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Radiation reaction

+ 2 posters

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RR Plunging phase and radial fall ?   ***Fundamental physics***

II. Classical problem of gravitation! (apple falling)

III. Radiation reaction not adiabatic!

IV. Most rr (in radial fall) between 4M - 2M

V. pN N/A (convergence, v/c)

$$\frac{dE}{dt} \propto \dddot{Q}^2$$

At 10 M pN = quadrupole

I. Entry problem, before tackling close and scattered orbits

II. Plunging and signal discrimination (known waveform)
No 17 parameters + etc. problem

IV. Plunging only visible (??) part of capture for low-end
LISA frequency spectrum   (10^{6-7} $M_{\text{Ÿ}}$)

V. All captures finish in a plunge

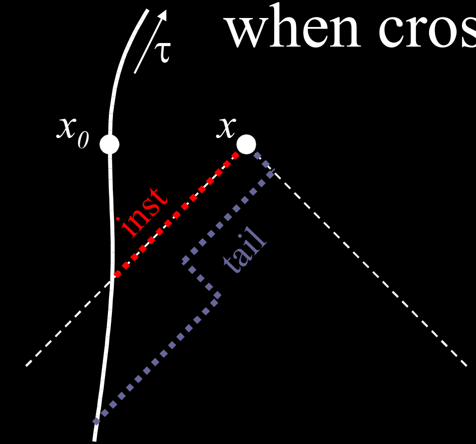
VI. Applicable to Last Phase in Coalescence (close limit)

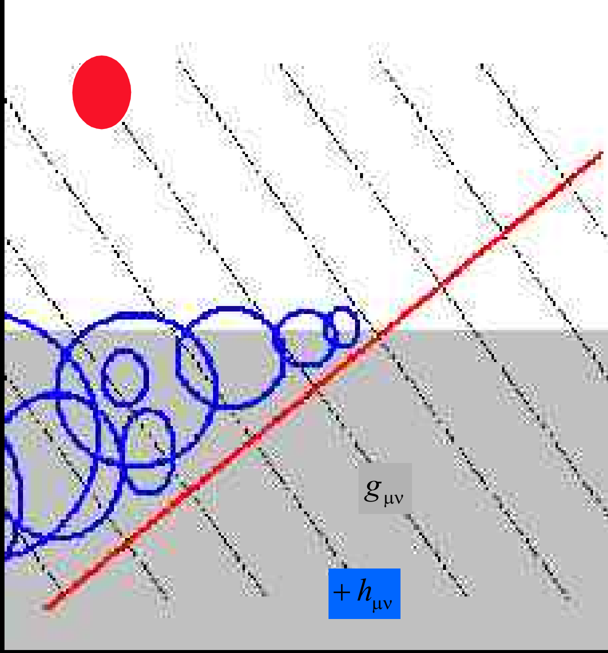
VII. It’s all quasinormal.

Review of what do we know and what are we doing in Nice

Self force

Some spacetimes determine tail effects when crossed by perturbations





$$F^{\mu}_{self} = \lim_{x \to x_0} \sum_{l=0}^{\infty} F^{\mu l}_{\text{tail}}(x) = \lim_{x \to x_0} \sum_{l=0}^{\infty} \left(F^{\mu l}_{\text{full}}(x) - F^{\mu l}_{\text{inst}}(x) \right)$$

$$\lim_{x \to 0} = \sum_{l=0}^{\infty} \left(F^{\mu l}_{full\pm} - A^{\mu}_{\pm} L - B^{\mu} - C^{\mu} / L \right) - D^{\mu}$$

Checked OK

PM sign indicates derivative forward or backward r_p

$L = l + 1/2$

F_{full} = waited waveform numerical simulation

$\bar{A}_{inst} = 0$, $B_{inst}, C_{inst} = 0$ checked OK

$D_{inst} = 0$ on-going

Work J.-Y. Vinet for determination ongoing → Poster

Perturbations

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

h << BH metric ※

Linearisation allows splitting of orders 0 and 1

Geometry = Matter ⟶ Perturbed Geometry = Perturbing Matter

Spherical symmetry → Rotationally invariant operator on h , equal to the variation of the energy momentum tensor, both expressed in spherical harmonics:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} \qquad \Rightarrow \qquad Q[h_{\mu\nu}] \propto \Delta T_{\mu\nu}$$

$$\frac{\partial^2 \Psi_l}{\partial r^{*2}} - \frac{\partial^2 \Psi_l}{\partial t^2} - V_l(r) = S_l(r,t)$$

$$r^* = r + 2M \ln\left(\frac{r}{2M} - 1\right) \qquad \lambda = \frac{(l+2)(l-1)}{2}$$

$$V_l(r) = \left(1 - \frac{2M}{r}\right) \frac{2\lambda^2(\lambda+1)r^3 + 6\lambda^2 M r^2 + 18\lambda M^2 r + 18M^3}{r^3(\lambda r + 3M)^2}$$

$$\kappa = \frac{4\sqrt{\pi}\,(9+8\lambda)^{1/4}}{\lambda+1} \frac{m}{\widetilde{E}}$$

$$S_l(r,t) = \kappa \frac{r\left(1 - \frac{2M}{r}\right)^3}{\lambda r + 3M} \left\{ \delta'[r - r_p(t)] - \frac{1}{r^2\left(1 - \frac{2M}{r}\right)^2} \left[(\lambda+1)r - \beta M - \frac{6Mr\widetilde{E}^2}{\lambda r + 3M}\right] \delta[r - r_p(t)] \right\}$$

$$\beta = 1(LP)3(MP)$$

$$t(r_p) = 4M\widetilde{E}\left[\frac{r_o}{4M}\sqrt{\frac{r_p}{2M}}\sqrt{1 - \frac{r_p}{r_0}} - \sqrt{\frac{r_0}{M}}\left(1 + \frac{r_0}{4M}\right)\arcsin\sqrt{\frac{r_p}{r_0}} - \frac{1}{4\widetilde{E}^2}\ln\left(\frac{1 + \frac{r_p}{2M} - 2\frac{r_p}{r_0} - 2\widetilde{E}\sqrt{\frac{r_p}{2M}}\sqrt{1 - \frac{r_p}{r_0}}}{1 + \frac{r_p}{2M} - 2\frac{r_p}{r_0} + 2\widetilde{E}\sqrt{\frac{r_p}{2M}}\sqrt{1 - \frac{r_p}{r_0}}}\right) + \frac{\pi}{2}\sqrt{\frac{r_0}{2M}}\left(1 + \frac{r_0}{4M}\right)\right]$$

The pragmatic approach (geodesic or minimal action)

$$\frac{d^2x^\mu}{ds^2} = -\Gamma^\mu_{\sigma\nu}\frac{dx^\sigma}{ds}\frac{dx^\nu}{ds}$$

$$\frac{d^2 r_e}{dt^2} = \Gamma^t_{rr}\left(\frac{dr_e}{dt}\right)^3 + (2\Gamma^t_{tr} - \Gamma^r_{rr})\left(\frac{dr_e}{dt}\right)^2 + (\Gamma^t_{tt} - 2\Gamma^r_{tr})\left(\frac{dr_e}{dt}\right) - \Gamma^r_{tt}$$

$$\Gamma^\mu_{\sigma\nu} = \frac{1}{2}g^{\mu\rho}\left(g_{\rho\sigma,\nu} + g_{\rho\nu,\sigma} - g_{\sigma\nu,\rho}\right)$$

1 $$r_e = r_p + \Delta r_p$$

2 $$g_{\mu\nu} = \eta_{\mu\nu} + h^{(1)}{}_{\mu\nu} \qquad \eta_{\mu\nu} = \text{Schwarzschild}$$

3 $$g_{\mu\nu}(r_e) = g_{\mu\nu}(r_p) + \Delta r_p \left.\frac{\partial g_{\mu\nu}}{\partial r}\right|_{r_p}$$

❁$_1$ correction

Extension to 6 new terms

$$\Delta\ddot{r}_p = \alpha_1\Delta r_p + \alpha_2\Delta\dot{r}_p + \alpha_3\Delta r_p^2 + \alpha_4\Delta\dot{r}_p^2 + \alpha_5\Delta r_p\Delta\dot{r}_p + \alpha_6 + \alpha_7\Delta r_p + \alpha_8\Delta\dot{r}_p + \alpha_9$$

$$\Delta\ddot{r}_p = \Delta\ddot{r}_p(\eta)_{\alpha_{1,\dots5}} + \Delta\ddot{r}_p(h)_{\alpha_{6-9}}$$

❁1-5 represent the unperturbed field at the perturbed trajectory

❁6 represent the perturbed field at the unperturbed trajectory

❁7-8 represent the perturbed field at the perturbed trajectory

❁9 represent quadratic first order perturbations - optional -)

Numerical simulation → + derivatives → H_0 H_1 → a terms → r_p radiation reaction

Work J.-Y. Vinet for determination ongoing → Poster

B

$$\alpha_0 = -\frac{M}{r}\left(\frac{r-2M}{r^2} - \frac{3M}{r-2M}\dot{r}_p^2\right)$$

$$\alpha_2 = \frac{6M}{r(r-2M)}\dot{r}_p^2$$

$$\alpha_4 = \frac{3M}{r(r-2M)}$$

C

A

$$\alpha_1 = -\frac{2M}{r^2}\left(\frac{3M}{r^2} - \frac{1}{r} + \frac{3(r-M)}{(r-2M)^2}\dot{r}_p^2\right)$$

$$\alpha_3 = \frac{4M^2}{r^3}\left(\frac{1}{r} + \frac{r-4M}{(r-2M)^3}\dot{r}_p^2\right)$$

$$\alpha_5 = -\frac{4M}{r(r-2M)}\left(\frac{M}{r(r-2M)} + \frac{2}{r} + \frac{1}{r-2M}\right)\dot{r}_p$$

$$\alpha_6 = \frac{1}{r-2M}\left[\frac{r^2}{2(r-2M)}\dot{H}_0 - \frac{M}{r-2M}H_1 - rH_1'\right]\dot{r}_p^3 - \frac{3}{2}H_0'\dot{r}_p^2 - 3\left(\frac{1}{2}\dot{H}_0 - \frac{M}{r^2}H_1\right)\dot{r}_p + \frac{r-2M}{r}\left[\frac{2M}{r^2}H_0 - \frac{1}{2}\frac{r-2M}{r}H_0' - \dot{H}_1\right]$$

$$\alpha_7 = -\frac{1}{r-2M}\left[\frac{2Mr}{(r-2M)^2}\dot{H}_0 - \frac{1}{2}\frac{r^2}{r-2M}\dot{H}_0' - \frac{2M}{(r-2M)^2}H_1 - \frac{M}{r-2M}H_1' + rH_1''\right]\dot{r}_p^3 - \frac{3}{2}H_0''\dot{r}_p^2 - \frac{3}{2}\left(\dot{H}_0' + \frac{4M}{r^3}H_1 - \frac{2M}{r^2}H_1'\right)\dot{r}_p$$

$$-\frac{1}{r}\left[\frac{4M(r-3M)}{r^3}H_0 - \frac{4M(r-2M)}{r^2}H_0' - \frac{1}{2}\frac{(r-2M)^2}{r}H_0'' + \frac{2M}{r}\dot{H}_1 + (r-2M)\dot{H}_1'\right]$$

$$\alpha_8 = \frac{3}{r-2M}\left[\frac{1}{2}\frac{r^2}{r-2M}\dot{H}_0 - \frac{M}{r-2M}H_1 - rH_1'\right]\dot{r}_p^2 - 3H_0'\dot{r}_p - \frac{3}{2}\dot{H}_0 + \frac{3M}{r^2}H_1$$

❁$_9$ (h^2)

2004.Class. Quantum. Grav., 21, S563.

Renormalisation

2004.Class. Quantum. Grav., 21, S563.

$$\overline{\Psi} \propto L^{-2.5}$$
$$\overline{\Psi}_{,r} \propto L^{-2.5}$$
$$\overline{\Psi}_{,rr} \propto L^{-0.5}$$
$$\overline{\Psi}_{,rrr} \propto L^{-0.5}$$
$$\overline{\Psi}_{,t} \propto L^{-2.5}$$
$$\overline{\Psi}_{,tr} \propto L^{-0.5}$$
$$\overline{\Psi}_{,trr} \propto L^{-0.5}$$

❁$_{6-9}$ are L dependent terms. Each ❁ is finite, but all together form a divergence (same pattern for L→ oo

$$L = l + \frac{1}{2}$$
$$\lambda = \frac{1}{2}L^2 - \frac{9}{8}$$

$$\alpha_6 = \sum_{l=0}^{\infty} \alpha_6^l \qquad \alpha_6^l = \alpha_6^a L^0 + \alpha_6^b L^{-2} + \alpha_6^c L^{-4} + 0\left(L^{-6}\right)$$

This term needs renormalisation

$$\zeta(s) = \sum_{l=1}^{\infty} l^{-s}$$ Riemann ▮ function

$$\zeta(s,a) = \sum_{l=0}^{\infty} (l+a)^{-s} \qquad a = 0.5$$

Hurwitz ▮ function

$$\zeta(-2,0.5) = 0 \quad \zeta(0,0.5) = 0$$
$$\zeta(2,0.5) = \frac{1}{2}\pi^2 \quad \zeta(4,0.5) = \frac{1}{6}\pi^4$$

$$\alpha_6 = \alpha_6^a \sum_{l=0}^{\infty} (l+0.5)^0 + \alpha_6^b \sum_{l=0}^{\infty} (l+0.5)^{-2} + \alpha_6^c \sum_{l=0}^{\infty} (l+0.5)^{-4} + \left[0(l+0.5)^{-6}\right] =$$
$$\frac{1}{2}\pi^2 a_6^b + \frac{1}{6}\pi^4 a_6^c + \left[0(l+0.5)^{-6}\right]$$

Similar behaviour for ❁$_7$ ❁$_8$ ❁$_9$

Brans-Dicke theory and capture (Poster B. Chauvineau, AS, J.-D. Fournier)

$$R_{\alpha\beta} = \frac{1}{\Phi}\nabla_\alpha \partial_\beta \Phi + \frac{\omega}{\Phi^2}\partial_\alpha \Phi \partial_\beta \Phi \qquad \text{Box}\,\Phi = 0 \qquad \partial\Phi = 0 \Rightarrow \text{GR}$$

BD non equal GR when regime is out of stationarity

Unconsidered effect so far (apart from monopole, dipole, less than c speed features)

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \qquad \Phi = 1 + \phi$$

$$\partial_\alpha\left(\sqrt{-\eta}\,\eta^{\alpha\beta}\partial_\beta \phi\right) = 0$$

$$R_{\alpha\beta}(\eta_{\alpha\beta}) + R_{\alpha\beta}(h_{\alpha\beta})$$

Schwarzschild

$$[A \text{ term}] = \left[\frac{\partial\phi}{r_{bh}}\right] \qquad [B \text{ term}] = \left[\omega\partial\phi^2\right] \qquad R_{\alpha\beta}(h_{\alpha\beta}) =$$

(B)

A(+B)

A **t**

$$\partial\phi > \frac{1}{\omega r_{bh}} \Rightarrow \text{B term (optional)}$$

$$\partial\phi < \frac{1}{\omega r_{bh}} \Rightarrow \text{A term}$$

$$\phi \propto e^{-\lambda t} R(r) Y(\theta,\varphi)$$

$$R \propto c_1\left(\frac{r}{r_g} - 1\right)^{\lambda r_g} + c_2\left(\frac{r}{r_g} - 1\right)^{-\lambda r_g} \quad r \to r_g \qquad R \propto \frac{e^{-\lambda r}}{r} \quad r \to \infty$$

$$T_{\alpha\beta} \propto e^{-2\lambda t} \quad \text{or} \quad e^{-\lambda t}$$

(initial) final

Slope change = (M)

Conclusions / Status

Two methods for radiation reaction:
Self-force: Partial confirmation of mode-sum renormalisation results of other groups (A,B,C instantaneous values)

Pragmatic: richer geodesic (addition of 6 terms)
Riemann-Hurwitz renormalisation extended (3 terms)

Connection self-force / pragmatic under investigation

Non radiative modes (L=0,1) analysis under going

Capture is a rich field for research
(Fundamental physics, Relativistic Astrophysics, Data Analysis)

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