Laser Interferometer Space Antenna (LISA)

Time-Delay Interferometry & The Zero-Signal Solution (ZSS)

Massimo Tinto

Jet Propulsion Laboratory,

California Institute of Technology

http://gwscience.jpl.nasa.gov

GSFC·JPL

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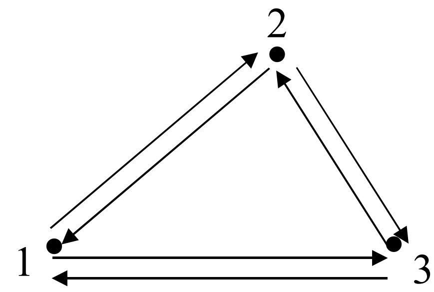
Time-Delay Interferometry (TDI)

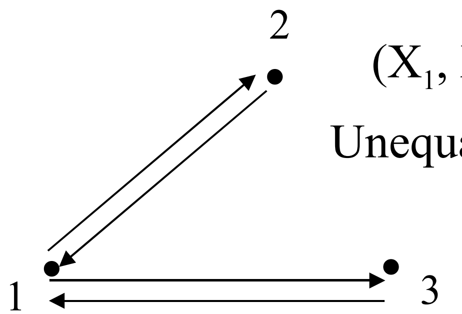
- We have heard excellent presentations (both experimental and theoretical) on new developments **in the field** of Time-Delay Interferometry.

- See, for instance, talks presented by D. Summers, D. Shaddock, I. Thorpe, and those by Nayak & Vallisneri later in this session.

TDI Combinations

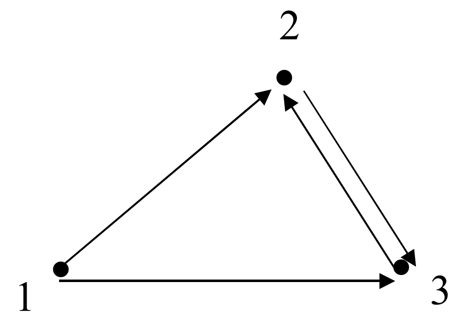


$(\alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3)$



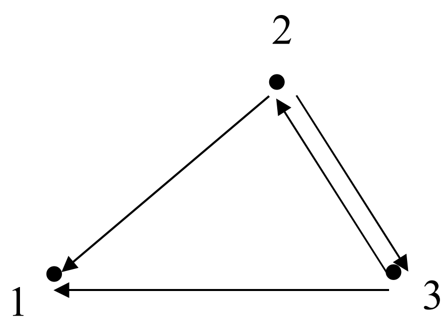
(X_1, X_2, X_3)

Unequal-arm Michelson



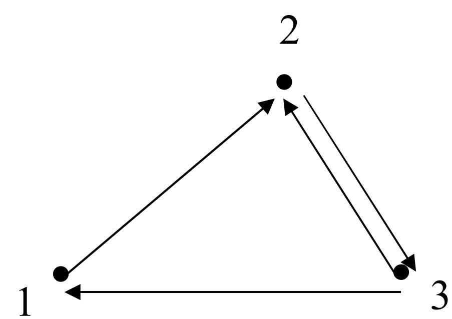
(P_1, P_2, P_3)

Beacon



(E_1, E_2, E_3)

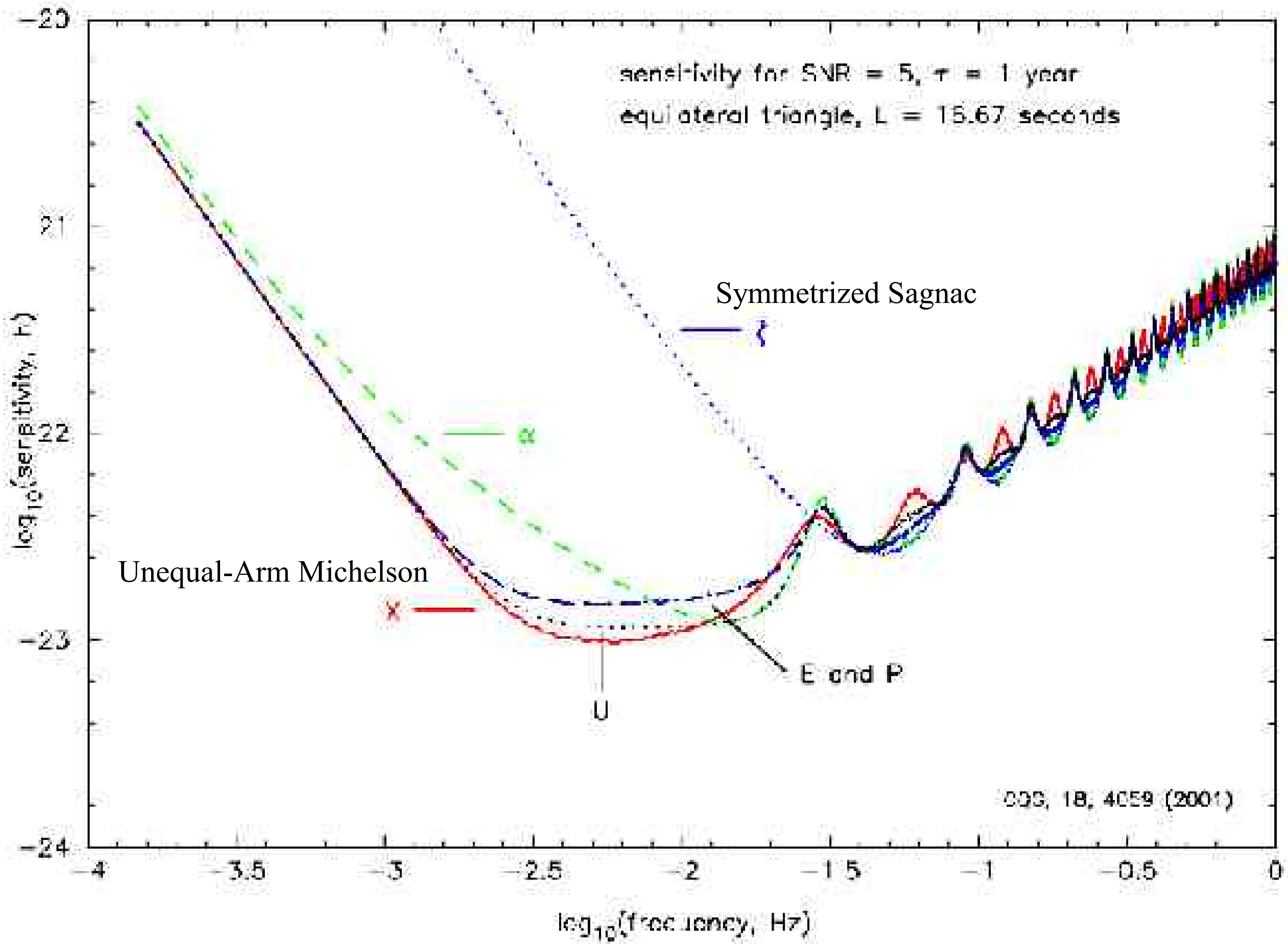
Monitor



(U_1, U_2, U_3)

Relay

Combinations' Sensitivities

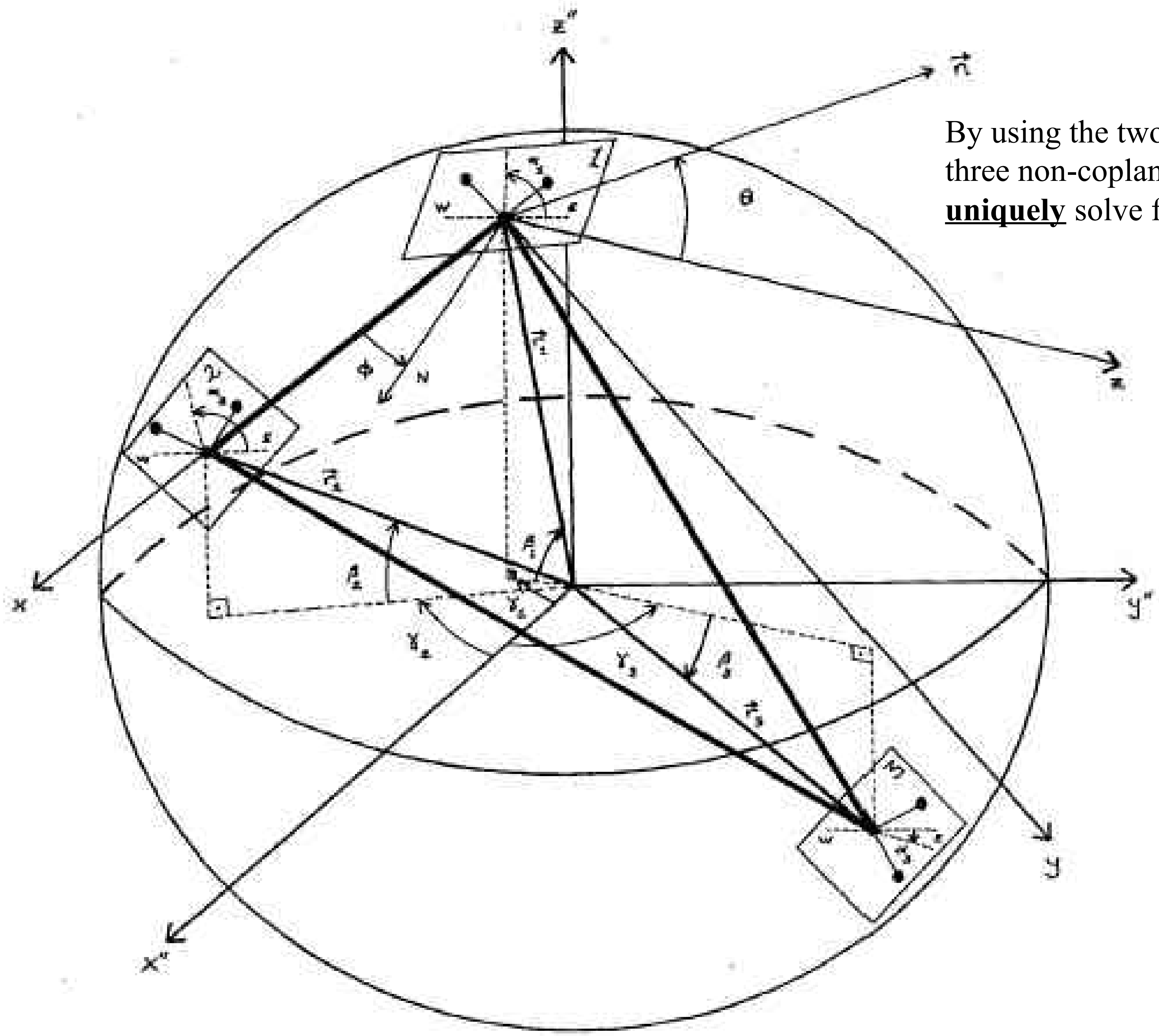


“Optimal” TDI Combinations

1. Is there a TDI combination that gives optimal SNR?
    - T. Prince, M. Tinto, S. Larson, J.W. Armstrong, *Phys. Rev. D*, **66**, 122002 (2002)
    - K.R. Nayak, S.V. Dhurandhar, A. Pai, J.-Y., Vinet, *Phys. Rev. D*, **68**, 122001 (2003)
2. What if a signal is so strong to stand up above ζ?
3. How do we solve the “inverse problem” for bursts with LISA?
4. How can we distinguish a signal from noise in the high-frequency part of the LISA band?

M. Tinto & S.L. Larson., *Phys. Rev. D*, accepted for publication; arXiv:gr-qc/0405147 v1

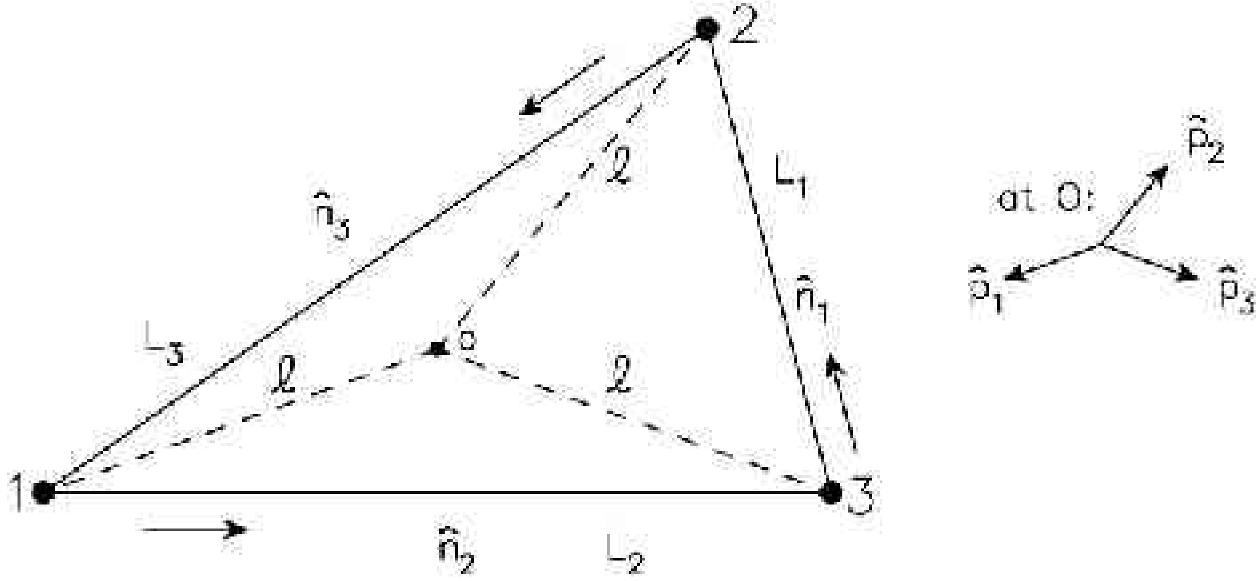
The Inverse Problem in GW Astronomy



By using the two independent time-delays, and the three non-coplanar detector responses, one can **uniquely** solve for the GW signal.

Y. Gursel & M.Tinto, *Phys. Rev. D*, **40**, 3884, (1989)

The Inverse Problem for LISA

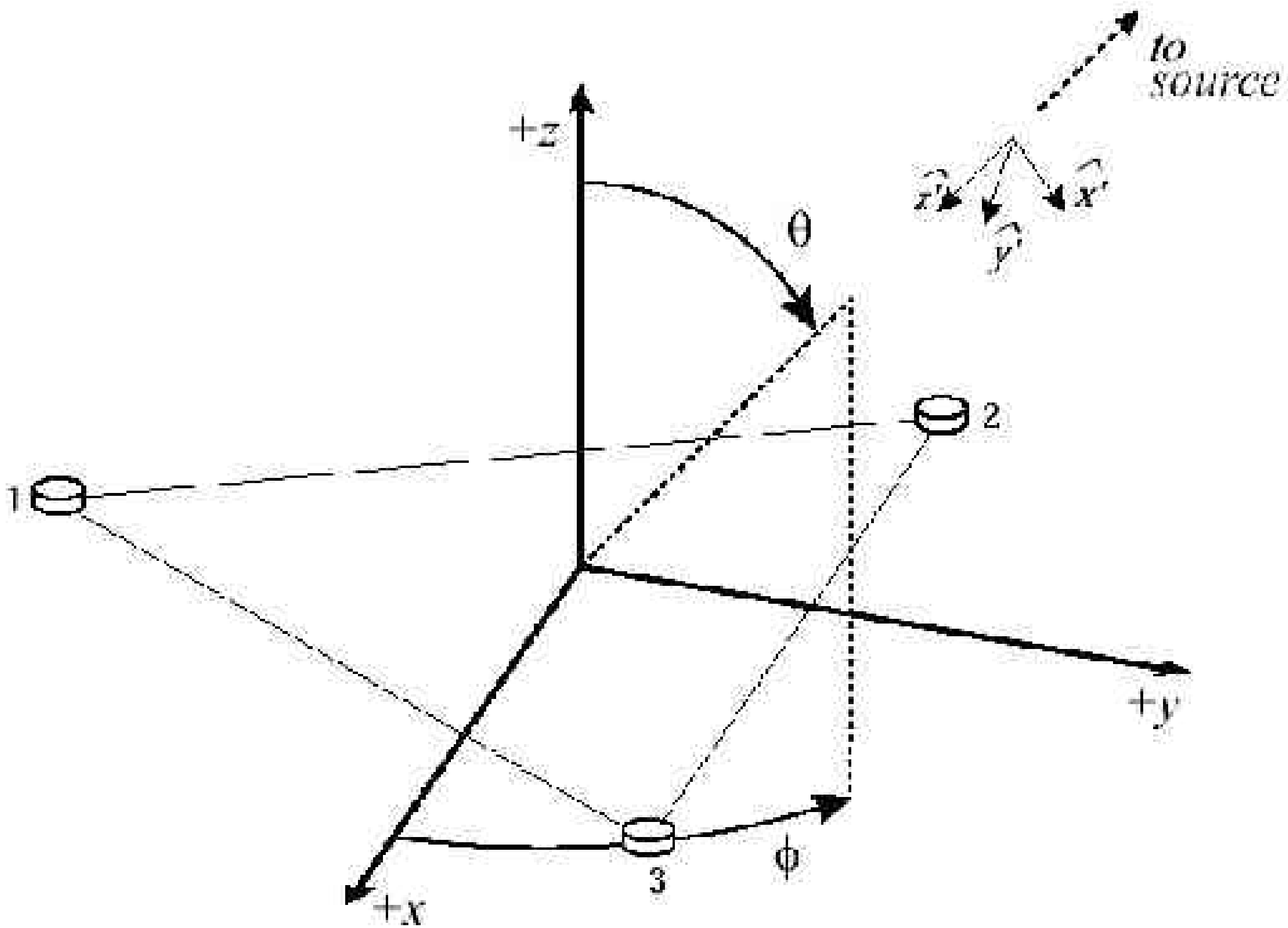


$$\mu_i \equiv \hat{k} \cdot \hat{p}_i$$

$$L_1 \hat{k} \cdot \hat{n}_1 = l(\mu_2 - \mu_3)$$

These are the LISA time-delays!

The Inverse Problem…



Let’s consider the signal responses of the three Sagnac combinations: α, β, γ

$$
\begin{aligned}
\alpha^{gw}(t) = & \left[1 - \frac{l}{L_1}(\mu_2 - \mu_3)\right] \left(\Psi_1(t - \mu_2\, l - L_1 - L_2) - \Psi_1(t - \mu_3\, l - L_2)\right) \\
& - \left[1 + \frac{l}{L_1}(\mu_2 - \mu_3)\right] \left(\Psi_1(t - \mu_3\, l - L_1 - L_3) - \Psi_1(t - \mu_2\, l - L_3)\right) \\
& + \left[1 - \frac{l}{L_2}(\mu_3 - \mu_1)\right] \left(\Psi_2(t - \mu_3\, l - L_2) - \Psi_2(t - \mu_1\, l)\right) \\
& - \left[1 + \frac{l}{L_2}(\mu_3 - \mu_1)\right] \left(\Psi_2(t - \mu_1\, l - L_2 - L_1 - L_3) - \Psi_2(t - \mu_3\, l - L_1 - L_3)\right) \\
& + \left[1 - \frac{l}{L_3}(\mu_1 - \mu_2)\right] \left(\Psi_3(t - \mu_1\, l - L_3 - L_1 - L_2) - \Psi_3(t - \mu_2\, l - L_1 - L_2)\right) \\
& - \left[1 + \frac{l}{L_3}(\mu_1 - \mu_2)\right] \left(\Psi_3(t - \mu_2\, l - L_3) - \Psi_3(t - \mu_1\, l)\right) .
\end{aligned}
$$

$$
\Psi_i(t) = \frac{1}{2} \; \frac{\hat{n}_i \cdot \mathbf{h}(t) \cdot \hat{n}_i}{1 - (\hat{k} \cdot \hat{n}_i)^2}
$$

There are six-pulses, determined by the two independent time-delays!

Following G&T, the idea is to find a suitable combination of the Sagnac responses that shows zero-signal response (ZSS)!

$$\tilde{\eta}^{gw}(f) \equiv a_1\, \tilde{\alpha}^{gw}(f) \; + \; a_2\, \tilde{\beta}^{gw}(f) \; + \; a_3\, \tilde{\gamma}^{gw}(f) = 0$$

$$\begin{aligned}
\tilde{\alpha}^{gw}(f) &= \alpha_+(f,\theta_s,\phi_s)\,\tilde{h}_+(f) \; + \; \alpha_\times(f,\theta_s,\phi_s)\,\tilde{h}_\times(f)\,, \\
\tilde{\beta}^{gw}(f) &= \beta_+(f,\theta_s,\phi_s)\,\tilde{h}_+(f) \; + \; \beta_\times(f,\theta_s,\phi_s)\,\tilde{h}_\times(f)\,, \\
\tilde{\gamma}^{gw}(f) &= \gamma_+(f,\theta_s,\phi_s)\,\tilde{h}_+(f) \; + \; \gamma_\times(f,\theta_s,\phi_s)\,\tilde{h}_\times(f)\,,
\end{aligned}$$

$$\begin{aligned}
\tilde{\eta}^{gw}(f) = & \left[a_1\,\alpha_+(f,\theta_s,\phi_s) \; + \; a_2\,\beta_+(f,\theta_s,\phi_s) \; + \; a_3\,\gamma_+(f,\theta_s,\phi_s)\right]\widetilde{h_+}(f) \\
& + \left[a_1\,\alpha_\times(f,\theta_s,\phi_s) \; + \; a_2\,\beta_\times(f,\theta_s,\phi_s) \; + \; a_3\,\gamma_\times(f,\theta_s,\phi_s)\right]\widetilde{h_\times}(f)
\end{aligned}$$

$$\begin{aligned}
a_1\,\alpha_+(f,\theta_s,\phi_s) \; + \; a_2\,\beta_+(f,\theta_s,\phi_s) \; + \; a_3\,\gamma_+(f,\theta_s,\phi_s) &= 0 \\
a_1\,\alpha_\times(f,\theta_s,\phi_s) \; + \; a_2\,\beta_\times(f,\theta_s,\phi_s) \; + \; a_3\,\gamma_\times(f,\theta_s,\phi_s) &= 0
\end{aligned}$$

$$a_1(f,\theta_s,\phi_s) = \beta_+(f,\theta_s,\phi_s)\,\gamma_\times(f,\theta_s,\phi_s) - \beta_\times(f,\theta_s,\phi_s)\,\gamma_+(f,\theta_s,\phi_s)\,,$$

$$a_2(f,\theta_s,\phi_s) = \gamma_+(f,\theta_s,\phi_s)\,\alpha_\times(f,\theta_s,\phi_s) - \gamma_\times(f,\theta_s,\phi_s)\,\alpha_+(f,\theta_s,\phi_s)\,,$$

$$a_3(f,\theta_s,\phi_s) = \alpha_+(f,\theta_s,\phi_s)\,\beta_\times(f,\theta_s,\phi_s) - \alpha_\times(f,\theta_s,\phi_s)\,\beta_+(f,\theta_s,\phi_s)\,.$$

$$\begin{aligned}\tilde{\eta} \equiv\ & [\beta_+(f,\theta,\phi)\,\gamma_\times(f,\theta,\phi) - \beta_\times(f,\theta,\phi)\,\gamma_+(f,\theta,\phi)]\,\tilde{\alpha}(f) \\ & + [\gamma_+(f,\theta,\phi)\,\alpha_\times(f,\theta,\phi) - \gamma_\times(f,\theta,\phi)\,\alpha_+(f,\theta,\phi)]\,\tilde{\beta}(f) \\ & + [\alpha_+(f,\theta,\phi)\,\beta_\times(f,\theta,\phi) - \alpha_\times(f,\theta,\phi)\,\beta_+(f,\theta,\phi)]\,\tilde{\gamma}(f)\,,\end{aligned}$$

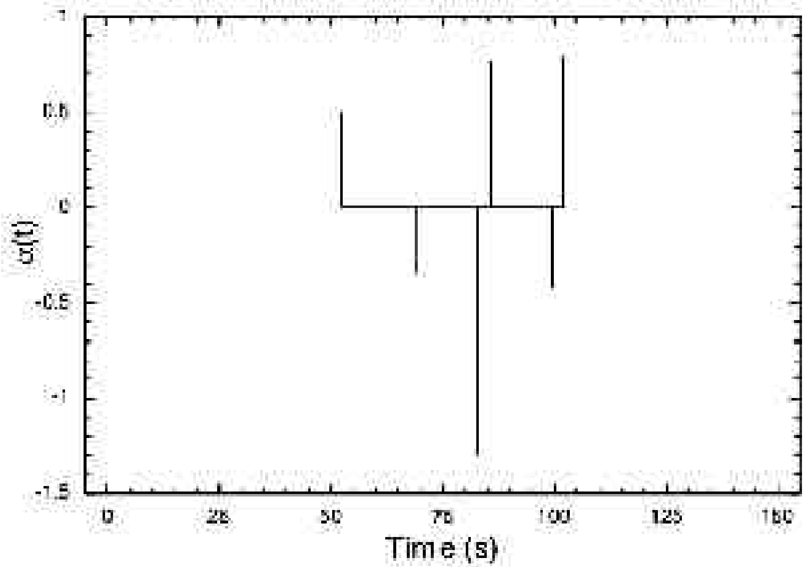
Since the antenna patterns are all co-planar, we can only identified two points in the sky (one of which corresponds to the source location!)

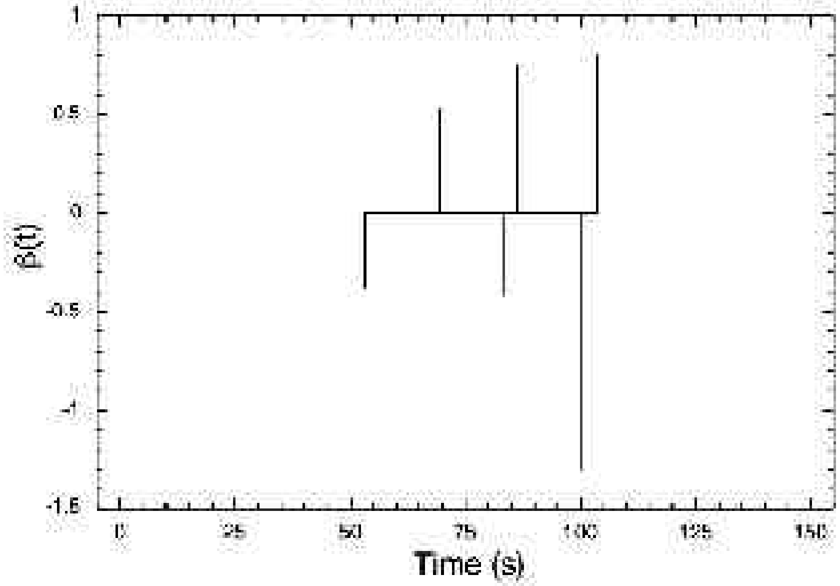
$$a_3(f,\theta,\phi) = \sum_{k=1}^{27} A_3^{(k)}(\theta,\phi)\; e^{2\pi i f \Delta_3^{(k)}}$$

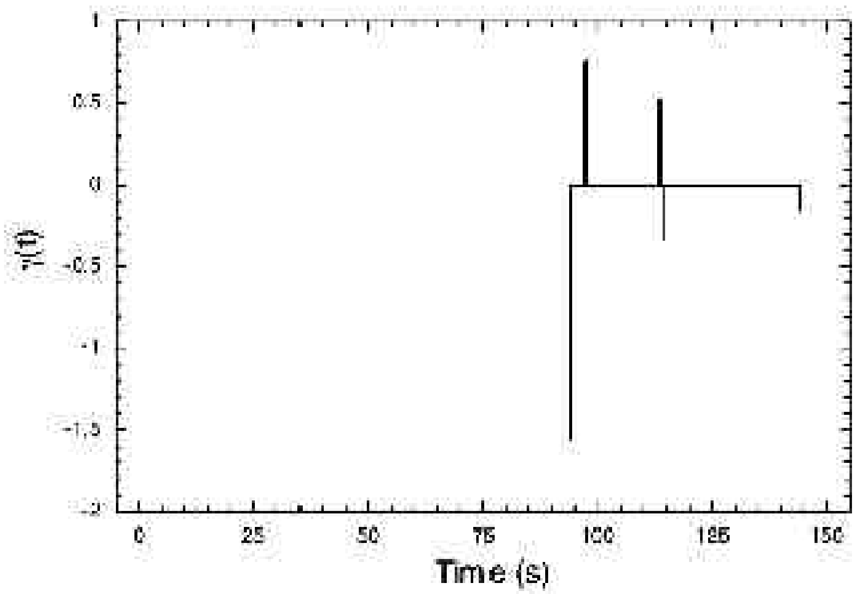
$$\eta(t,\theta,\phi) = \sum_{k=1}^{27} [A_1^{(k)}\; \alpha(t-\Delta_1^{(k)}) + A_2^{(k)}\; \beta(t-\Delta_2^{(k)}) + A_3^{(k)}\; \gamma(t-\Delta_3^{(k)})]$$

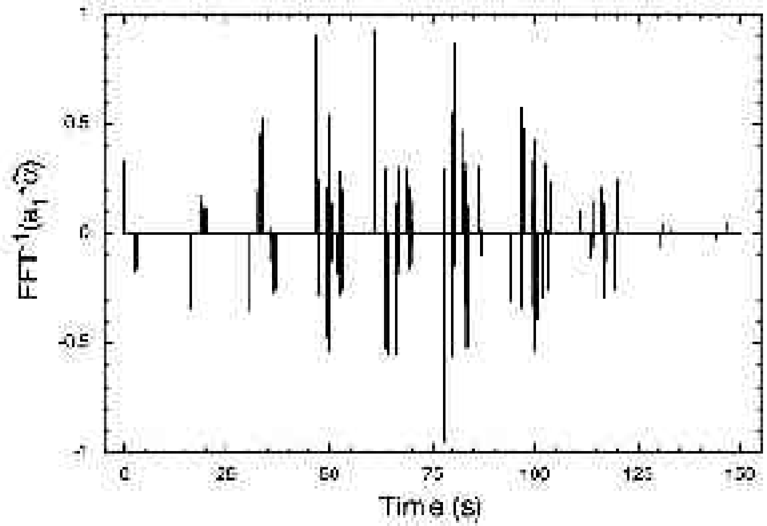
The ZSS is obtained by applying TDI to the TDI observables!!

Example Application

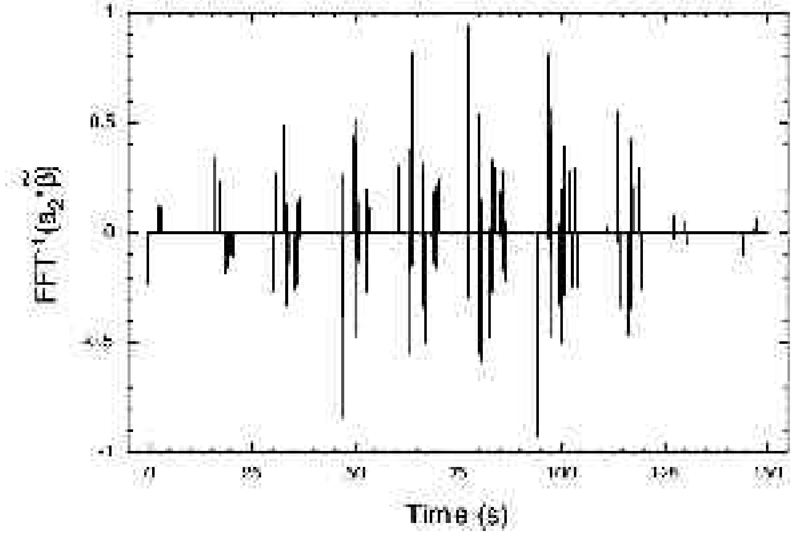




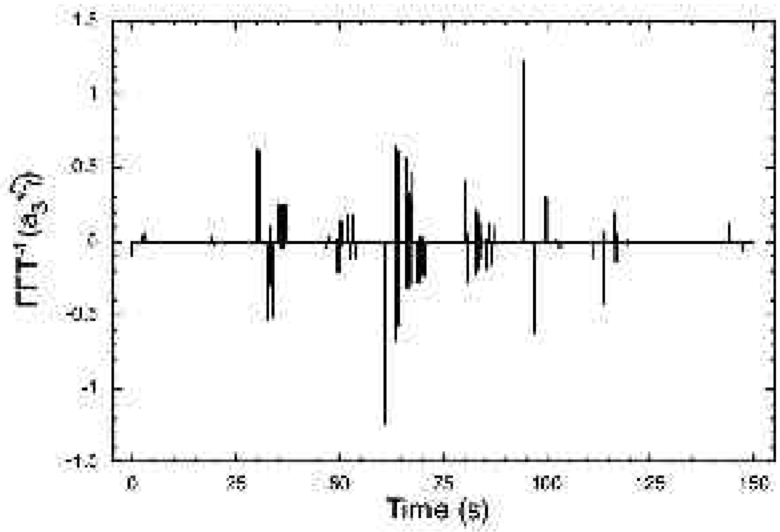




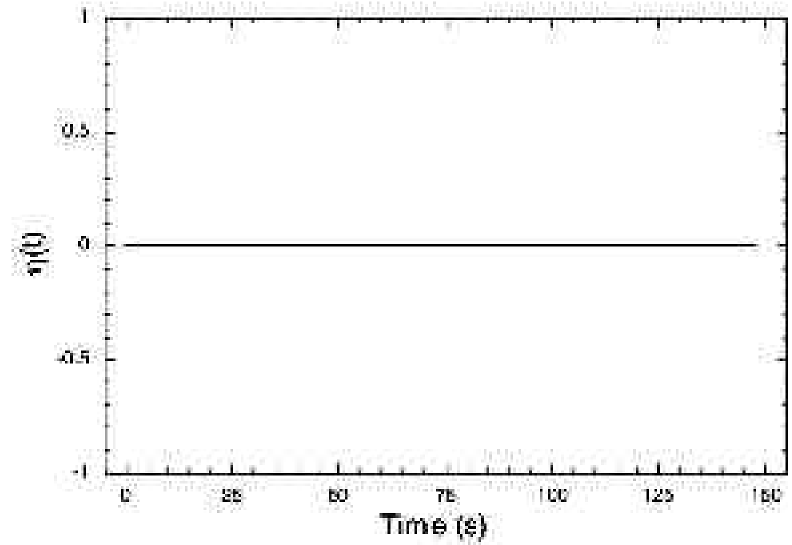
1
0.5
0
-0.5
-1
0
25
50
75
100
125
150
Time (s)



FFT-1(a2*β)
1
0.5
0
-0.5
-1
Time (s)



1.5
1
0.5
0
-0.5
-1
-1.5
0
25
50
75
100
125
150
Time (s)



η(t)
1
0.5
0
-0.5
-1
0
25
50
75
100
125
150
Time (s)

Accuracies

- We are in the process of performing a Monte Carlo-like simulation for estimating the accuracies in the determination of the source location with the ZSS.
- However, there exist a simple analytic formula for the error box in the sky where the source must have come from:

$$\Delta\Omega = \frac{2\, c^2}{\pi^2\, A\, \cos\theta\, f_0^2\, SNR^2}$$

- For a burst of central frequency $f_0 = 1$ Hz, SNR = 10, => $\Delta\Omega = 3.4 \times 10^{-5}$ sr.

The Long-Wavelength Limit

$$\tilde{\eta}(f,\theta,\phi) \approx (2\,\pi f\,L)^4\,\left[(F_1^+F_2^\times - F_1^\times F_2^+) - (F_2^+F_3^\times - F_2^\times F_3^+) + (F_3^+F_1^\times - F_3^\times F_1^+)\right]$$
$$\times\left\{\frac{i\,(2\,\pi\,f\,L)^3}{4}\left[[(\hat{k}^{(s)}-\hat{k})\cdot\hat{n}_1](\hat{n}_1\cdot\widetilde{\mathbf{h}^{(s)}}(f)\cdot\hat{n}_1) - [(\hat{k}^{(s)}-\hat{k})\cdot\hat{n}_2](\hat{n}_2\cdot\widetilde{\mathbf{h}^{(s)}}(f)\cdot\hat{n}_2)\right.\right.$$
$$\left.\left.+[(\hat{k}^{(s)}-\hat{k})\cdot\hat{n}_3](\hat{n}_3\cdot\widetilde{\mathbf{h}^{(s)}}(f)\cdot\hat{n}_3)\right] + \tilde{\alpha}_n + \tilde{\beta}_n + \tilde{\gamma}_n\right\}\;,$$

$$\tilde{\zeta}(f) \sim \frac{1}{12}\,(2\pi i f L)^3\,\left[(\hat{k}^{(s)}\cdot\hat{n}_1)(\hat{n}_1\cdot\widetilde{\mathbf{h}^{(s)}}(f)\cdot\hat{n}_1) + (\hat{k}^{(s)}\cdot\hat{n}_2)(\hat{n}_2\cdot\widetilde{\mathbf{h}^{(s)}}(f)\cdot\hat{n}_2)\right.$$
$$\left.+ (\hat{k}^{(s)}\cdot\hat{n}_3)(\hat{n}_3\cdot\widetilde{\mathbf{h}^{(s)}}(f)\cdot\hat{n}_3)\right] + \frac{1}{3}\left[\tilde{\alpha}_n + \tilde{\beta}_n - \tilde{\gamma}_n\right]\;,$$

The ZSS should be regarded as an improvement over ζ.

Conclusions

- The ZSS allows one to identify two points in the sky where the burst signal must have come from.
- The ZSS can be implemented with sinusoidal signals (integrating data on shorter time scales!)
- It does not rely on any assumptions about the gravitational waveform, and in fact it work for waveforms of any kind.
- It provides a way for testing whether one has observed a signal (or only noise) in the entire LISA band.