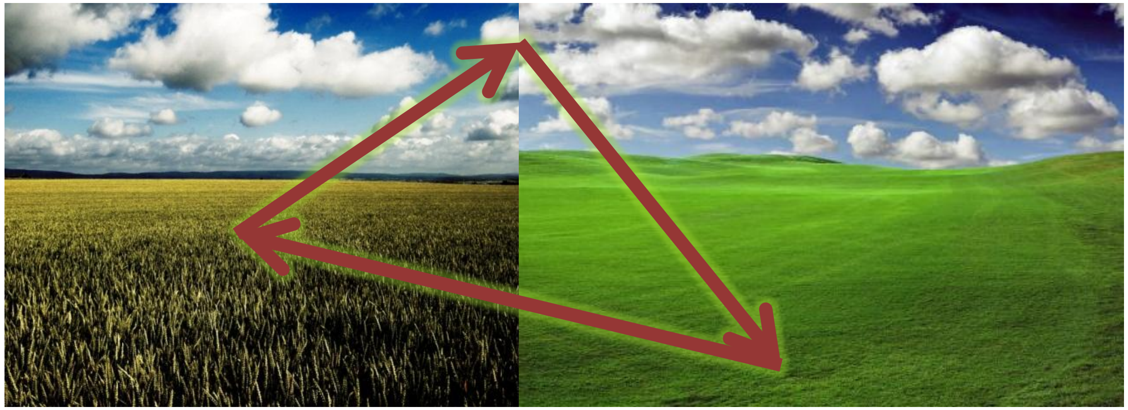


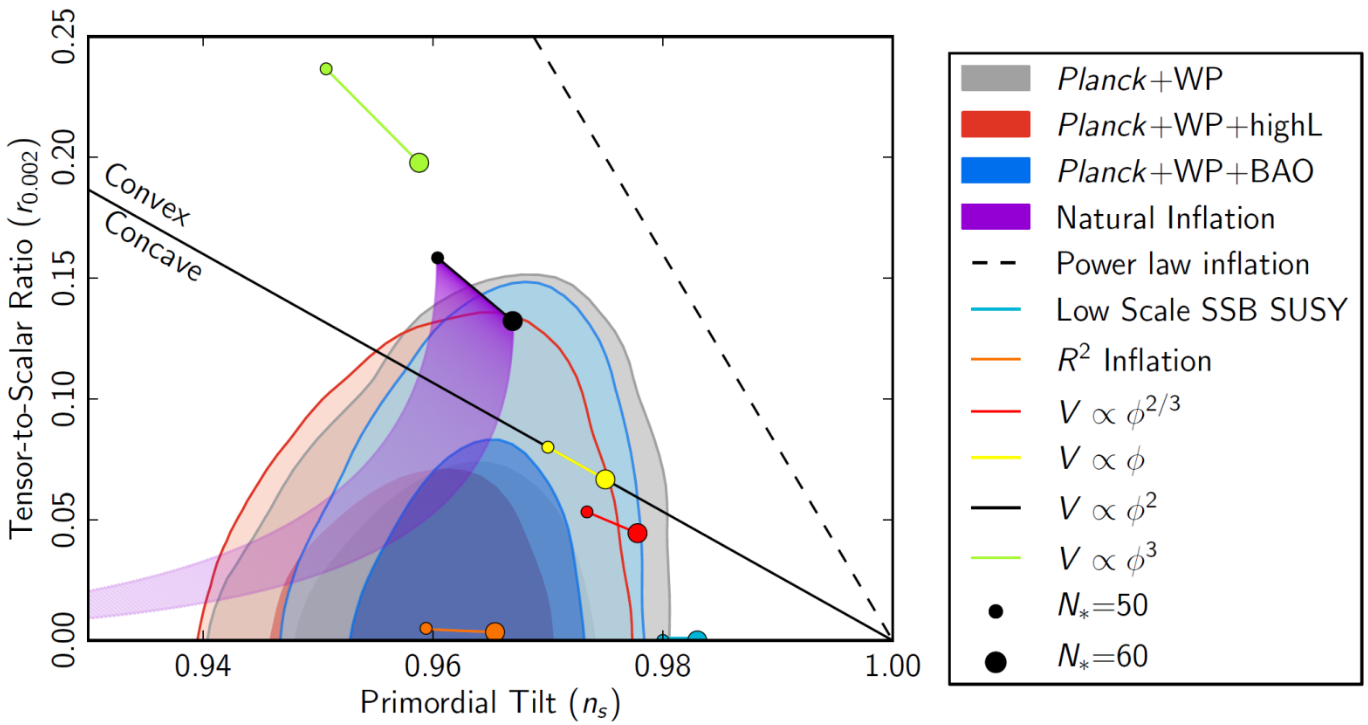
Non-Gaussianity and the Adiabatic Limit

Joel Meyers
Canadian Institute for Theoretical Astrophysics

The Universe as Seen by Planck
ESA/ESTEC
April 4, 2013

Based on arXiv:1011.4934, 1104.5238 with Navin Sivanandam
And 13xx.xxxx with Ewan Tarrant

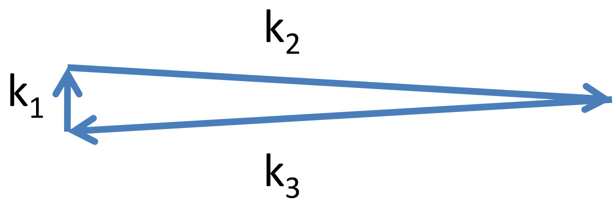
Power Spectrum and Inflation

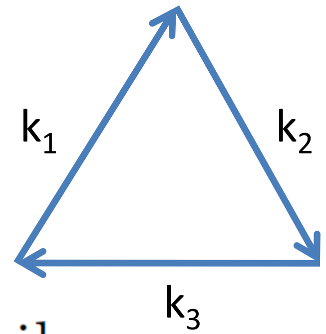


Planck (2013)

- Observations already rule out some models of inflation, but many models remain viable

Non-Gaussianity





Planck (2013)

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8 \qquad f_{\rm NL}^{\rm equil} = -42 \pm 75$$

$$f_{\rm NL}^{\rm orhto} = -25 \pm 39$$

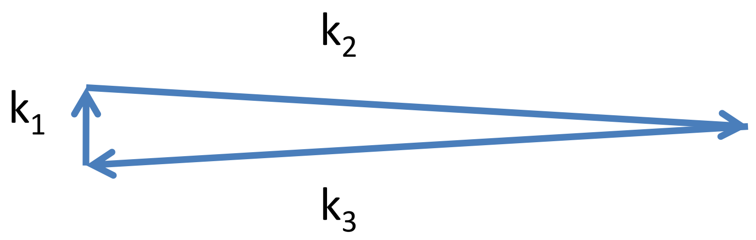
- Planck has provided us with excellent constraints on several forms of non-Gaussianity

Single Field Consistency Relation

- $f_{\rm NL}^{\rm local}$ is always small in single field inflation

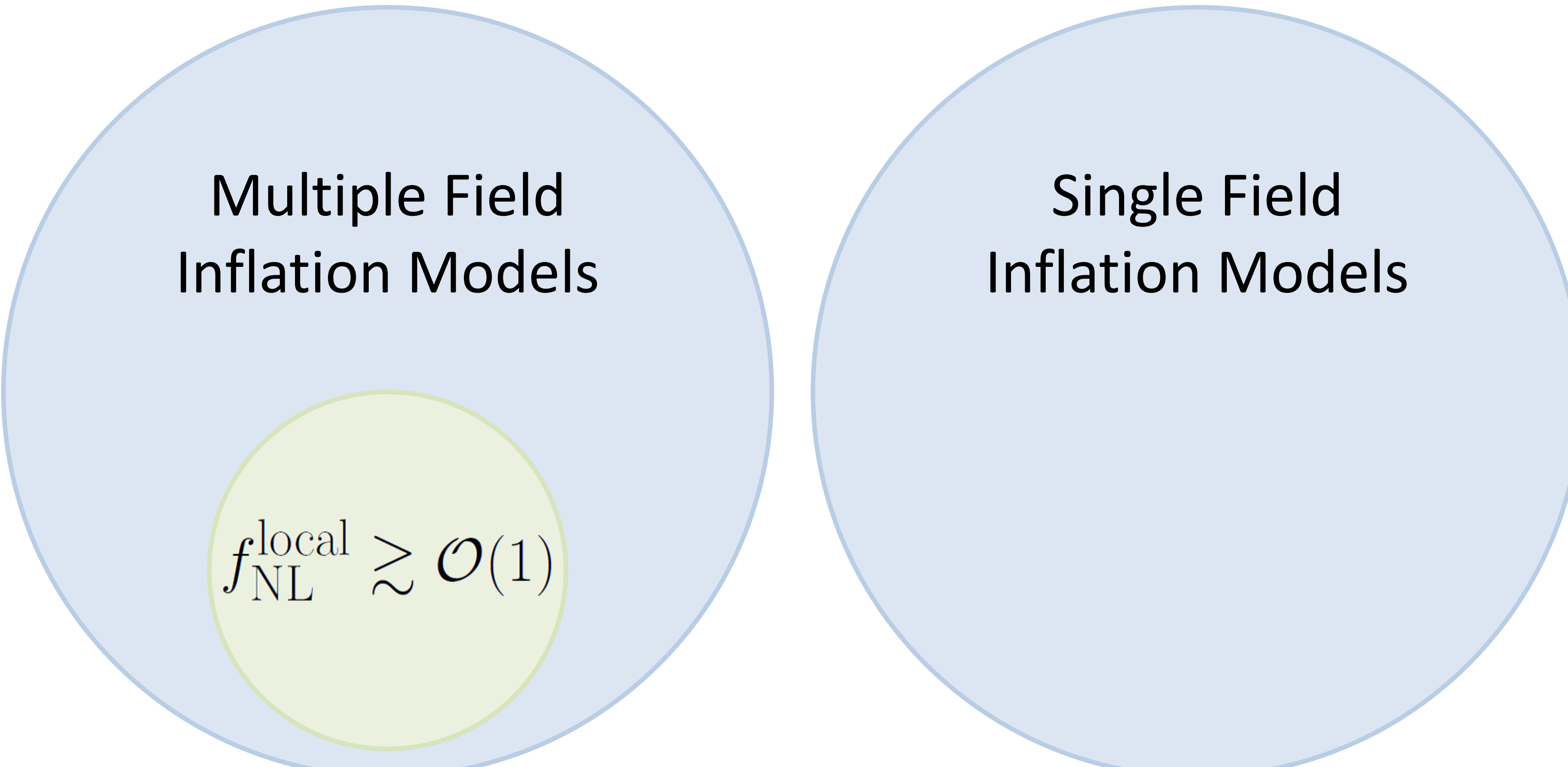
$$f_{\rm NL}^{\rm local} = \frac{5}{12}(1 - n_s)$$

Maldacena (2002)
Creminelli, Zaldarriaga (2004)
Ganc, Komatsu (2010)

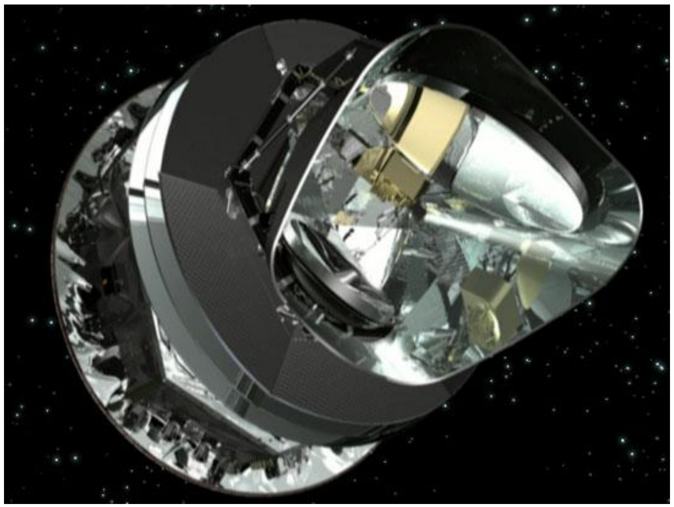


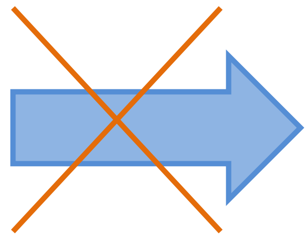
- A convincing detection of $f_{\rm NL}^{\rm local}$ would rule out *all* models of single field inflation

Inflation and Non-Gaussianity



Lessons From Data



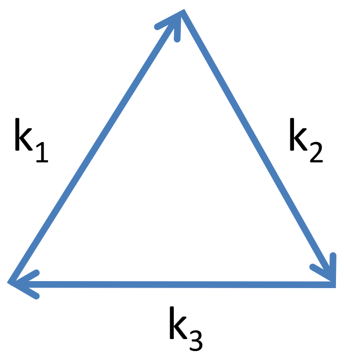


$$V(\phi^I)$$

- It is unlikely that we will *ever* be able to pin down exactly which model of inflation was responsible for the universe we see
- Observations can, however, teach us something about the physical principles which governed inflation

Equilateral Non-Gaussianity

- The equilateral bispectrum measures the speed of sound during inflation



$$f_{\rm NL}^{\rm equil} \sim \frac{1}{c_s^2}$$

- A small sound speed indicates new physics near the inflationary scale

Baumann, Green (2011)

The Other Consistency Relation

- All models of single field inflation predict a relation between the tensor-to-scalar ratio and the tensor spectral index

$$r = -8 c_{\mathrm{S}} n_{\mathrm{T}}$$

Gruzinov (2004)

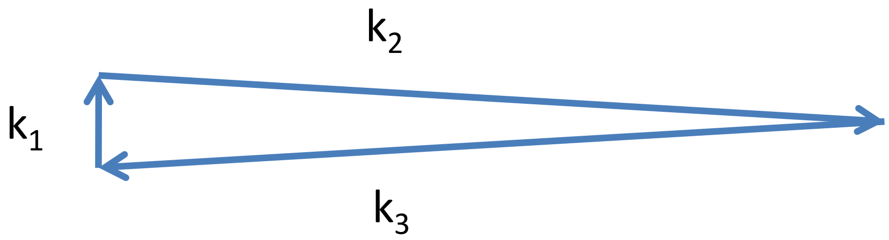
- Constraints on non-Gaussianity and future polarization data could provide evidence which rules out single field inflation

$$c_{\mathrm{S}} \geq 0.02$$

Planck (2013)

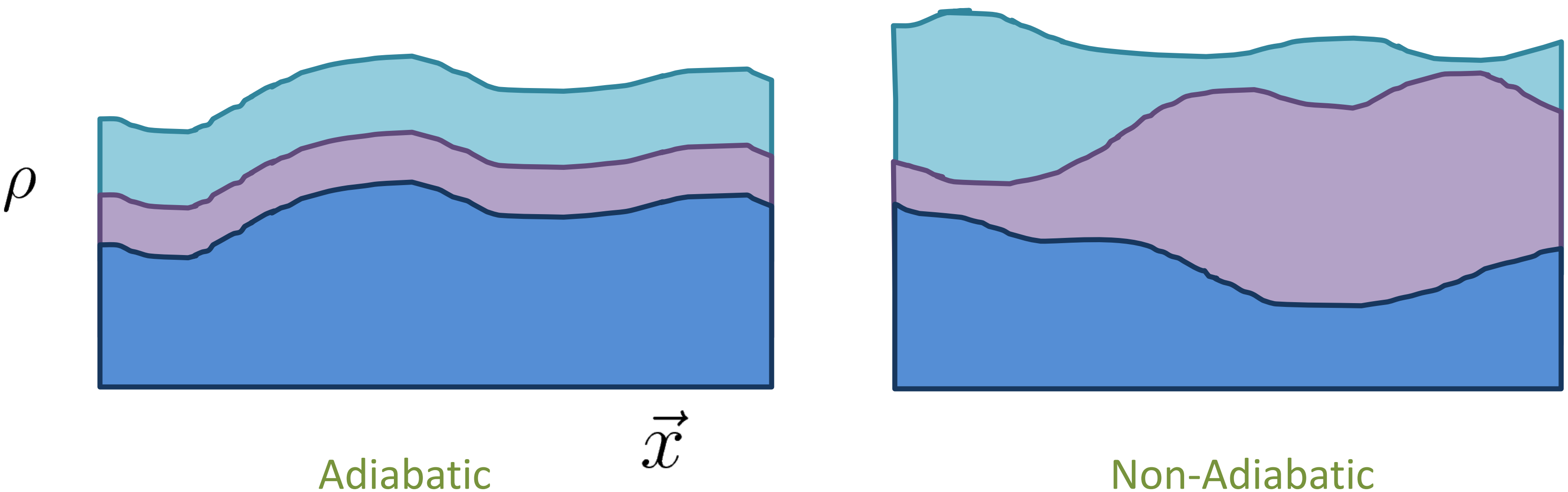
Local Non-Gaussianity

- The local bispectrum can only be produced from multiple field inflation



- What else can we learn from observational constraints on local non-Gaussianity?

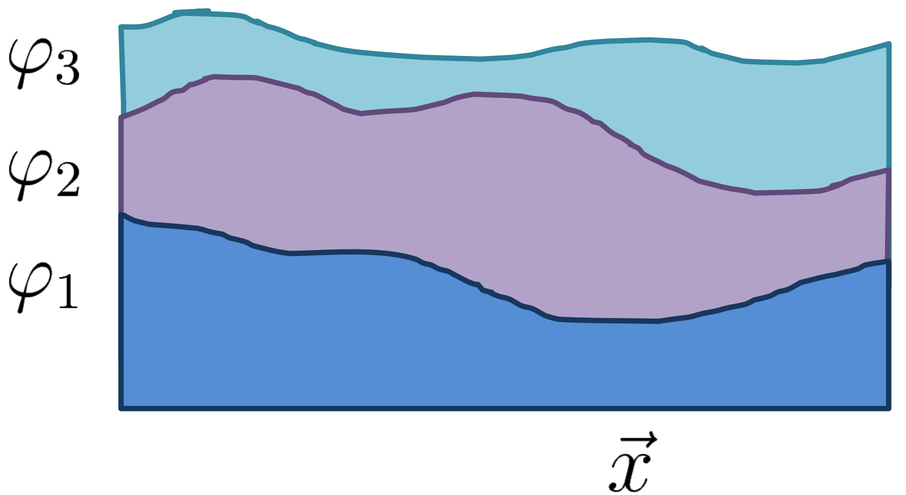
Adiabaticity



- The curvature perturbation is conserved outside the horizon when the fluctuations are adiabatic
- Single field inflation always produces purely adiabatic fluctuations

Weinberg (2003), (2004a), (2008)

Adiabaticity

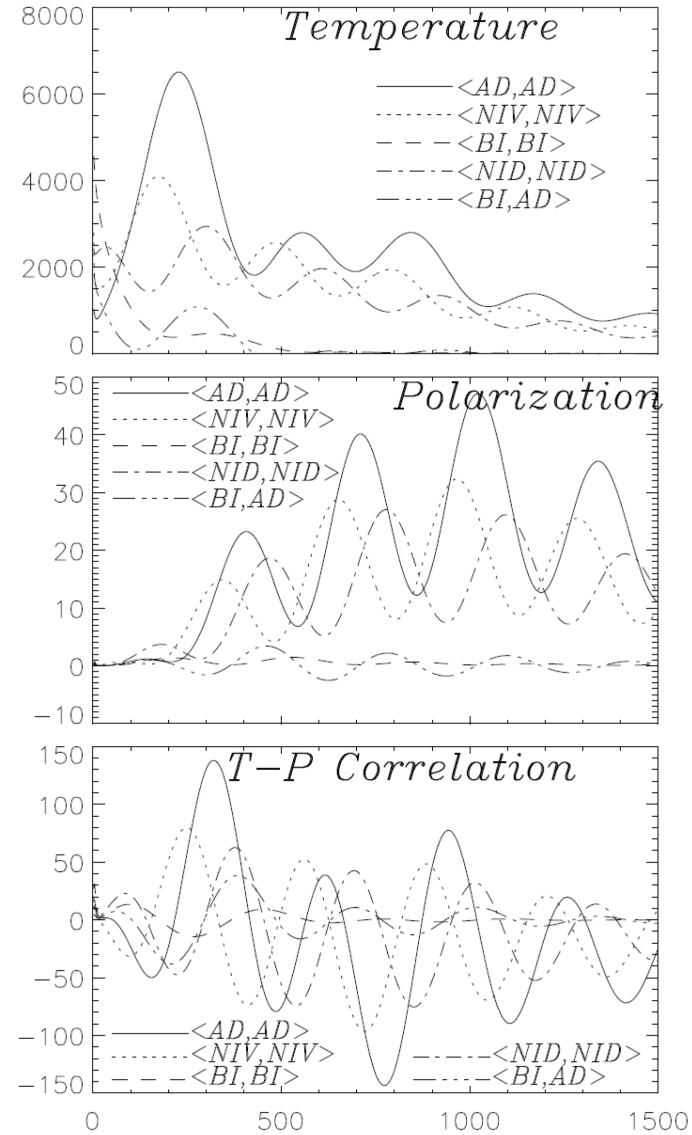


- The curvature perturbation can evolve on superhorizon scales in the presence of non-adiabatic fluctuations
- Multiple field inflation naturally produces non-adiabatic fluctuations

Adiabaticity

- Non-adiabatic fluctuations which persist through the radiation-dominated era produce observable effects
- Observations show no evidence for non-adiabatic fluctuations
  - Uncorrelated: $\alpha_{\mathcal{RR}} = [0.98 : 1]$
  - Correlated: $\alpha_{\mathcal{RR}} = [0.97 : 1]$
  - Anti-correlated: $\alpha_{\mathcal{RR}} = [1 : 1.06]$

Bucher, Moodley, Turok (2001); Planck (2013)



Approach to Adiabaticity

- Non-adiabatic fluctuations may become adiabatic in at least two ways:

Effectively single field inflation

Local thermal equilibrium

Weinberg (2004b), (2008a), (2008b); JM (2012)

Approach to Adiabaticity

Any model with multiple dynamical fields is *incomplete* without an understanding of the evolution of the cosmological perturbations until they become adiabatic, or until they are observed.

Adiabaticity and Non-Gaussianity

- Are there general features of multiple field inflation models which predict observable local non-Gaussianity and a purely adiabatic power spectrum?

- We will focus on two field inflation models which pass through a short phase of effectively single field inflation before reheating

δN Formalism

- We use the δN formalism to calculate the evolution of observables outside the horizon

$$N = \int_*^c H\, dt \qquad\qquad N_{,I} \equiv \frac{\partial N}{\partial \phi_*^I}$$

$$\zeta = \delta N \simeq \sum_I N_{,I} \delta\phi_*^I + \sum_{IJ} N_{,IJ} \delta\phi_*^I \delta\phi_*^J$$

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,I} N_{,J} N_{,IJ}}{\left(\sum_K N_{,K}^2\right)^2}$$

Sasaki, Stewart (1996); Lyth, Rodriguez (2005)

Results

- For a separable potential: $W(\phi,\chi) = U(\phi) + V(\chi)$

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\frac{x^2}{\epsilon_*^\phi}\left(2 - \frac{x\eta_*^\phi}{\epsilon_*^\phi}\right) + \frac{y^2}{\epsilon_*^\chi}\left(2 - \frac{y\eta_*^\chi}{\epsilon_*^\chi}\right)}{\left(\frac{x^2}{\epsilon_*^\phi} + \frac{y^2}{\epsilon_*^\chi}\right)^2} + \frac{2\frac{(U_c+V_c)^2}{(U_*+V_*)^2}\left(\frac{x}{\epsilon_*^\phi} - \frac{y}{\epsilon_*^\chi}\right)^2 \frac{\epsilon_c^\phi \epsilon_c^\chi}{\epsilon_c}\left(\frac{\eta_c^{ss}}{\epsilon_c} - 1\right)}{\left(\frac{x^2}{\epsilon_*^\phi} + \frac{y^2}{\epsilon_*^\chi}\right)^2}$$

- Where we have used the definitions

$$x \equiv \frac{1}{U_* + V_*}\left(U_* + \frac{V_c\epsilon_c^\phi - U_c\epsilon_c^\chi}{\epsilon_c}\right) \qquad y \equiv \frac{1}{U_* + V_*}\left(V_* - \frac{V_c\epsilon_c^\phi - U_c\epsilon_c^\chi}{\epsilon_c}\right)$$

$$\eta^{ss} \equiv \frac{\epsilon^\chi \eta^\phi + \epsilon^\phi \eta^\chi}{\epsilon}$$

Vernizzi, Wands (2006)

Effectively Single Field Inflation

- As the inflaton rolls through a steep valley, characterized by $(\eta^{ss} > 1)$ we find:
  - Entropy perturbations: $|\delta s| \sim \exp\left[-\frac{3}{2}\int H\, dt\right]$
  - Non-Gaussianity:

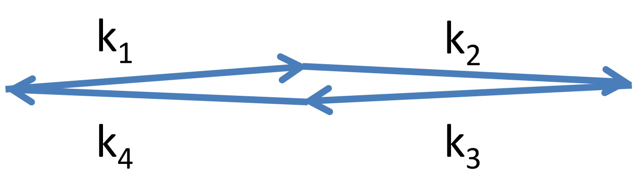
$$f_{\rm NL}^{\rm local} \sim \mathcal{O}(\varepsilon_*) + \mathcal{O}(1) \times \eta^{ss} \exp\left[-2\int C_\eta H \eta^{ss}\, dt\right]$$

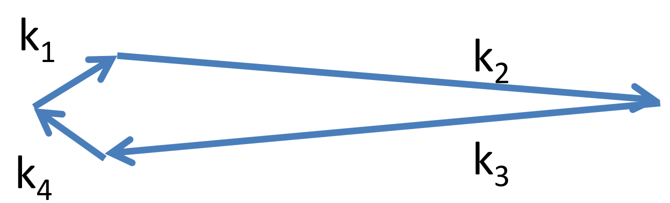
JM, Sivanandam (2010); Watanabe (2012)

Higher Point Statistics

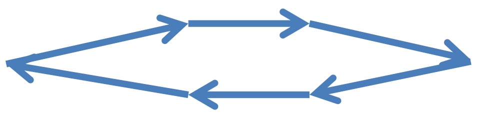
- Similar results also apply to the trispectrum after passing through a steep valley:

$$\tau_{\rm NL} \sim \mathcal{O}(\varepsilon_*) \qquad g_{\rm NL} \sim \mathcal{O}(\varepsilon_*)$$

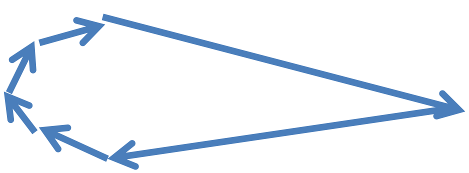




- And in fact to all local form n-point statistics:



$$F^{(n)}_{{\rm NL},i} \sim \mathcal{O}(\varepsilon_*)$$



JM, Sivanandam (2011)

Observables in Adiabatic Limit

- After adiabaticity is achieved, the observables take the following form

$$\frac{6}{5} f_{\rm NL}^{\rm local} \simeq \left(\frac{rx}{16\epsilon_*^\phi} \right)^2 (2\epsilon_*^\phi - x\eta_*^\phi) + \left(\frac{ry}{16\epsilon_*^\chi} \right)^2 (2\epsilon_*^\chi - y\eta_*^\chi)$$

$$n_s - 1 = -2\epsilon_* - 2\left(\frac{rx}{16\epsilon_*^\phi} \right) (2\epsilon_*^\phi - x\eta_*^\phi) - 2\left(\frac{ry}{16\epsilon_*^\chi} \right) (2\epsilon_*^\chi - y\eta_*^\chi)$$

- Recall the definitions

$$x \equiv \frac{1}{U_* + V_*} \left(U_* + \frac{V_c\epsilon_c^\phi - U_c\epsilon_c^\chi}{\epsilon_c} \right) \qquad y \equiv \frac{1}{U_* + V_*} \left(V_* - \frac{V_c\epsilon_c^\phi - U_c\epsilon_c^\chi}{\epsilon_c} \right)$$

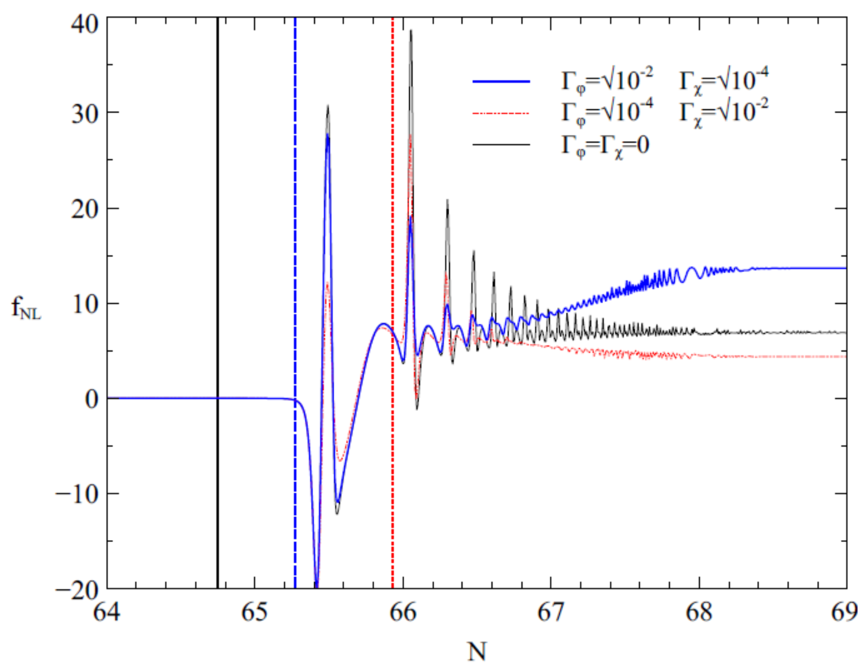
Conditions for Observable f_{NL}

- Generating local non-gaussianity which is preserved in upon passing through an effectively single field phase seems to require (at least for simple potentials):
  - One *very* slowly rolling field at horizon exit
  - A finely-tuned trajectory through field space
  - One field with negligible contribution to the energy density at horizon exit

Byrnes, Choi, Hall (2008); Kim, Liddle, Seery (2010); Elliston, et al. (2011); JM, Sivanandam (2010)

Reheating After Multiple Field Inflation

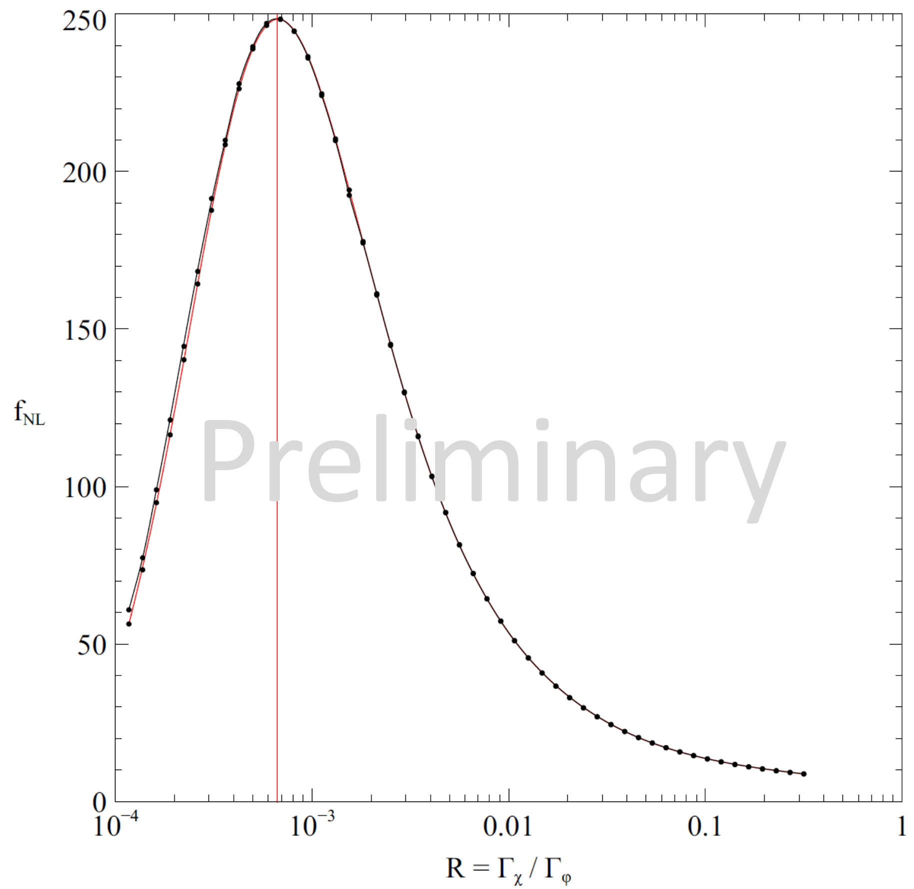
- If the adiabatic limit is not reached during inflation, the dynamics of reheating may significantly affect the final value of $f_{\mathrm{NL}}^{\mathrm{local}}$



Leung, Tarrant, Byrnes, Copeland (2012)

Reheating and Non-Gaussianity

- We must understand the effects of reheating in order to draw conclusions about multiple field inflation from constraints on non-Gaussianity



JM, Tarrant (In Prep.)

Conclusions

- Tensor modes or isocurvature fluctuations may still provide evidence for multiple field inflation
- Understanding the implications of observational constraints on non-Gaussianity requires that we understand the predictions of multiple field inflation
- Sharp predictions require an understanding of the evolution until fluctuations become adiabatic, or until they are observed