0.01)

**Exact spectral shape depends on discharging scheme & variability of e.g. mean charging rate** (Shaul et al, *CQG* '04)

# Stiffness: GRS Requirement: $-2\times10^{-8}$ -> $8\times10^{-8}\ s^{-2}$

| | LISA, solar min | LISA, solar max | LISA PF, solar min | LISA PF, solar max | Within Limit? |
|---|---|---|---|---|---|
| Stiffness, $s_{Qx}$ ($s^{-2}$) | -1.11E-08 | -5.27E-09 | -1.00E-08 | -4.93E-09 | ✓ |

$$s_{Qk} = -m\frac{\partial a_{Qk}}{\partial k}$$

$$= \frac{Q}{C_T}\sum_{i=1}^{N-1} V_i \frac{\partial^2 C_{i,N}}{\partial k^2} + \frac{2QV_T}{C_T^2}\left(\frac{\partial C_T}{\partial k}\right)^2 + \frac{Q^2}{C_T^3}\left(\frac{\partial C_T}{\partial k}\right)^2 - \frac{2Q}{C_T^2}\frac{\partial C_T}{\partial k}\sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k} - \frac{QV_T}{C_T}\frac{\partial^2 C_T}{\partial k^2} - \frac{Q^2}{2C_T^2}\frac{\partial^2 C_T}{\partial k^2}$$

~V (100mV)

Terms that dominate are ~ independent of $\Delta k$

~$V_T$ (100mV)

Independent of voltage

# Coulomb Noise

| For f ~ $10^{-4}$Hz (Charge noise~1/f) | *LISA, solar min* | *LISA, solar max* | *LISA PF, solar min* | *LISA PF, solar max* |
|---|---|---|---|---|
| ***Displacement noise ($ms^{-2}Hz^{-0.5}$)*** | 1.70E-17 | 8.54E-18 | 1.55E-17 | 8.02E-18 |
| ***Charge noise ($ms^{-2}Hz^{-0.5}$)*** | 3.97E-16 | 3.07E-16 | 4.04E-16 | 3.09E-16 |
| ***Voltage noise ($ms^{-2}Hz^{-0.5}$)*** | 4.62E-16 | 2.39E-16 | 4.25E-16 | 2.25E-16 |

$$\delta a_{Qk}^2 = \left(\frac{\partial a_{Qk}}{\partial k}\right)^2 \delta k^2 + \sum_{i=1}^{N-1}\left(\frac{\partial a_{Qk}}{\partial V_i}\right)^2 \delta V_i^2 + \left(\frac{\partial a_{Qk}}{\partial Q}\right)^2 \delta Q^2$$

$$\frac{\partial a_{Qk}}{\partial k} = -\frac{1}{m} s_{Qk}$$

$$\delta x = 2.5\,\mathrm{nmHz}^{-0.5}$$

$$\frac{\partial a_{Qk}}{\partial Q} = \frac{Q}{C_T^2 m}\frac{\partial C_T}{\partial k} + \frac{V_T}{mC_T}\frac{\partial C_T}{\partial k} - \frac{1}{mC_T}\sum_{i=1}^{N-1} V_i \frac{\partial C_{i,N}}{\partial k}$$

: all terms same order of magnitude

$$\frac{\partial a_{Qk}}{\partial V_p} = \frac{Q}{mC_T^2}\frac{\partial C_T}{\partial k} C_{p,N} - \frac{Q}{mC_T}\frac{\partial C_{p,N}}{\partial k}$$

$$\delta V = 10/\sqrt{2}\,\mu\mathrm{VHz}^{-0.5}$$

Dominant term

# Lorentz Noise

| For f ~ $10^{-4}$Hz (Lorentz noise~$1/f^{1.6}$) | *LISA, solar min* | *LISA, solar max* | *LISA PF, solar min* | *LISA PF, solar max* |
|---|---|---|---|---|
| ***Lorentz noise ($ms^{-2}Hz^{-0.5}$)*** | 1.22E-16 | 6.32E-17 | 1.12E-16 | 5.95E-17 |

$\overline{B}_{IP} = 3\times10^{-8}\,\mathrm{T}$ $\delta B_{IP} = 3\times10^{-7}\,\mathrm{THz}^{-0.5}$ $\overline{B}_{SC} = 8\times10^{-7}\,\mathrm{T}$ $\delta B_{SC} = 1\times10^{-7}\,\mathrm{THz}^{-0.5}$

Lorentz accn:

$$\boldsymbol{a}_{\mathrm{L}} \approx \left(\eta\overline{\dot{Q}}t\overline{\boldsymbol{V}}_I \times \overline{\boldsymbol{B}}_{IP} + \boxed{\eta\overline{\dot{Q}}t\overline{\boldsymbol{V}}_I \times \delta\boldsymbol{B}_{IP}} + \boxed{\eta\overline{\dot{Q}}t\delta\boldsymbol{V}_I \times \overline{\boldsymbol{B}}_{IP} + \overline{\dot{Q}}t\delta\boldsymbol{V}_{SC} \times \overline{\boldsymbol{B}}_{SC}} + \boxed{\eta\delta Q\overline{\boldsymbol{V}}_I \times \overline{\boldsymbol{B}}_{IP}}\right)\Big/m$$

Fluctuations in $B_{IP}$

Fluctuations in TM velocity

Charging "shot noise"

Accn Noise

Dominant term by > 3 orders of magnitude so ~ insensitive to range of $B_{IP}$

# Magnitude of Charging Disturbances

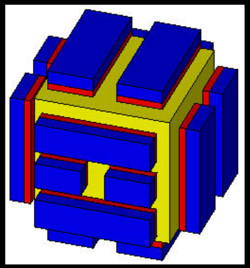
| | *LISA, solar min* | *LISA, solar max* | *LISA PF, solar min* | *LISA PF, solar max* | Within Limit? |
|---|---|---|---|---|---|
| *Net charging rate (GEANT GCR) (+e/s)* | *100* | *47* | *88* | *43* | |
| *"Worst case" net charging rate assumed (+e/s)* | *191* | *99* | *176* | *93* | |
| *Effective charging rate (+e/s)* | *708* | *462* | *746* | *469* | |
| *$e_x(t)$ (S/N)* | 4,833.57 | 2,498.52 | 4,447.51 | 2,351.47 | |
| *$f_x(t)$ (S/N)* | 850.87 | 227.35 | 720.38 | 201.38 | |
| *$l_x(t)$ (S/N)* | 0.57 | 0.30 | 0.53 | 0.28 | |
| *Stiffness, $s_{Qx}$ ($s^{-2}$)* | -1.11E-08 | -5.27E-09 | -1.00E-08 | -4.93E-09 | ✓ |
| *Displacement noise ($ms^{-2}Hz^{-0.5}$)* | 1.70E-17 | 8.54E-18 | 1.55E-17 | 8.02E-18 | |
| *Charge noise ($ms^{-2}Hz^{-0.5}$)* | 3.97E-16 | 3.07E-16 | 4.04E-16 | 3.09E-16 | |
| *Voltage noise ($ms^{-2}Hz^{-0.5}$)* | 4.62E-16 | 2.39E-16 | 4.25E-16 | 2.25E-16 | |
| *Lorentz noise ($ms^{-2}Hz^{-0.5}$)* | 1.22E-16 | 6.32E-17 | 1.12E-16 | 5.95E-17 | |
| *Total noise ($ms^{-2}Hz^{-0.5}$)* | 6.22E-16 | 3.95E-16 | 5.98E-16 | 3.87E-16 | ~✓ |

Noise Allocation =20% of LISA noise budget = $0.6\times10^{-15}\left[1+\left(f/3\mathrm{mHz}\right)^2\right]\mathrm{ms}^{-2}\mathrm{Hz}^{-0.5}$

- Reduce T by ~2 hrs => noise within spec
- If no contribution of secondaries -> net charging rate, total noise ⇩~10-20%
- If V=1mV, the e(t) ⇩ ~55% , stiffness ⇩ ~90%, total noise ⇩ ~20-30%

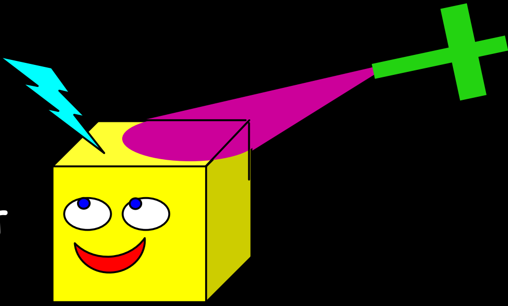
# Comparison to //plate

- // plate approx overestimates
  - Coulomb noise, stiffness, $e_x(t)$ by ~10%
  - $f_x(t)$ by ~40%
- BUT LEVEL OF AGREEMENT IS DEPENDENT ON SENSOR GEOMETRY
- E.g. For similar "Trento torsion" sensor design, FE results (validated in expt: **Carbone *et al* 2003 Physical Review Letters** 91 **151101**) & // plate approx give similar levels of agreement
  - But if exclude guard rings for this design, agreement ⇩:
    - $C_T(FE) = C_T(//) \times 1.56$ (from 1.30)
    - agreement of 2nd derivatives wrt *x*, *y* & *z* ~same (~20-30%)
    - //plate approx overestimates:
      - noise, stiffness, $e_x(t)$ by ~30-40%
      - $f_x(t)$ by ~110%
      - guard rings minimise fringing fields
      - exact level of agreement between derived quantities also dependent on other parameters e.g. V



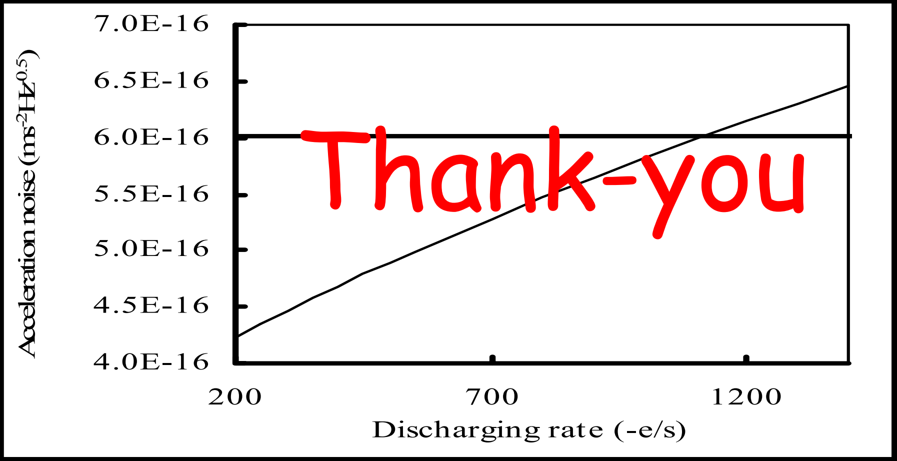
=> USE CAUTION WHEN EMPLOY // PLATE APPROX

# Management of disturbances



- UV light => discharge the TMs via pe effect.
- Nominally, T ~ 1 day => accn noise & stiffness within budget
- But coherent charging signals above noise target
- To remove charging signals:
  - spectral analysis?
  - continuous discharging of the TMs at a rate exceeding the charging rate? Disadvantage = increased noise

Variation of total noise for LISA solar min, with discharging rate, assuming Q=0




⇒Discharging rate = ~6 x Charging rate, to reach nominal noise budget

⇒Variation in charging rate between solar min & max ~ 50% => plausible that discharging rate could be set high enough to cope with variations in the GCR flux, without exceeding budgets

•Still need to quantify e.g. impact of restoration of equilibrium of charging and discharging currents following changes in the mean charging rate