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2D Particle-in-Cell Simulations of the Pulsar Polar-Cap Discharge

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Introduction

Induced electric fields and $e^{\pm}$ discharge above polar caps are usually invoked in models of observed pulsar emission, from radio waves to X-rays. The discharge is mainly controlled by the dimensionless parameter $\alpha = \mathbf{j}_B/c\rho_{\rm GJ}$, where $\rho_{\rm GJ}$ is the co-rotational (Goldreich-Julian) charge density and $\mathbf{j}_B = (4\pi/c)\nabla\times\mathbf{B}$ is the electric current that is required to sustain the toroidal magnetic field induced from the light cylinder. The existing detailed discharge model (e.g. Arons & Scharleman 1979) pictures a steady state with a special value of α close to unity. In fact, the discharge is expected to occur in a poorly understood unsteady regime with α that may be far from unity (e.g. Levinson et al. 2005; Timokhin 2006; Beloborodov 2008). We have developed a code that can help understand how the discharge operates.

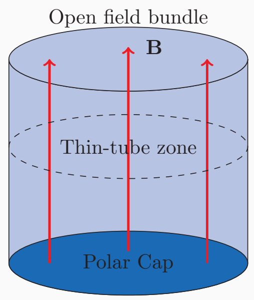
Problem Formulation

The plasma is modeled from first principles, as a collection of many ($N \sim 10^7$) individual particles that move in the self-consistent electro-magnetic field. We use the positions and velocities of the particles to calculate the charge and current densities, which are used to solve the time-dependent Maxwell equations in the rotating frame (e.g. Levinson et al. 2005):

$$\partial_t\mathbf{E} = c\nabla\times\mathbf{B} - 4\pi(\mathbf{j}-\mathbf{j}_R), \quad \partial_t\mathbf{B} = -c\nabla\times\mathbf{E}$$

1D Charge-Separated Flow: The system can be approximated as 1D close to the stellar surface. In the 1D approximation fields depend only on the vertical coordinate z. One can show that in this approximation $\mathbf{B}$ and j_B are constant in time, the electric field is potential and completely determined by Gauss's Law.

2D Axisymmetric Cylinder: We then extended the code to handle 2D axisymmetric cylindrical geometry. We simulate the cylinder above the polar cap, up to altitude several times the polar cap radius.



The boundary of the cylinder separates the open and closed field lines and therefore we assume the following boundary conditions: A perfect conductor is assumed outside the cylinder, and the electric potential is set to zero at the bottom and vertical boundaries. Charges are lifted from the lower boundary according to the local value of j_B. Free escape is assumed at the upper boundary. We model pair creation by allowing the particles to emit photons at high energies (through inverse-Compton scattering or curvature radiation) and following the photons until they convert to $e^{\pm}$ pairs or escape. The code runs on GPUs.

References

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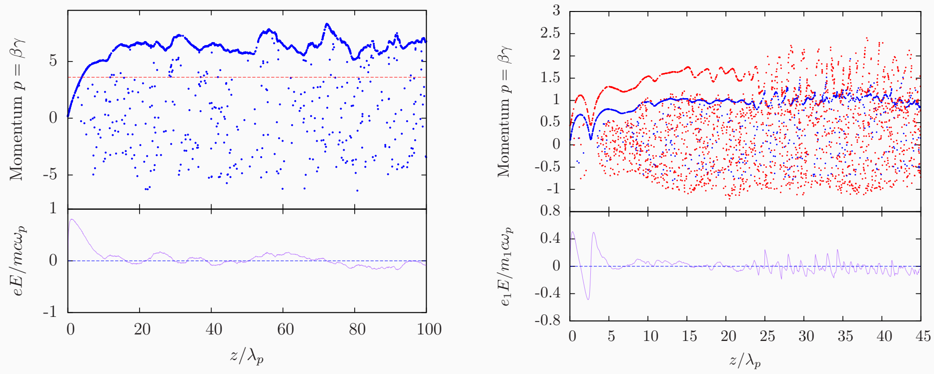
1D Charge-Separated Flow

The simplest numerical model of the polar cap accelerator was calculated for 1D charge-separated flows (Chen & Beloborodov 2013; Timokhin & Arons 2013). When $0 < \alpha < 1$ the accelerated particles can achieve the maximum Lorentz factor of $\gamma \sim (1+\alpha^2)/(1-\alpha^2)$, which is modest, so no pair creation is initiated. The plasma flow relaxes to the quasi-steady state shown below.



Left: Phase-space snapshot of the quasi-steady state for 1D charge-separated flow with $\alpha = 0.8$. Red dashed line shows the maximum momentum predicted by the analytical steady-state model (Beloborodov 2008). Distance is measured in units of plasma skin depth $\lambda_p = c/\omega_p$ where $\omega_p = \sqrt{4\pi\rho_{\rm GJ}e/m_e}$ is the plasma frequency corresponding to $\rho_{\rm GJ}$. Right: Phase-space snapshot of the quasi-steady state for 1D flow with two ion species, H^+(red) and He^{2+}(blue), $\alpha = 0.4$.

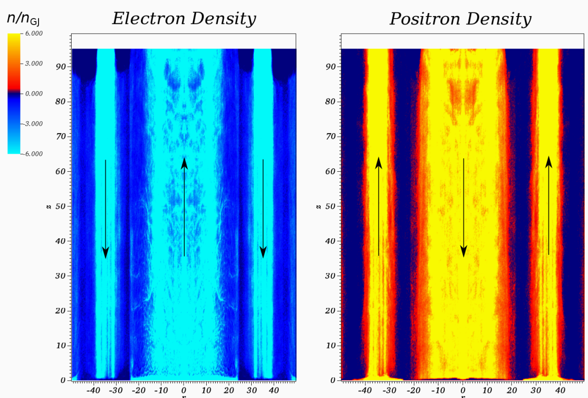
- The flow is separated into two components. A fraction ($\approx \alpha$) of the particles flow in a cold stream with v close to c, while the rest of the particles has a broad momentum distribution with a small mean momentum. The two components are distributed so that the flow can maintain $\rho = \rho_{\rm GJ}$ and $j = j_B$, as required for a quasi-steady state.
- We also studied the possibility of having two ion species in the flow. The result resembles the single species case, except that a strong two-stream instability develops and completely destroys the stream of lighter ions.
- In all models with $0 < \alpha < 1$ there is no significant particle acceleration. The polar cap is "dead" — unable to produce electron-positron pairs.
- There is significant particle accleration when $\alpha > 1$ or $\alpha < 0$.

Axisymmetric Polar Cap with $e^{\pm}$ Discharge

The full axisymmetric polar cap problem is qualitatively different from the 1D charge-separated flow in the following aspects

- Above an axisymmetric polar cap the potential can only grow up to an altitude H comparable to the polar cap size. The region above $z \sim r_{\rm pc}$ can be called the "thin-tube" zone (see figure in Problem Formulation). If one imagines a global steady state with $j = j_B$ then the potential in this region is constant $\Phi = S_\perp(\rho - \rho_{\rm GJ})$ where $S_\perp$ is the cross sectional area of the open tube, therefore $\mathbf{E}_\parallel \sim 0$ and particle acceleration stops in this zone. This potential gives an upper bound on the Lorentz factor achievable by the particles through polar cap acceleration, even when $\alpha > 1$ or $\alpha < 0$.
- In the axisymmetric problem the magnetic field becomes an additional degree of freedom. It allows the generation and propagation of Alfvén waves as a result of the voltage fluctuations, which is accompanied by twisting of the magnetc field lines (Beloborodov 2008). In addition to regulating the local discharge behavior, the Alfvén waves may give insight on how radio emission is generated from pulsars.

We set up the 2D axisymmetric simulation with $e^{\pm}$ pair creation. The current $\mathbf{j}_B$ changes sign over the polar cap so that the net current is zero. A quasi-steady state is established after several light-crossing times. In this state, $\rho \approx \rho_{\rm GJ}$ and $\mathbf{j} \approx \mathbf{j}_B$ are maintained on average, although both significantly fluctuate in time. We found that when $0 < \alpha < 1$ the 2D results are in agreement with the 1D simulations shown in the figures above. As expected, in regions where $\alpha > 1$ or $\alpha < 0$ significant particle acceleration occurs and $e^{\pm}$ discharge is initiated. Below is a snapshot of the density distribution for electrons and positrons from one of such simulations. Arrows indicate the direction of the particle flow corresponding to $\mathbf{j}_B$.



In this simulation $\rho_{\rm GJ}$ has the same sign as the electron charge. In the central column $\alpha > 1$ is assumed; as a result, pairs are created near the stellar surface. In the outer column $\alpha < 0$ and the gap develops closer to the outer boundary of the computational box. In the column where $\alpha < 0$ current is mainly conducted by electrons flowing back to the polar cap. The multiplicity of $e^{\pm}$ pairs created in the simulation is ~ 20.

We also observed large amplitude Alfvén waves carrying a fraction of the plasma kinetic energy. Possible implications of Alfvén waves for the radio emission mechanism will be a topic for further study.