Post-processing implementation of time-delay interferometry with interpolation

Daniel Shaddock, Brent Ware, Bob Spero and Michele Vallisneri

14 July 2004

Daniel.Shaddock@jpl.nasa.gov

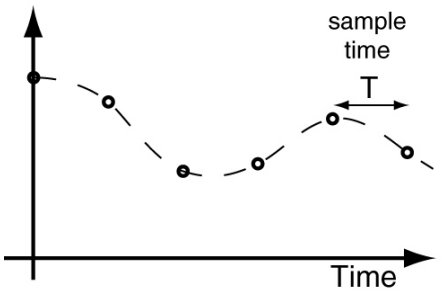
Fractional Delay

- Timing accuracy of phase measurements for time-delay interferometry (TDI) implementation: $\delta t < 50$ ns
- Phase measurements transmitted at ~10 Hz.
- Current approach is to trigger phase measurements at the correct times
- Alternative approach is to sample phase at a constant rate and interpolate in post-processing (back on Earth).
  - Previous calculations have shown this alternative approach to be not feasible (several months of continuous data needed to interpolate to required accuracy).
  - New results indicate that it is very feasible (~2 seconds of data needed for interpolation).

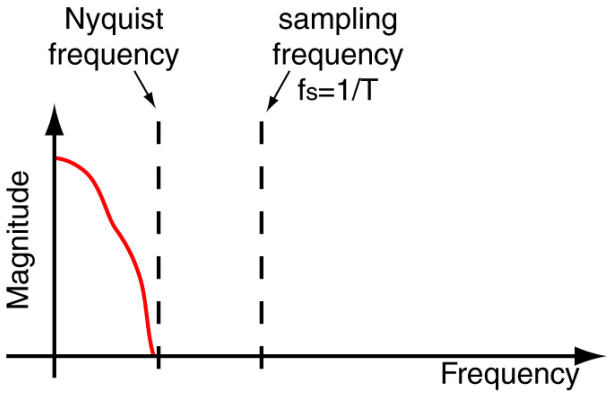
Interpolation Accuracy Requirement

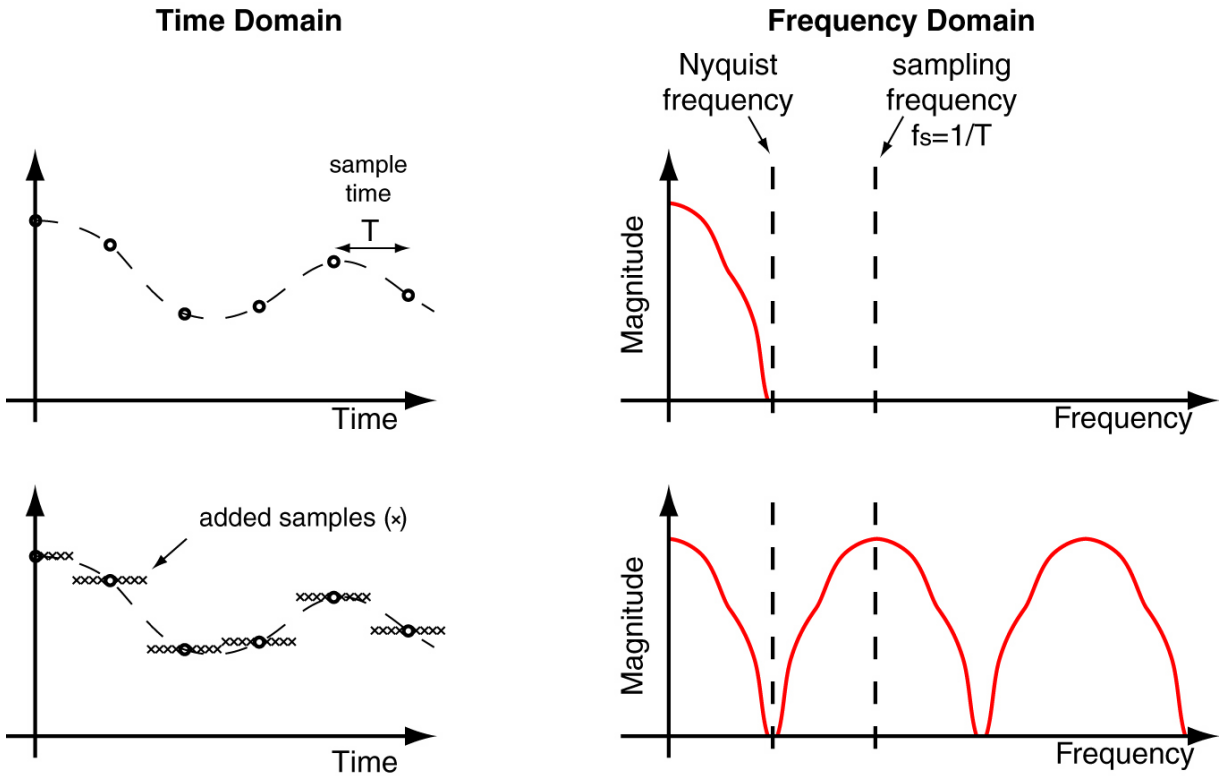
- Assume 100 cycles/√Hz at the output of the photodetector.
- Allocate error of 1 μcycle/√Hz to interpolation (~1/10 of shot noise)
- Requirement: Interpolation error < 10^{-8} for frequency components from 1 mHz to 1 Hz.

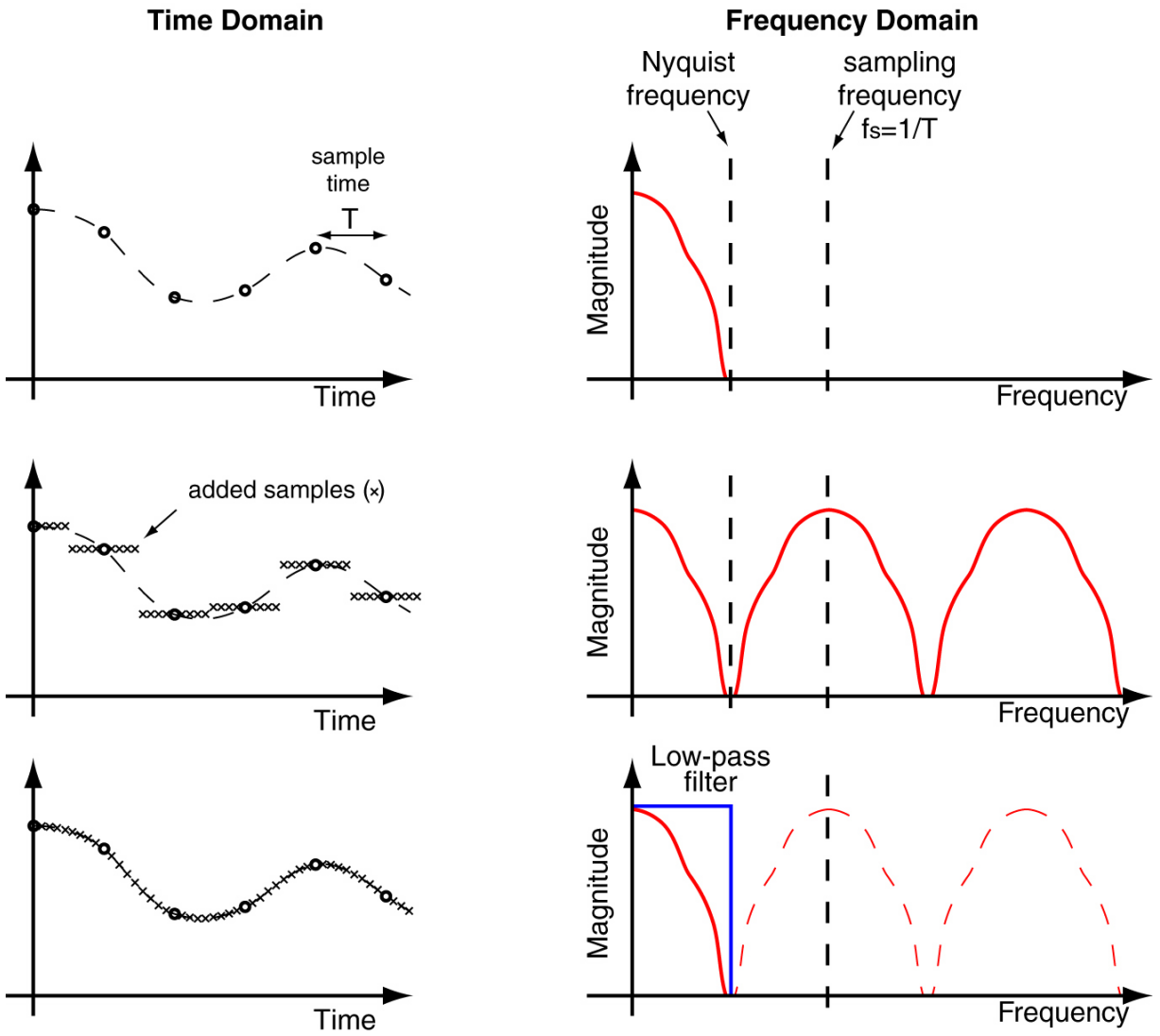
Time Domain



Frequency Domain

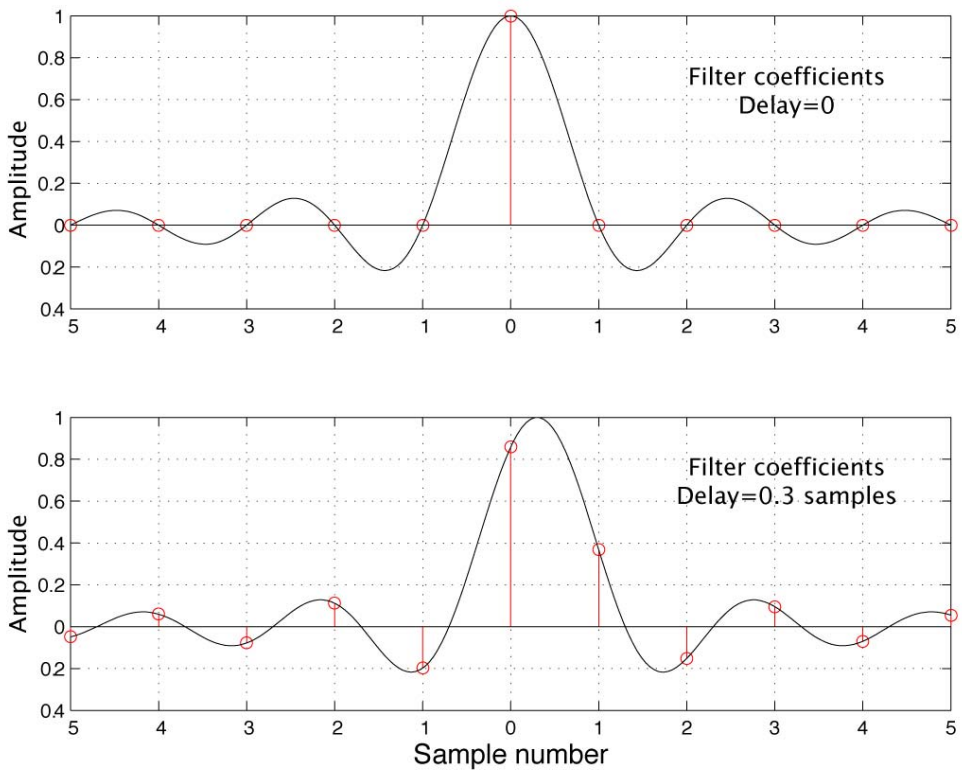



Time Domain
Frequency Domain
sample time
T
Time
Nyquist frequency
sampling frequency
fs=1/T
Magnitude
Frequency
added samples (×)
Time
Magnitude
Frequency


Time Domain
Frequency Domain
sample time
T
Time
Nyquist frequency
sampling frequency
fs=1/T
Magnitude
Frequency
added samples (×)
Time
Magnitude
Frequency
Time
Low-pass filter
Magnitude
Frequency

Low-pass filter design

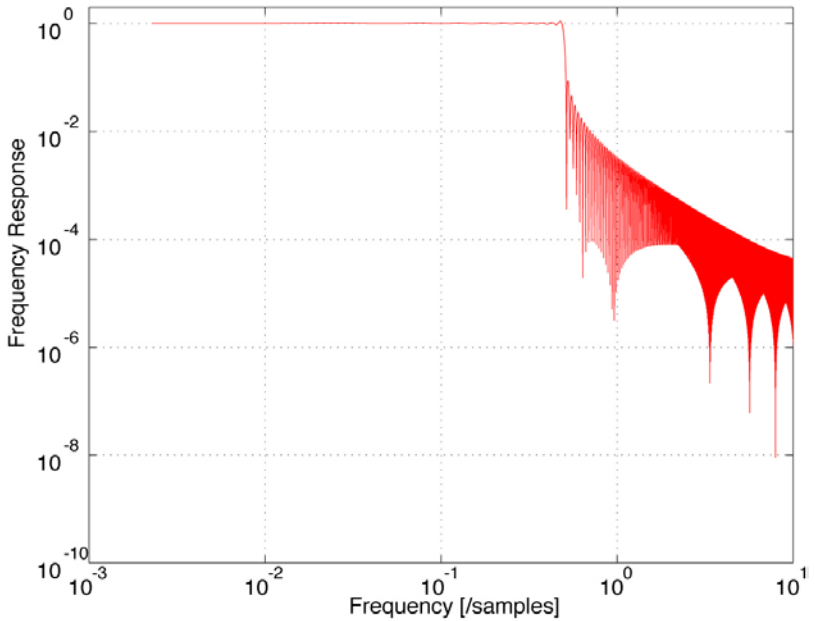
- The filter kernel (impulse response) of a perfect (brick wall) low-pass filter is a sampled sinc function.

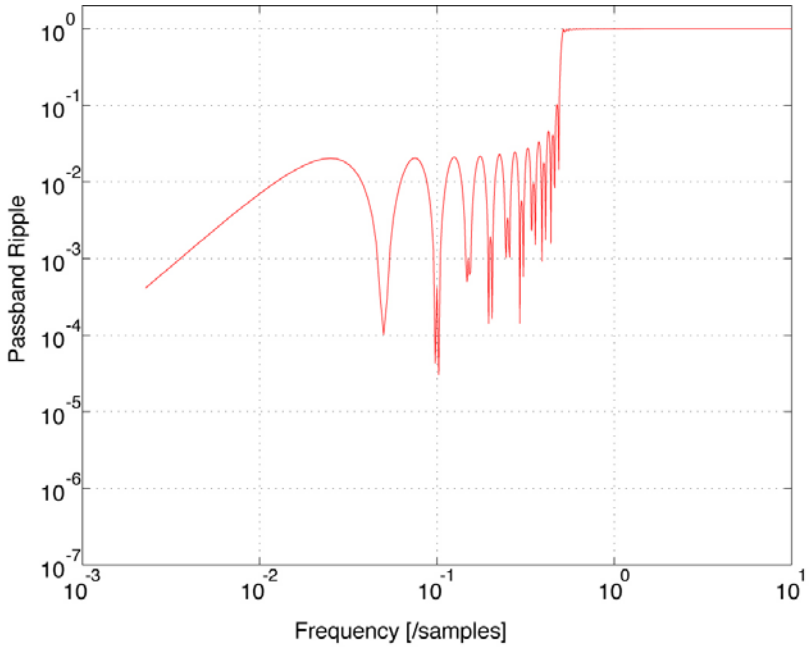


- Sinc function has infinite length and cannot be realized in practice.

Finite length sinc

- Truncating sinc changes the frequency response of the filter.
  - Adds ripple in the pass-band.
  - Finite stop-band suppression.
  - Not suitable for use in LISA (requires several months of continuous data) [1].

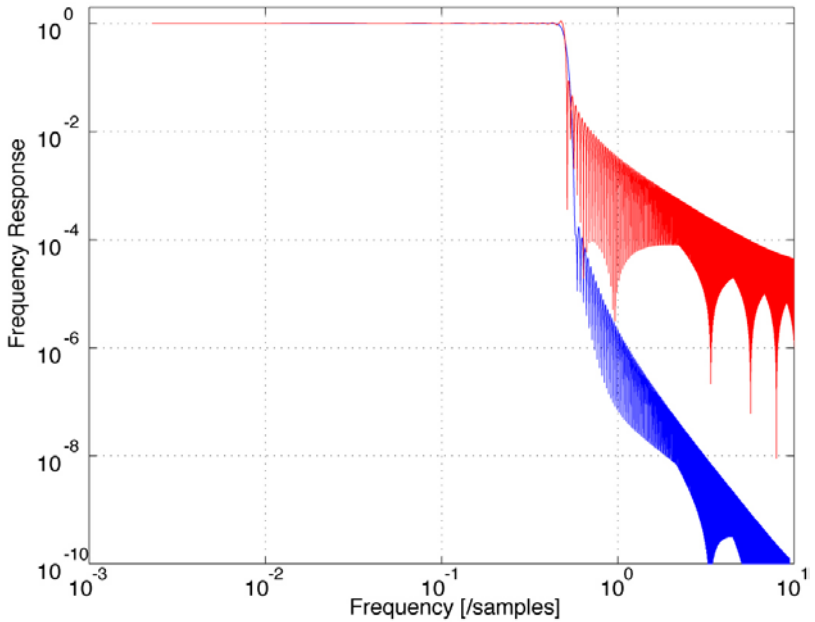


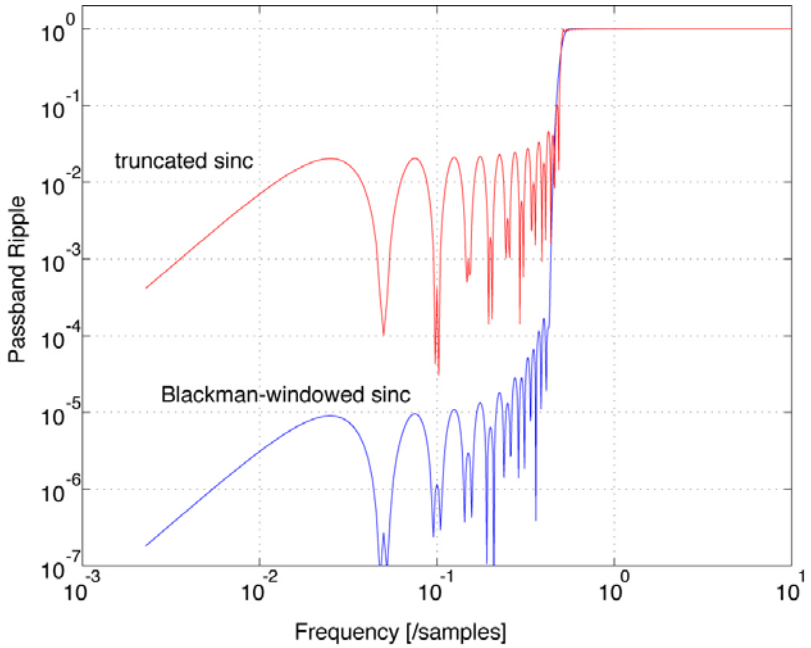


[1] M. Tinto, D.A. Shaddock, J. Sylvestre, J.W. Armstrong, “Implementation of time-delay interferomtery” Phys. Rev. D **67** 122003 (2003).

Windowed sinc

- Windowing the sinc function provides significantly better performance (many orders of magnitude).
  - Provides significantly better pass-band ripple and stop-band attenuation.
  - Approximately 30 seconds of data required to interpolate.

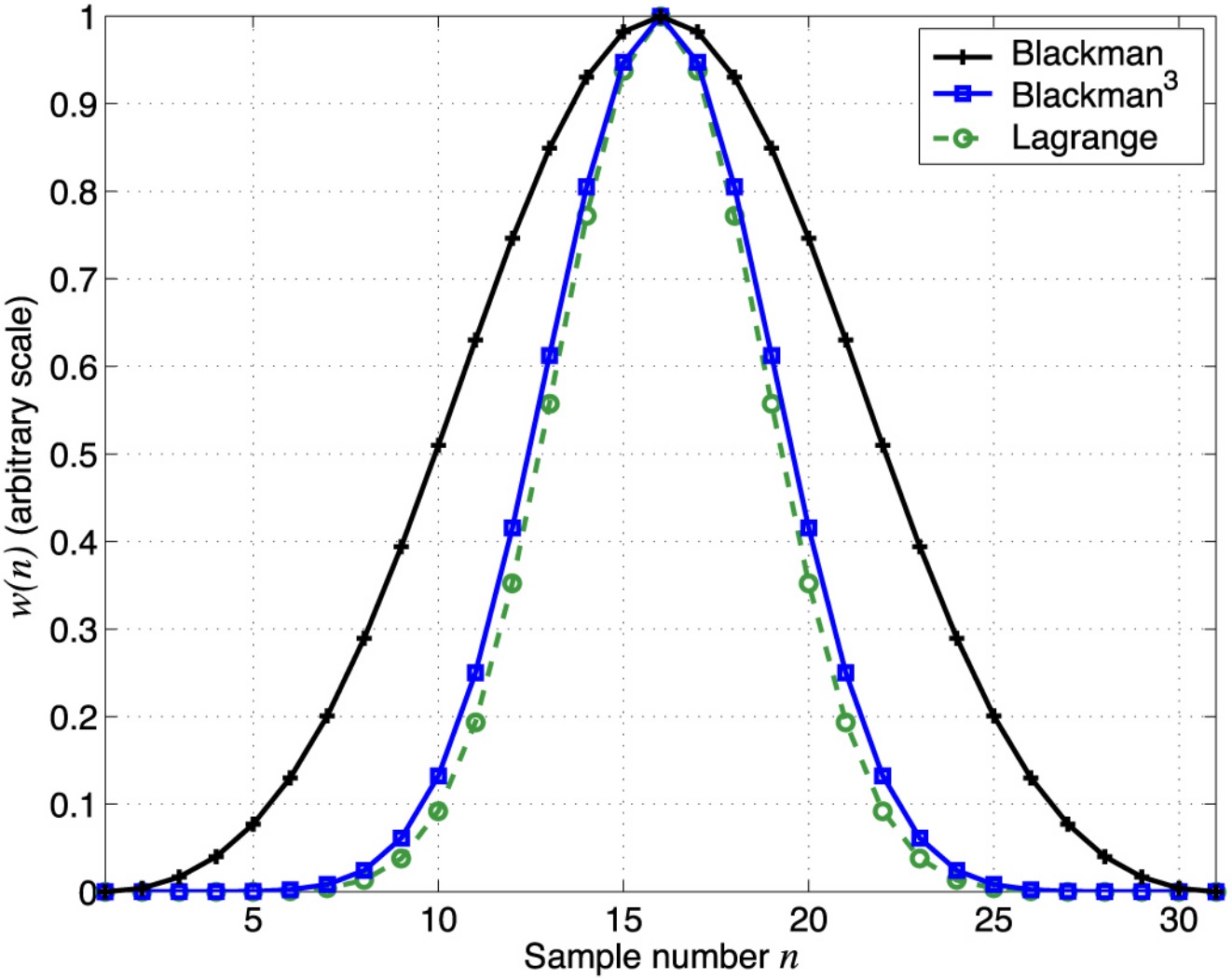




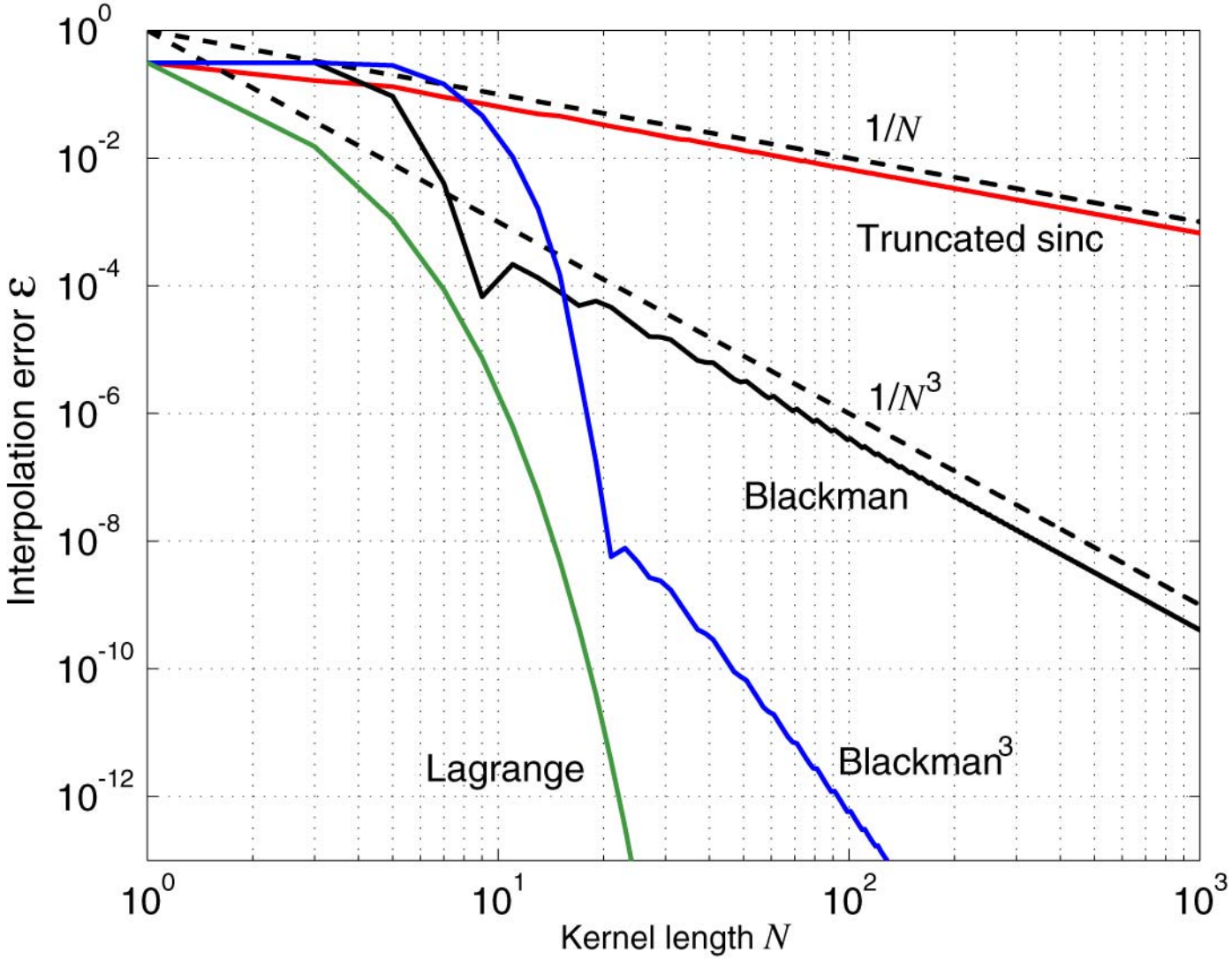
Different windows

- Many different types of windows considered.
- Boxcar (no window) - several months (Tinto et al 2002).
- Blackman window- 33 seconds
- Blackman^3 window- 2.1 seconds
- “Lagrange” window- 1.6 seconds
  - Lagrange window can be found by solving for a maximally flat filter frequency response at low frequencies.
  - Mathematically equivalent to Lagrange polynomial interpolation.

Windows


Blackman
Blackman^3
Lagrange
$w(n)$ (arbitrary scale)
Sample number n
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
5
10
15
20
25
30

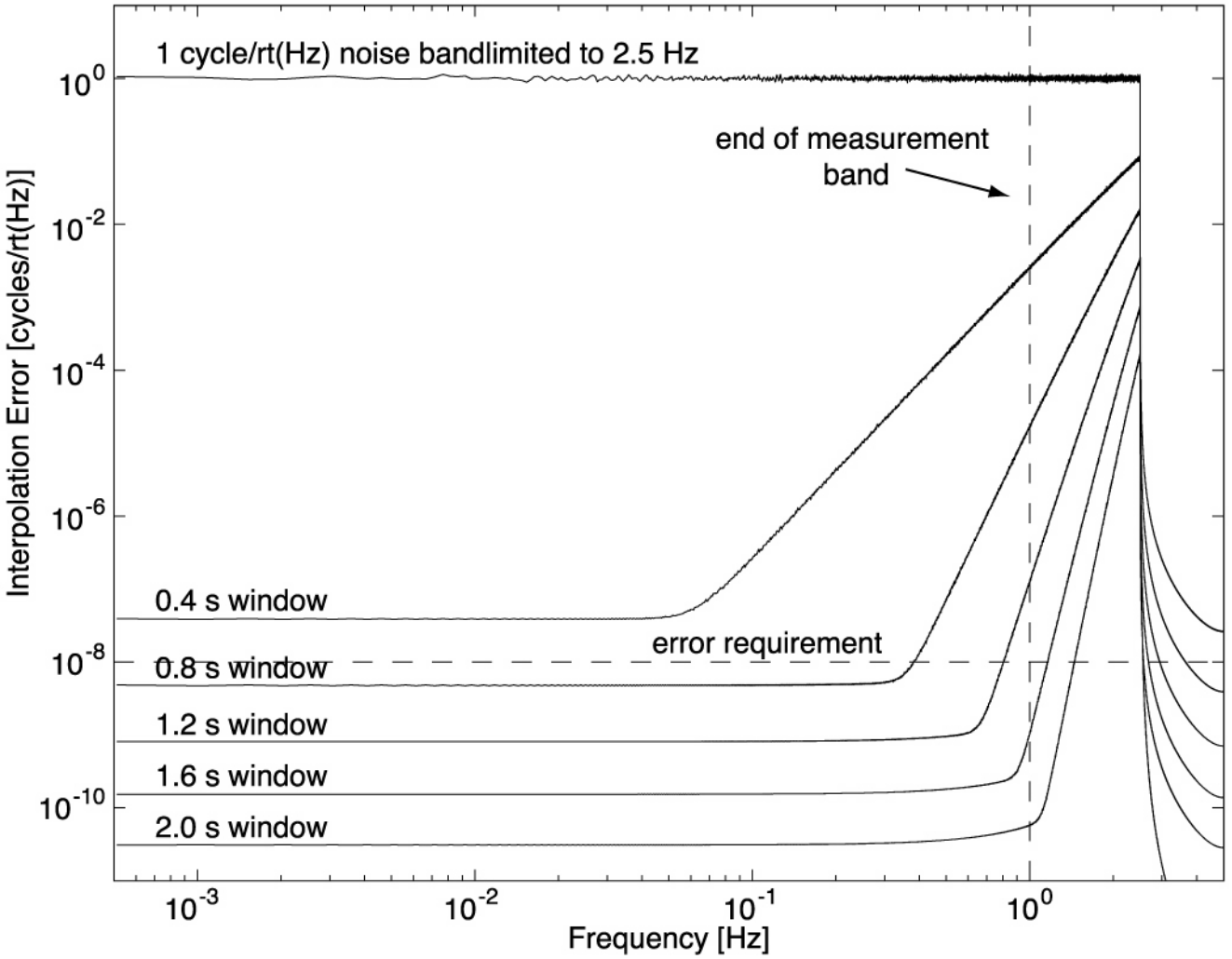
Error vs filter length



Testing and Simulation

- Tested by interpolation a variety of functions
  - sinusoidal signals
  - band limited white and 1/f-type noise
  - blind tests (thanks to John Armstrong) of both band limited and non band limited signals.
- Slow variation of LISA arm lengths requires slowly changing the interpolation time shift.
  - Simulated this by changing delay from 0.05 to 0 seconds (0.5 to 0 samples) over 500,000 seconds.

Simulation results

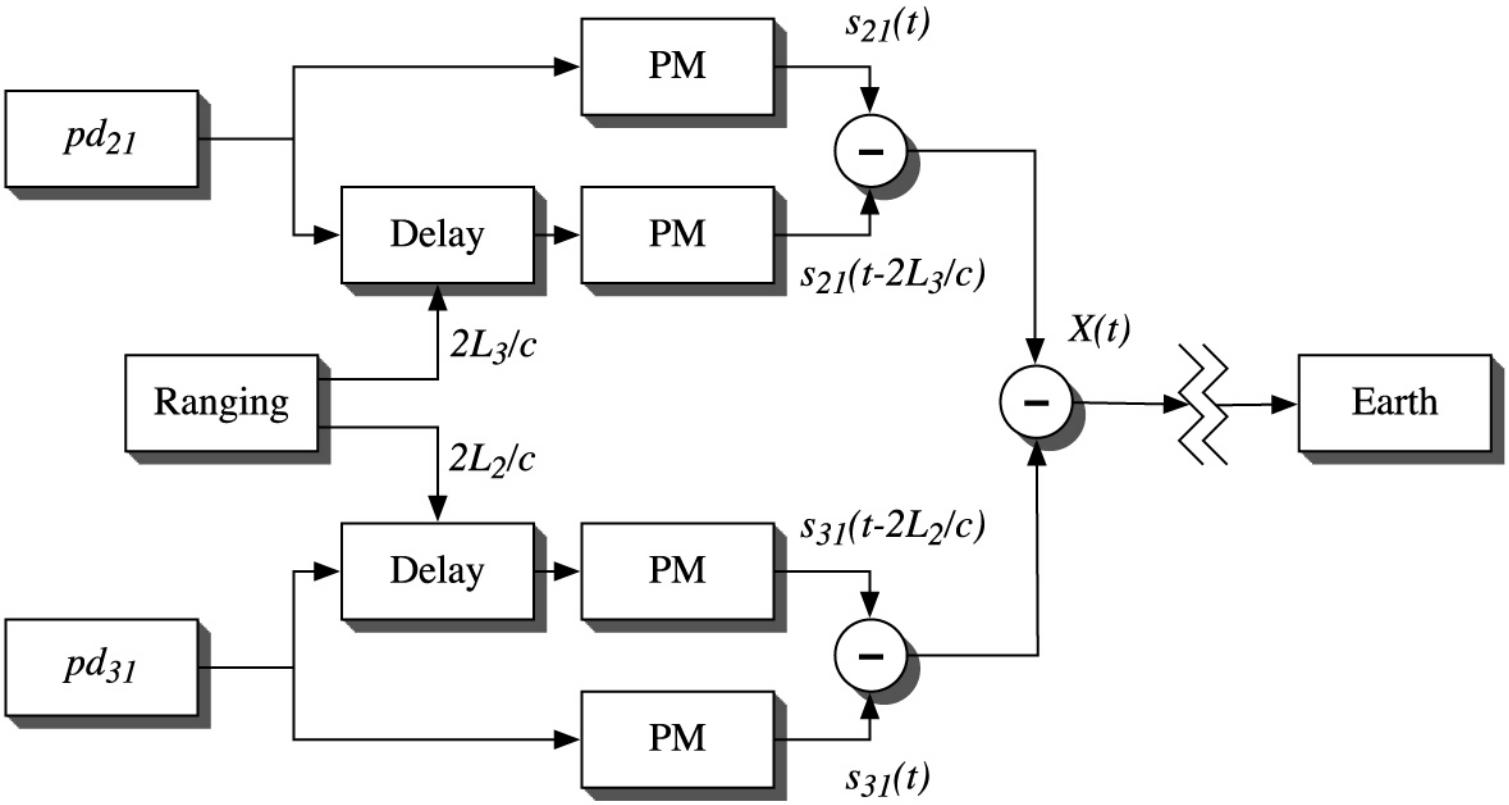


Implications: Real-time TDI

- Current plan is to implement TDI on board by accurately triggering the phase measurements.
- TDI is a function performed on S/C. Real-time TDI.
- Arm length knowledge of 30m needed at time of measurement.
  - Knowledge of past and future arm lengths needed.
  - Some TDI combinations require knowledge of opposite arm.
- Clocks in different spacecraft need to be synchronized to ~100 ns.
- Errors in arm length knowledge, clock synchronization or phasemeter triggering will irreversibly impose laser frequency noise on the TDI combinations.
- Only TDI combinations are transmitted to Earth, not raw phase measurements.

Real-time TDI

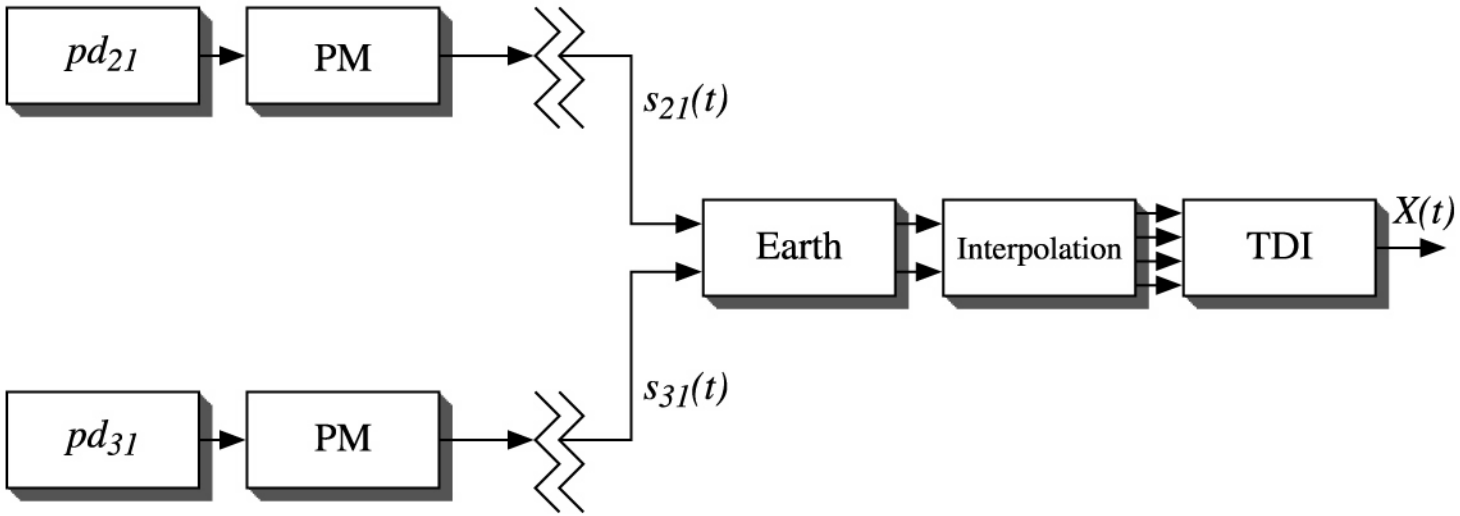
- Consider simplest possible TDI combination. “First generation” TDI combination X (assuming all lasers locked to one master laser)



$$X(t)=s_{21}(t)-s_{31}(t)-[s_{21}(t-2L_3/c)-s_{31}(t-2L_2/c)]$$

Post-processed TDI

- Consider same combination obtained using interpolation referred to as post-processed TDI.



$$X(t)=s_{21}(t)-s_{31}(t)-[s_{21}(t-2L_3/c)-s_{31}(t-2L_2/c)]$$

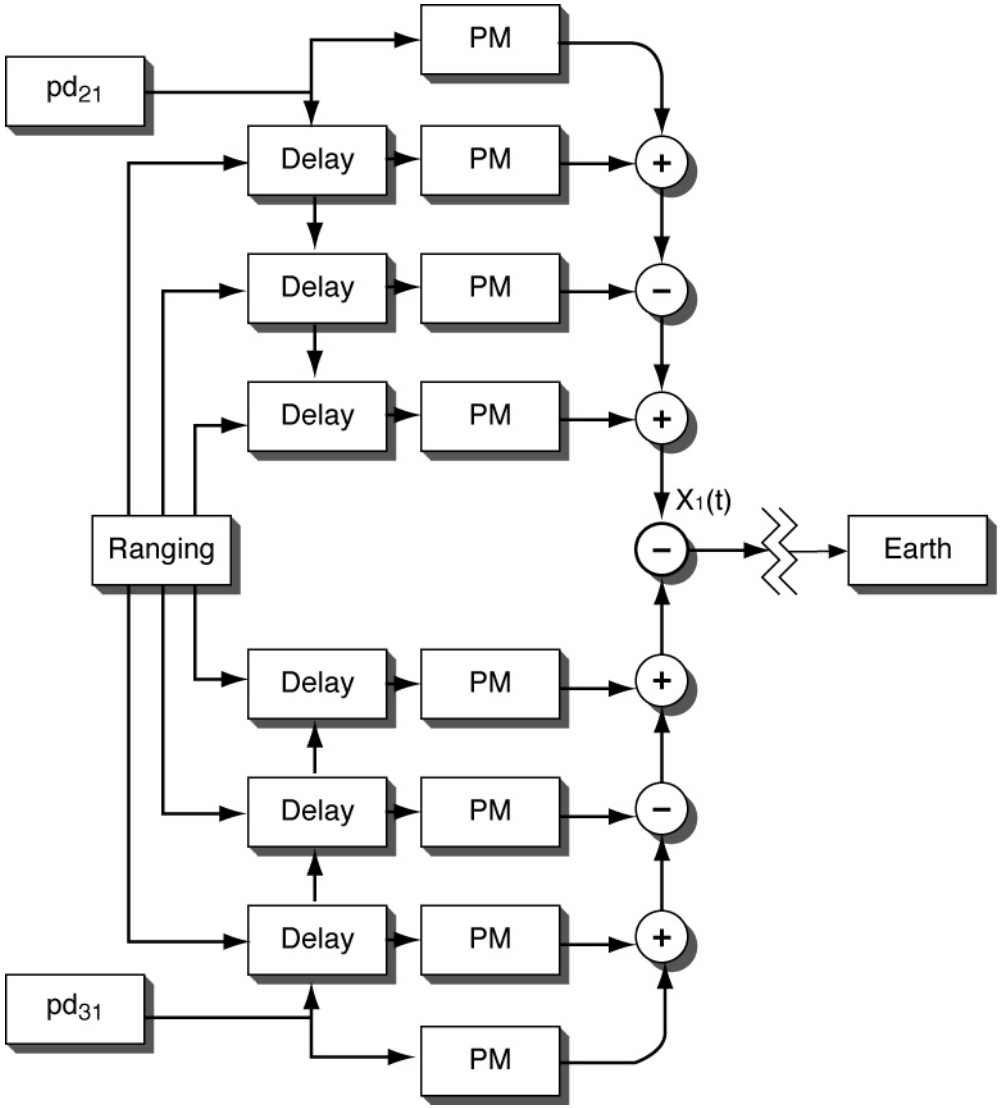
Post-processed TDI

- With interpolation, delays are implemented in post-processing.
  - Eliminates the need for nanosecond-scale triggering of the phase measurements.
  - Phasemeters sample at a constant rate time tagged by local clock.
  - No arm length knowledge needed on-board (by phasemeters, payload computer.)
  - S/C clocks do not need to be synchronized in real-time. A correction can be applied in post-processing.
- Time-delay interferometry combinations can be completely constructed on Earth.
  - Gives scientists access to raw data.
  - Allows flexibility to change algorithms construct 1st or 2nd generation TDI variables. X, Y, Z, X_1, X_2, X_3, ζ, α, β, γ, α_1, α_2, α_3, A, E, T, monitor, beacon, etc (even combinations not realized until after data is in hand).
  - Small increase in data transmitted to Earth (signals transmitted to Earth require larger dynamic range).

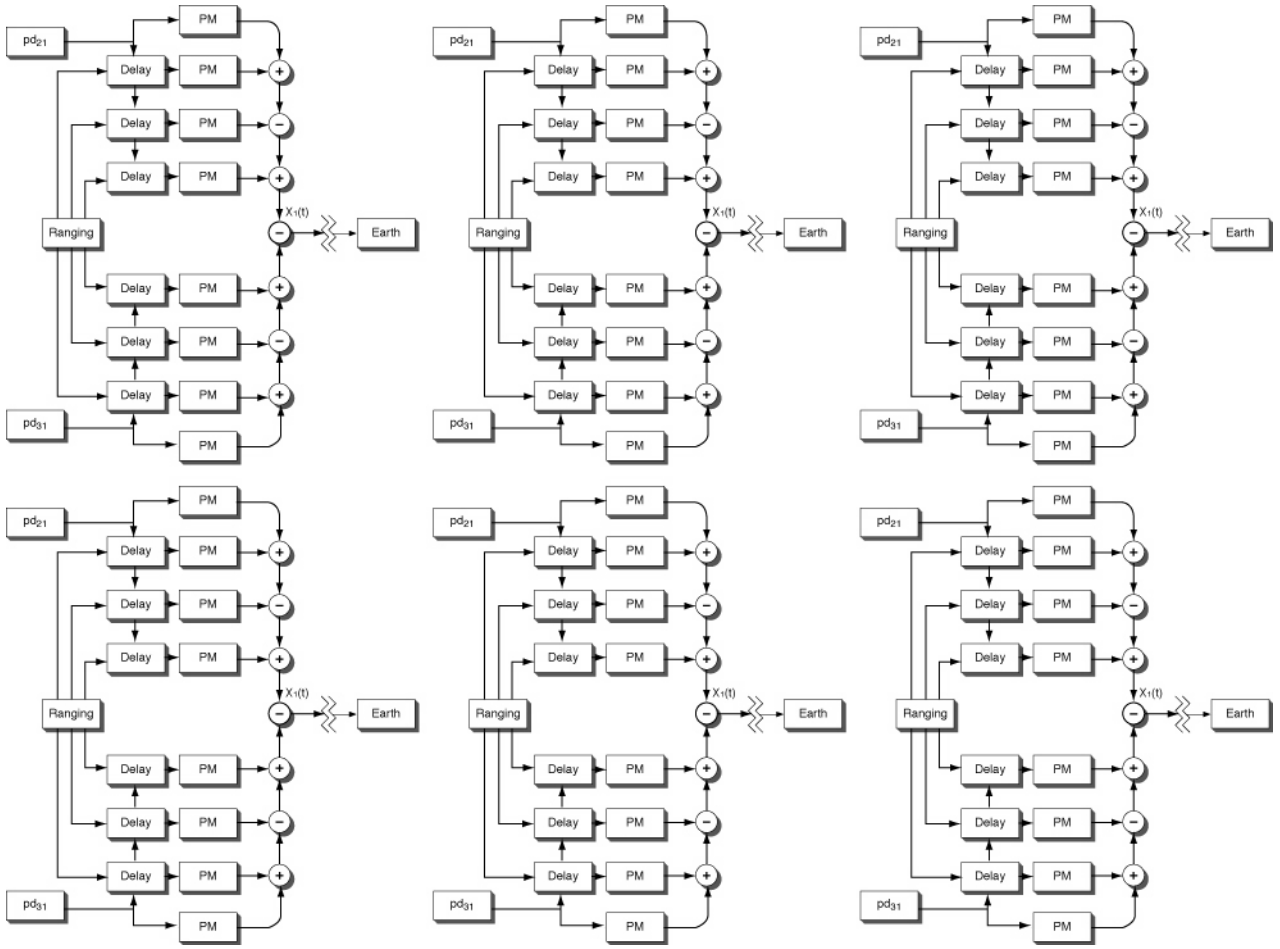
Post-processed TDI

- Simplifies hardware.
  - Fewer phasemeter channels (factor of ~10 reduction).
  - Reduction in, or removal of, inter-spacecraft communications (one data stream per phasemeter).
  - Removal of explicit ranging or clock synchronization systems.
- Potentially improved frequency noise suppression.
  - Estimates of arm length and clock synchronization/correction may be improved by use of post-processing techniques (e.g. auto-correlation) and adjusted after the fact.

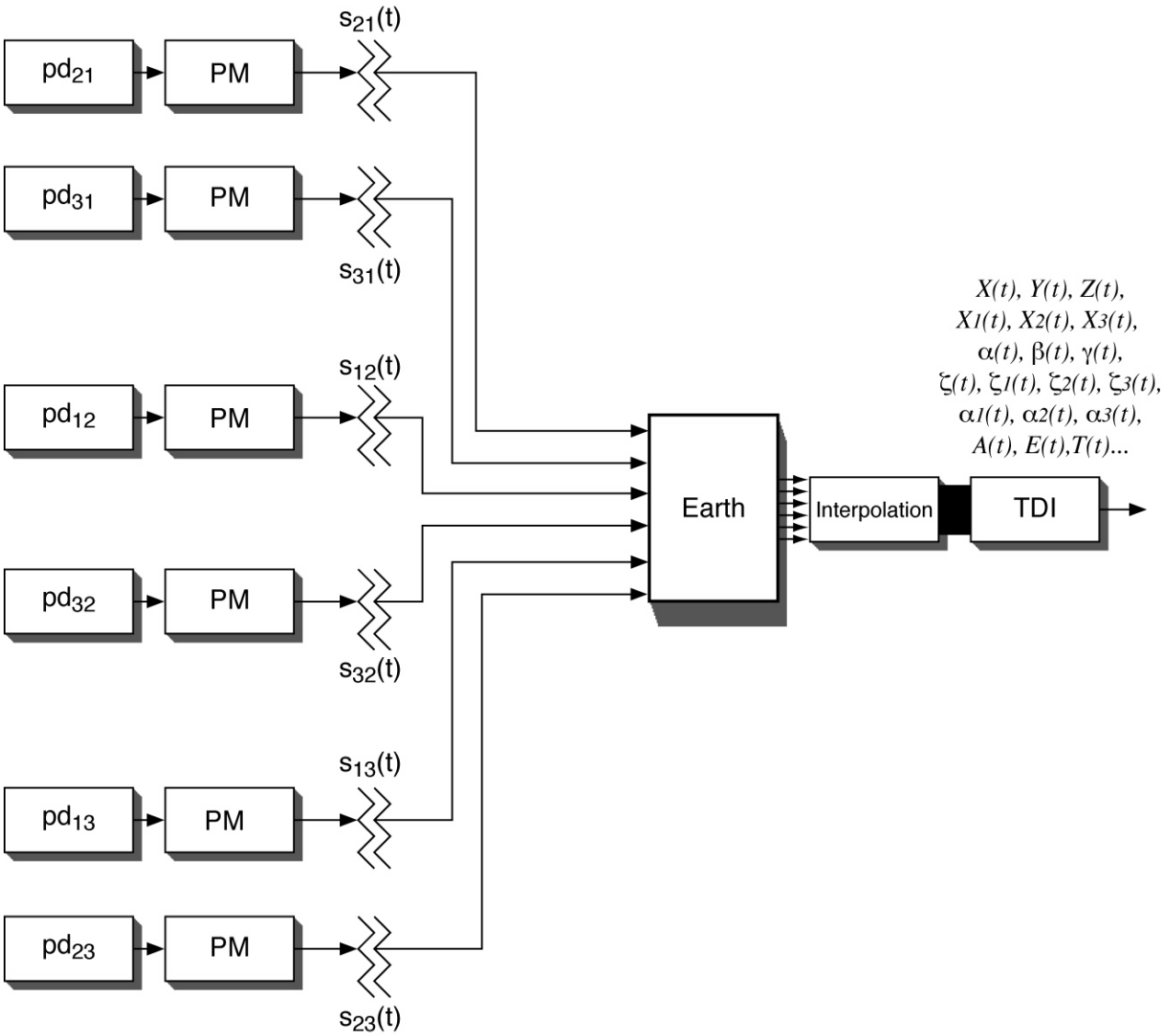
Real-time TDI: $X_1(t)$



Real-time TDI



Post-processed TDI



Conclusions

- Post-processing TDI is feasible. Interpolation meets requirements with only a few seconds of data.
  - Significantly simplified
  - More flexible
  - Interpolation algorithms presented here may not be optimal but are good enough to proceed.
- Slight increase in data rates to Earth.