General formula for the running of f_{NL}

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CB & Gong; 1210.1851
CB, Kari Enqvist, Nurmi & Tomo Takahashi; 1108.2708
CB, Enqvist, Takahashi; 1007.5148
CB, Mischa Gerstenlauer, Nurmi, Tasinato & Wands; 1007.4277
CB, Nurmi, Tasinato & Wands; 0911.2780

ESTEC Planck meeting, Noordwijk, 4th April 2013, 12 minutes

With large data sets, its neither practical nor desirable to search for every possible signal

To avoid endless discussions about posterior detections and anomalies

Theorists are needed to motivate template searches

We need as many observables as possible, to break the degeneracy between the many models

We all know theorists need observers, but observers also need theorists

SDSS

The non-linearity of the perturbations is expected to depend on the scale

- In the simplest local model, f_{NL} looks constant $\zeta = \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2$
- But analogous to the power spectrum, f_{NL} should have a scale dependence

Equilateral non-Gaussianity: Chen '05

Local non-Gaussianity: CB, Choi & Hall '08; CB, Nurmi, Tasinato & Wands '09

- The power spectrum is not scale invariant at 5-sigma significance: WMAP9 & Planck
- Reflects evolution/dynamics during inflation (e.g. it ends)
- Breaks degeneracy between early universe models
  - As well as the trispectrum
  - f_{NL} constraints mean g_{NL} likely to be unobservable (because it generates a “local” f_{NL})

Nelson & Shandera '12; Nurmi, CB & Tasinato '13

- Observers have shown great interest, many forecasts and recently the first constraints have been made   Becker & Huterer '12

Why is this useful?

- Can distinguish between different non-Gaussian scenarios, not just between Gaussian and non-Gaussian models
- The amplitude of f_{NL} can be tuned in most non-Gaussian models, so a precise measurement of f_{NL} wont do this
- In contrast the scale dependence often can not be tuned independently of:
  - f_{NL}
  - spectral index of the power spectrum
- Shows the power of consistency relations between observables, test or rule our whole classes of models
- Despite years of study, there may still be important observable signatures to be calculated

Definition of scale dependent f_{NL}

For the equilateral triangle (one k)

$$n_{f_{NL}} \equiv \frac{\partial \log |f_{NL}|}{\partial \log k}$$

- In general f_{NL} trivariate function, so definition needs care
- However $n_{f_{NL}}$ is independent of the shape provided one scales the triangle preserving the shape
  - Hence the above definition is a useful definition of a new observable
  - Not much change if the shape and size of triangle are changed together, unless one goes deep into the interesting squeezed limit

Byrnes, Nurmi, Tasinato and Wands, '09

Sources of scale dependent f_{NL}

- Scale dependence arises in three ways:

 1) Non-trivial field space metric

 2) Multiple fields contributing to the curvature perturbation

 3) Self-interactions of a light field during inflation

CB, Gerstenlauer, Nurmi, Tasinato & Wands '10

 – From 3), valid for "single-source" scenarios, includes many of the models studied to date, may be observable

$$n_{f_{NL}} \sim \frac{\sqrt{r_T}}{f_{NL}} \frac{V'''}{3H^2} \qquad r_T = \frac{P_T}{P_\zeta}$$

- Only known way to probe third derivative of a "modulaton" field
- E.g. self-interacting curvaton scenario, even the non-linearity from a weak self-interaction creates a large effect

CB, Enqvist & Takahashi '10; CB, Enqvist, Nurmi & Takahashi '11

Multivariate extension of local f_{NL}

- The multivariate local model

$$\zeta(x) = \zeta_{G,\phi}(x) + \zeta_{G,\chi}(x) + f_\chi \zeta^2_{G,\chi}(x) + g_\chi \zeta^3_{G,\chi}(x)$$

phi is the Gaussian inflaton field,
chi generates non-Gaussianity (uncorrelated to phi)

applies to mixed inflaton and curvaton/modulated reheating scenarios, provided f_χ is a constant

- Bispectrum has the usual local shape – not changed

$$P_\zeta(k) = P_{\zeta_\phi}(k) + P_{\zeta_\chi}(k), \quad P_\zeta \propto k^{n-4}, \; P_{\zeta_\chi} \propto k^{n_\chi - 4}$$

$$f_{NL}(k) = \frac{5}{3}\frac{B_{\zeta_\chi}}{3P_\zeta^2} = \frac{5}{3}\frac{P_{\zeta_\chi}(k)^2}{P_\zeta(k)^2} f_\chi \propto \left(\frac{k}{k_p}\right)^{2(n_\chi - n)}$$

- So a scale dependence of f_{NL} is simple and natural
- Trispectrum $$n_{\tau_{NL}} = n_{g_{NL}} = \frac{3}{2} n_{f_{NL}} = 3(n_\chi - n)$$

Mixed inflaton-curvaton scenario

- The inflaton phi has Gaussian perturbations,
  the curvaton field sigma (quadratic potential) is non-Gaussian
  phi and sigma have different spectral indices
  assume a small field model of inflation $\epsilon \ll |\eta_{\phi\phi}|$

New consistency relation

$$n_{f_{NL}} = -2(n_s - 1) \simeq 0.1$$

- Gives an idea of what sort of scale dependence we might expect from a simple model
- Is it big enough to be observable? Depends on the amplitude of the non-Gaussianity

A new source: curved field space

- Fundamental theories, such as SUSY, “often” predict not only multiple fields, but also a non-trivial field space metric (warped/curved field space manifold)

$$\mathcal{L} = -\frac{1}{2}\gamma_{ab} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V$$

Elliston, Seery & Tavakol '12

- Field metric reduces to the trivial case if $\gamma_{ab} = \delta_{ab}$

- Given a model, may also learn about the geometry of the field space

$$n_{f_{\rm NL}} \equiv \frac{1}{f_{\rm NL}} \frac{D f_{\rm NL}}{d \log k} = -\frac{N_a N_b N_c w^{abc}}{N_{de} N^d N^e} + 4 w^{ab} \left(\frac{N_a N_b}{N_d N^d} - \frac{N_{ac} N_b N^c}{N_{de} N^d N^e} \right)$$

$$w_{ab} = u_{(a;b)} + \frac{R_{c(ab)d}}{3} \frac{\dot\phi_0^c}{H} \frac{\dot\phi_0^d}{H}, \quad u_a = -\frac{V_{;a}}{3H^2}$$

$$w_{a(bc)} = u_{(a;bc)} + \frac{1}{3} \left[R_{(a|de|b;c)} \frac{\dot\phi_0^d}{H} \frac{\dot\phi_0^e}{H} - 4 R_{a(bc)d} \frac{\dot\phi_0^d}{H} \right]$$

Most general formula to date: CB & Gong '12

Prospect for observations

- Used to look quite good, Planck was forecast to reach $\sigma_{n_{f_{NL}}} = 0.1$ for a fiducial f_{NL}=50

CMB: Sefusatti, Ligouri, Yadav, Jackson, Pajer; '09

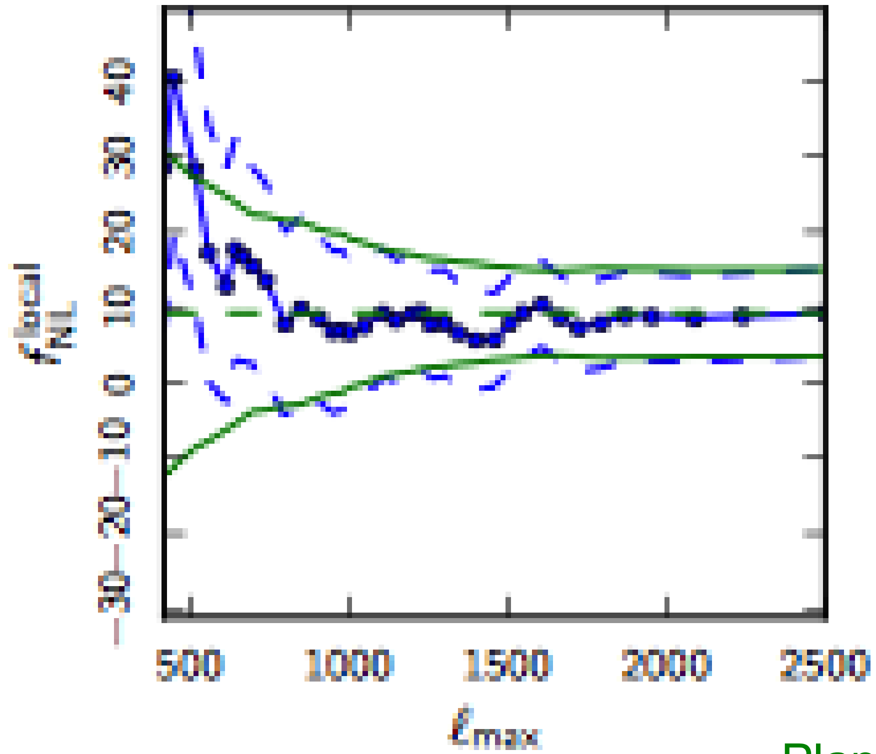
- The new constraints limit the discovery potential
- Models (e.g. self interacting curvaton) could produce parameters which would allow non-Gaussianity and its scale dependence to be observable with Planck

CB, Enqvist, Nurmi, Takahashi '11; Kobayashi & Takahashi '12

- The constraints are inversely proportional to the fiducial value of f_{NL}
- However, that the Universe is >99.95% Gaussian is an amazing result, and powerful at constraining alternative models to the simplest inflation

Any tiny hint from Planck?

Probably not, but a scale dependent fit hasn't been made yet



Planck XXIV: Non-Gaussianity

f_{NL} as a function of l_{max}, consistent with WMAP bias towards positive values

Focus will move to LSS

- Euclid is forecast to make a significantly tighter f_{NL} constraints than Planck (from scale dependent bias, hence only for the local model)
- However it has a shorter lever arm, the forecast is $\sigma_{n_{f_{NL}}} = 0.1$ for f_{NL}=30 (c.f. 50 with Planck), a bit stronger

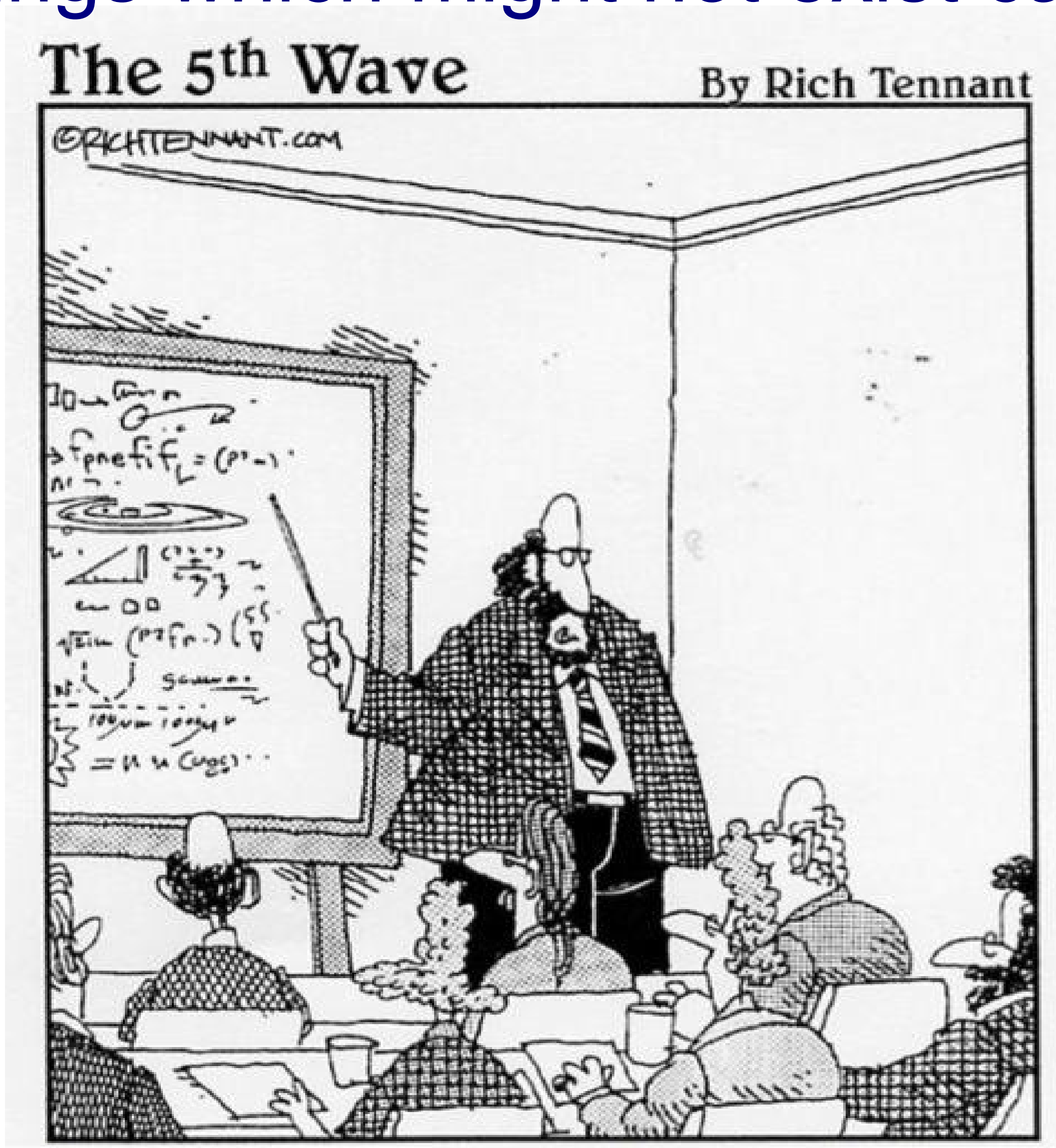
Euclid forecasts: Giannantonio et al '11
First LSS simulations: Shandera, Dalal & Huterer '10

- Here really probing the squeezed limit of the bispectrum, some modification to the formalism is required

Dias, Ribeiro & Seery '13

- Future: cosmic mu distortions (17 efold lever arm from CMB), 21cm, how far can we go?

Even things which might not exist can matter



"After the discovery of 'antimatter' and 'dark matter', we have just confirmed the existence of 'doesn't matter', which does not have any influence on the Universe whatsoever."

Conclusions

- Non-Gaussianity is naturally scale dependent, similarly to the power spectrum
- However, its scale dependence does not have to be slow-roll suppressed, even in slow-roll models
- A detection would allow us to discriminate between non-Gaussian scenarios, in a way which measuring the amplitude to any accuracy can not

- How much further can large scale structure surveys or other probes push this field?

Interacting curvaton scenario: Intro

$$V(\sigma) = \frac{1}{2}m^2\sigma^2 + \lambda\sigma^n$$

Strength of self interaction (at horizon exit, *)

$$s = 2\lambda \frac{\sigma_*^{n-2}}{m^2}$$

In the limit of s=0 recover scale invariance - because the quadratic curvaton perturbation has a linear equation of motion

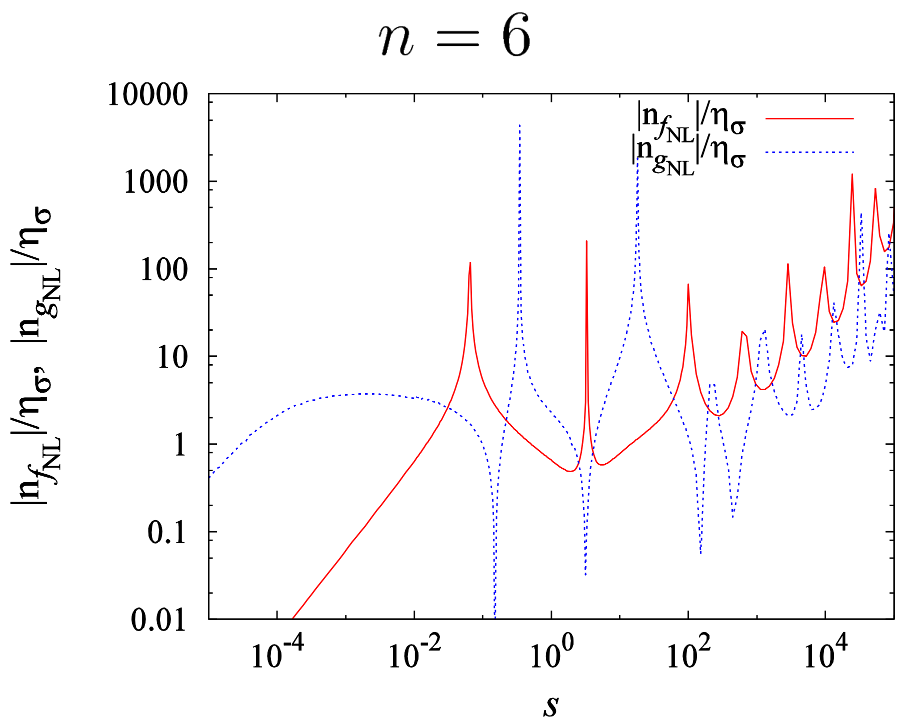
Energy density of the curvaton is subdominant during inflation, but it grows relative to that of radiation (from the decayed inflaton) while it oscillates about the (quadratic) minimum of its potential

Energy density of curvaton at time of decay

$$r_{dec} \equiv \frac{3}{4}\frac{\rho_\sigma}{3H^2}\Big|_{decay} \qquad f_{NL} \sim \frac{1}{r_{dec}}$$

CB, Enqvist, Nurmi, Takahashi; '11

Scale dependence can be very large



Small s regime:
CB, Enqvist, Takahashi '10

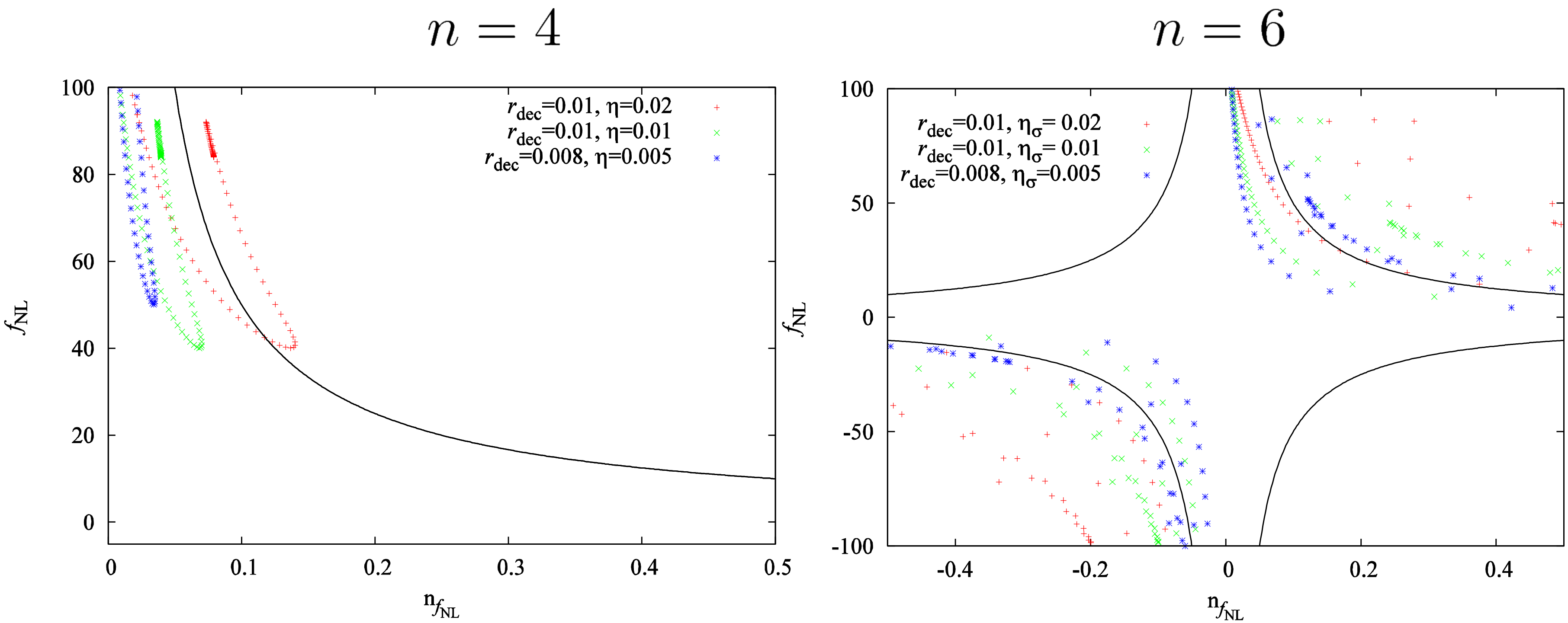
Any s (self-interaction) regime: CB, Enqvist, Nurmi, Takahashi '11

Axionic curvaton potential: Huang '10

See: Riotto & Sloth '10 for a step-function like f_{NL}

Typically the scale dependence grows with the interaction strength, but there are large spikes even for s<1. However spikes tend to correspond to small values of the non-linearity parameters.
No scale dependence for s=0

f_{NL} and its scale dependence

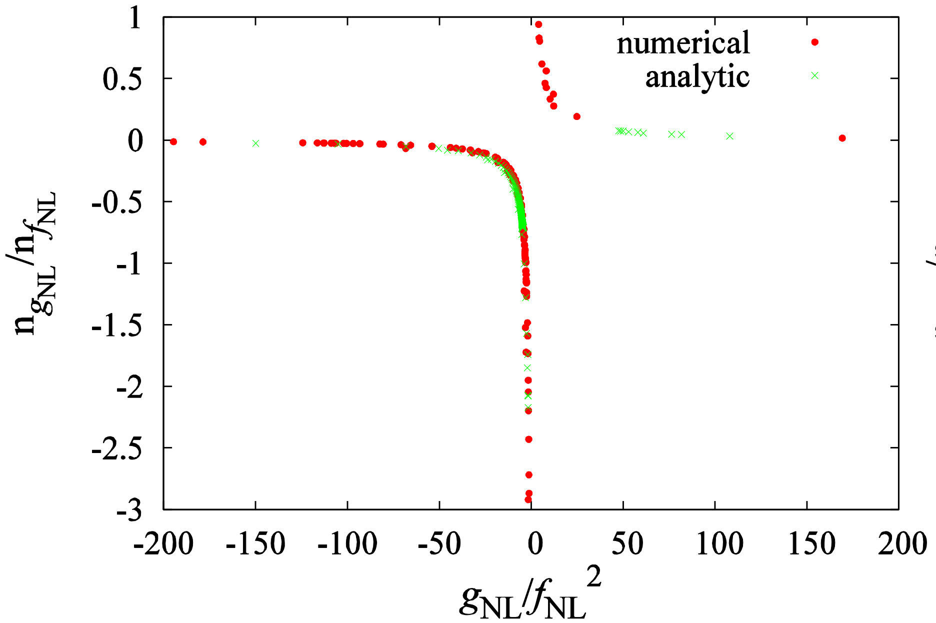


$$s \sim \frac{\text{self interaction}}{\text{quadratic}} \qquad r_{dec} \sim \Omega_{\sigma,decay}$$

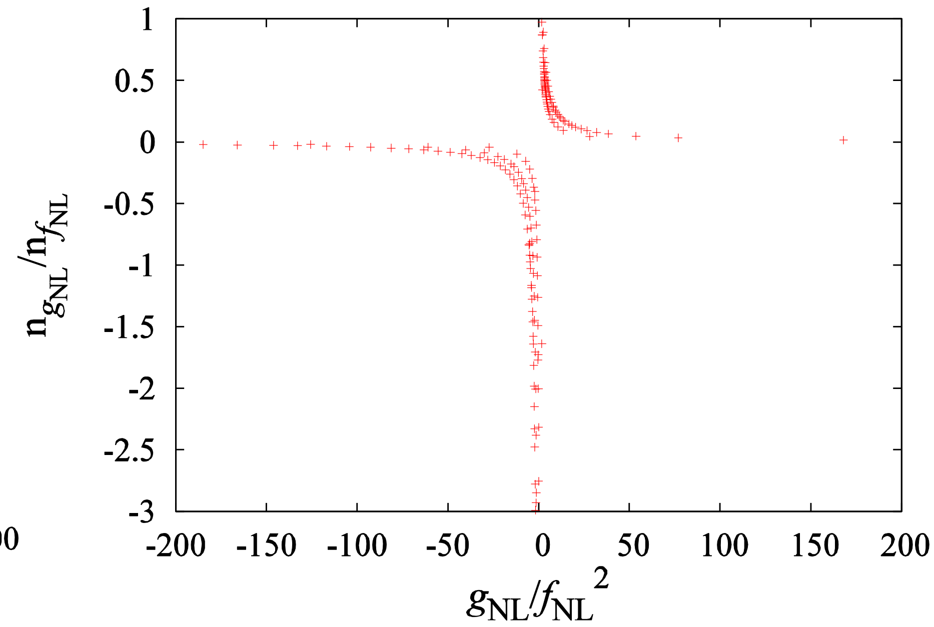
The black lines show $|f_{NL} n_{f_{NL}}| = 0.1$, the regions outside of these lines are "detectable" with Planck at 1-sigma and CMBPol/CORE at 2-sigma. Better chance of a detection if n>4.

This complex model can be ruled out

$n = 4$



$n = 6$



In spite of the many free parameters (compared to the quadratic model), observation of f_{NL} and g_{NL} with scale dependence can rule out the model, most regions of the plots cannot be realised for any parameter values

Large self-interaction limit

- Consider s>>1, i.e. potential is dominated by the self-interaction term during inflation
- It will eventually oscillate in a quadratic minimum before decaying
- n=4: $n_{f_{NL}} = 4\eta_\sigma, \; n_{g_{NL}} = 20\eta_\sigma$
- n=6: non-linearity parameters are small
- n=8: $n_{f_{NL}} = -12\eta_\sigma, \; n_{g_{NL}} = 28\eta_\sigma$
- So scale dependence is an order of magnitude larger than the spectral index $n_s - 1 = -2\epsilon + 2\eta_\sigma$

  which makes this topic very interesting
  - We could probe the self interactions of a field which is always subdominant