Algebraic Approach to Time-Delay Data Analysis for Orbiting LISA

K. Rajesh Nayak and J-Y. Vinet

Dept. Artemis
Observatoire de la Côte d'Azur,
Nice.

TDI- Approach to supress Laser Noise!

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- Elegant Approach to cancel the overwhelming ($\frac{\Delta\nu}{\nu_0} = 10^{-14} Hz/\sqrt{Hz}$) laser noise in the space based gravitational wave detectors.

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Algebraic Approach to TDI Analysis

S. V. Dhurandhar, K. R. Nayak and J-Y. Vinet, *Phys. Rev.* **D 65**, 102002(2002).

- Systematic approach based on computational commutative algebra.
- All the noises cancellation data combination can be generated from a set of 4 generators.
- This formalism can easily be extend to alternative geometry or the follow on missions.
- In the present work, we extend the formalism to include the motion of LISA(Sagnac effect)

Sagnac Effect in LISA

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- As a result up and down light travel time for the laser beam along same arm is different(by about few Km)
- This results in large residual laser noise, Hence set of new TDI solution were proposed.

- We apply the algebraic approach by taking the up and down path lengths are different for same arm and obtain a set of generators.

Delay operators used in TDI are defined as,

$$E_{ij}A(t) = A(t - L_{ij})$$

in the present case $L_{ij} \neq L_{ji}$ With this extra constraint, we follow the formalism given for stationary case.

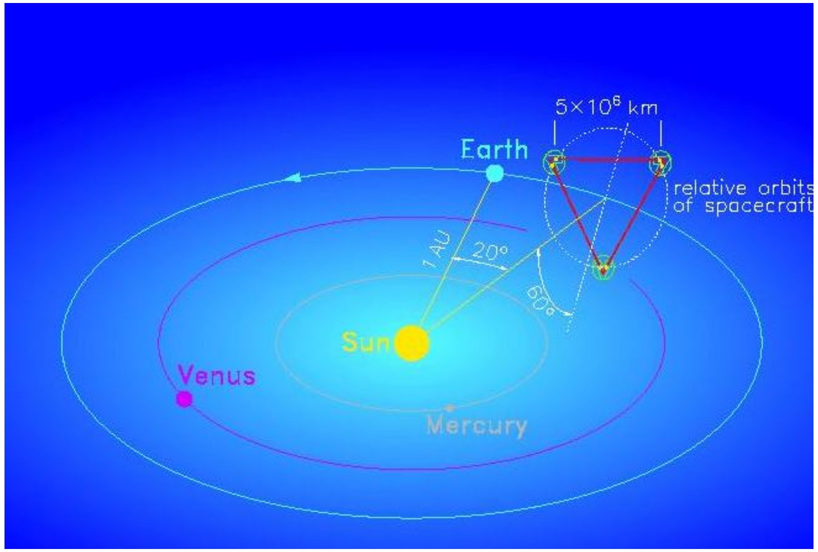
- Earlier results assume a simple module in which LISA rotates only about its own axis!!

- In reality the motion of LISA is much more complex and the our study shows that the main for Sagnac effect comes form orbital motion.

Sagnac effect in LISA system

Metric in LISA frame, $O(\Omega)$

$$
\begin{aligned}
g_{00} &= 1 + O(\Omega^2) \\
g_{01} &= R_\odot \Omega \cos \Omega t - \frac{\Omega}{2} y - \frac{\sqrt{3}}{2} \Omega z \cos \Omega t, \\
g_{02} &= R_\odot \Omega \sin \Omega t + \frac{\Omega}{2} x - \frac{\sqrt{3}}{2} \Omega z \sin \Omega t, \\
g_{03} &= \frac{\sqrt{3}}{2} \Omega \left(x \cos \Omega t + y \sin \Omega t \right), \\
g_{11} &= -1 = g_{22} = g_{33}
\end{aligned}
$$



Metric is time dependent!

Sagnac Phase for LISA Beams

$$R_\odot \simeq 150 \times 10^6 Km$$

$$L \simeq 5 \times 10^6 Km$$

$$\Omega \simeq 10^{-7} sec^{-1}$$

$$\Delta\phi_{orbit}(t)/\Delta\phi_L \simeq R/L$$

$$
\begin{aligned}
\Delta\tau_{U1} &= L\left(1 + \overbrace{\Omega R_\odot \sin\Omega t} - \frac{\Omega L}{\underbrace{4\sqrt{3}}}\right), \\
\Delta\tau_{U2} &= L\left(1 + \overbrace{\Omega R_\odot \sin\left(\Omega t - \frac{\pi}{3}\right)} - \frac{\Omega L}{\underbrace{4\sqrt{3}}}\right), \\
\Delta\tau_{U3} &= L\left(1 - \overbrace{\Omega R_\odot \sin\left(\Omega t + \frac{\pi}{3}\right)} - \frac{\Omega L}{\underbrace{4\sqrt{3}}}\right), \\
\Delta\tau_{V1} &= L\left(1 - \overbrace{\Omega R_\odot \sin\left(\Omega t - \frac{\pi}{3}\right)} + \frac{\Omega L}{\underbrace{4\sqrt{3}}}\right), \\
\Delta\tau_{V2} &= L\left(1 + \overbrace{\Omega R_\odot \sin\left(\Omega t + \frac{\pi}{3}\right)} + \frac{\Omega L}{\underbrace{4\sqrt{3}}}\right), \\
\Delta\tau_{V3} &= L\left(1 - \overbrace{\Omega R_\odot \sin\Omega t} + \frac{\Omega L}{\underbrace{4\sqrt{3}}}\right).
\end{aligned}
$$

The Generators for the Module

We apply the formalism similar to the stationary case and obtain the generators,

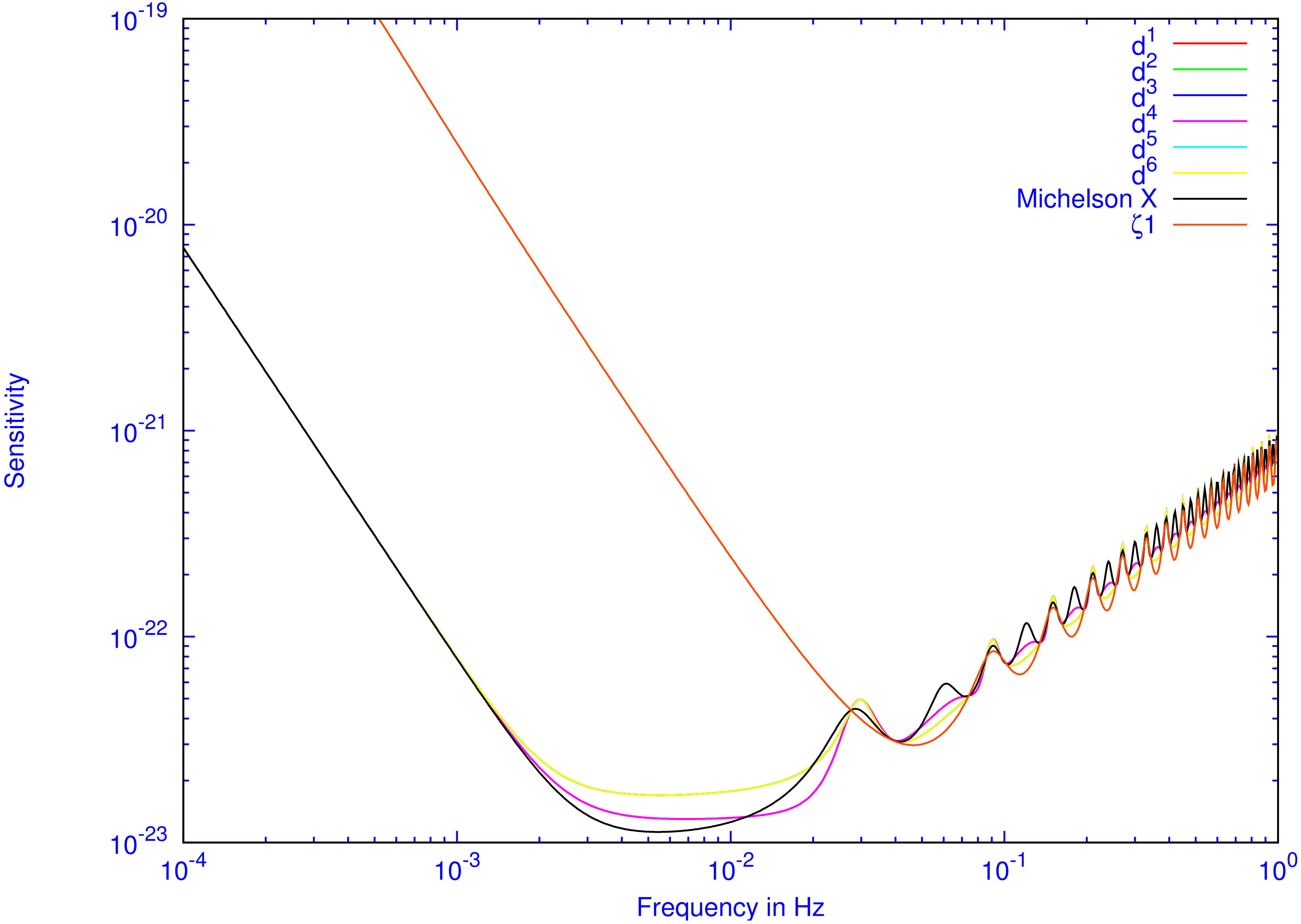
$$
\begin{aligned}
d^{(1)} &= \left(E_{13}(1-E_{23}E_{32}), \quad E_{21}E_{13}-E_{23}, \quad 1-E_{23}E_{32}, \quad 0, \quad 0, \quad 1-E_{21}E_{13}E_{32} \right), \\
d^{(2)} &= \left(E_{23}(1-E_{13}E_{31}), \quad 0, \quad E_{21}-E_{31}E_{23}, \quad E_{23}-E_{21}E_{13}, \quad 0, \quad E_{21}(1-E_{13}E_{31}) \right), \\
d^{(3)} &= \left(0, \quad 1-E_{31}E_{13}, \quad E_{32}-E_{12}E_{31}, \quad E_{12}-E_{13}E_{32}, \quad 1-E_{13}E_{31}, \quad 0 \right), \\
d^{(4)} &= \left(E_{12}-E_{13}E_{32}, \quad E_{12}E_{21}-1, \quad E_{32}(E_{21}E_{12}-1), \quad 0, \quad E_{21}E_{13}E_{32}-1, \quad 0 \right), \\
d^{(5)} &= \left(E_{23}E_{32}-1, \quad E_{31}E_{23}-E_{21}, \quad 0, \quad E_{23}E_{32}-1, \quad 0, \quad E_{21}E_{32}-E_{31} \right), \\
d^{(6)} &= \left(E_{13}-E_{12}E_{23}, \quad 0, \quad 1-E_{12}E_{21}, \quad 0, \quad E_{23}-E_{21}E_{13}, \quad 1-E_{12}E_{21} \right).
\end{aligned}
$$

The Sagnac and Symmetric Sagnac solutions do not form a generating set. Any noise cancellation data combination X can be expressed as,

$$X = \sum_{I=1}^{6} \alpha_{(I)} d^{(I)}$$

where $\alpha_{(I)}$ are polynomial coefficients in six variable E_{ij}.

The Sensitivity Curves



Conclusions

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Conclusions

- The contribution from Sagnac effect is much lager than earlier predicted.
- In this case also laser noise cancellation solutions form the first module of syzygies over polynomial ring in six time delays E_{ij}.
- The analysis can be extended to cancel optical bench noise in a straight forward manner.
- The generators are non-trivial in this case. The Sagnac and Symmetric Sagnac combinations do not form a generating set as in the stationary case.
- The formalism is useful as set of all solution can be parame-terised in terms of coefficients of generators.

Applications(Stationary case)

- ⋆ Maximisation of SNR: Since each noise cancellation solution has different signal response, we can determine the data combination with maximum SNR

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- ⋆ For a source in given direction: One can get the optimal SNR same as the above case but the lower bound is zero!

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