Non-parametric reconstruction of the primordial power spectrum with *Planck*

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4 April 2013, Estec Planck Conference

Underlying question: conventional parameterization

What is the underlprimordial power spectrum?

- For lack of a fundamental theory, expand in powers of $\ln(k)$

$$
\begin{aligned}
\ln\left(\mathcal{P}(\ln k)\right) &= \mathcal{P}_0\left(\ \ln(k/k_{piv})\right)^0 + \mathcal{P}_1\left(\ \ln(k/k_{piv})\right)^1 + \mathcal{P}_2\left(\ \ln(k/k_{piv})\right)^2 + \ldots \\
\mathcal{P}(k) &= A(k/k_{piv})^{(n_s-1)} \\
&\text{or} \\
\mathcal{P}(k) &= A(k/k_{piv})^{(n_s-1)+\alpha\ln(k/k_{piv})+\ldots}
\end{aligned}
$$

- *Planck* seems to be telling us that the first two terms suffice, and just the first term can be ruled out a respectable statistical significance. $n_S \neq 1$ implies exact scale invariance needs to be downgraded to an approximate symmetry. No statistically significant evidence for running of the spectral index.

Underlying question: searching for features

- Two approaches
  - Parameterized approaches : make Ansätze with a small number of extra parameters compare quality of fit to simpler model to determine whether extra parameters are justified by the data (Aikake Information Criterion, Bayesian Information Criterion, Bayesian Evidence, . . .). Approach followed in Planck paper XXII, section 8)
  - Non-parameterized approaches: penalized likelihoods,.... [Details of approach followed in Planck XXII paper follow: Gauthier, Christopher; Bucher, Martin; Reconstructing the primordial power spectrum from the CMB, JCAP 10, 050 (2012) (arXiv:1209.2147)

Tentative conclusion of Planck Parameters paper

> *To the extremely high accuracy afforded by the Planck data, the power spectrum at high multipoles is compatible with the predictions of the base six parameter ΛCDM cosmology.* This is the main result of this paper. Fig. 1 does, however, suggest that the power spectrum of the best-fit base ΛCDM cosmology has a higher amplitude than the observed power spectrum at multipoles $\ell \lesssim 30$. We will return to this point in Sect. 7.

- Conclusion based on looking at overall χ^2 with a very large number of degrees of freedom.
- We want to examine whether this conclusion is really justified.

Penalized likelihood

Let $\mathcal{P}_0(k) = A_\mathrm{s}(k/k_*)^{n_\mathrm{s}-1}$ be the best fit power spectrum of the six parameter model. We define a general ansatz for the power spectrum in terms of a fractional variation, $f(k)$, relative to this fiducial model, so that

$$\mathcal{P}_\mathcal{R}(k) = \mathcal{P}_0(k)\Big[1 + f(k)\Big]. \tag{1}$$

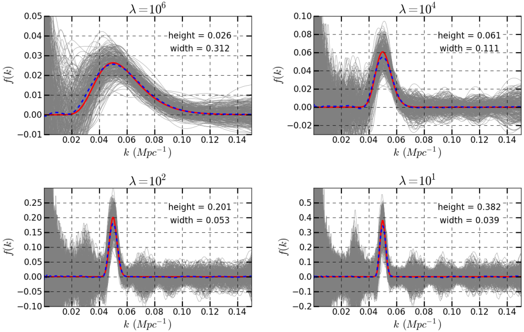
Any features are then described in terms of $f(k)$.

In this analysis we use the Planck+WP likelihood supplemented by the following prior, which is added to $-2\ln\mathcal{L}$:

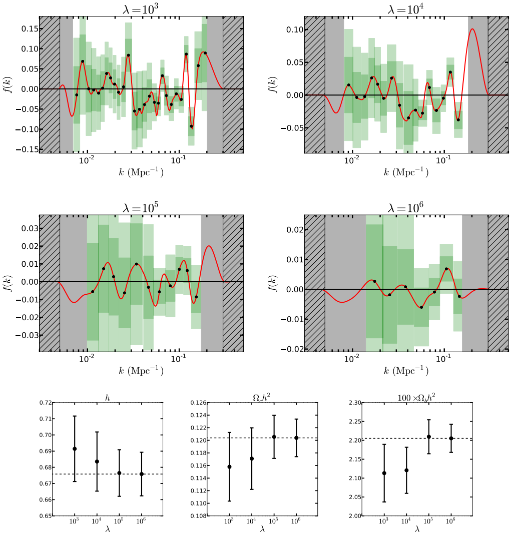
$$\begin{aligned}\mathbf{f}^T\mathbf{R}(\lambda,\alpha)\mathbf{f} = {} & \lambda\int \mathrm{d}\kappa\ \left(\frac{\partial^2 f(\kappa)}{\partial\kappa^2}\right)^2 \\ & + \alpha\int_{-\infty}^{\kappa_\mathrm{min}} \mathrm{d}\kappa\ f^2(\kappa) + \alpha\int_{\kappa_\mathrm{max}}^{+\infty} \mathrm{d}\kappa\ f^2(\kappa).\end{aligned} \tag{2}$$

where $\kappa = \ln k$.

Validation of method

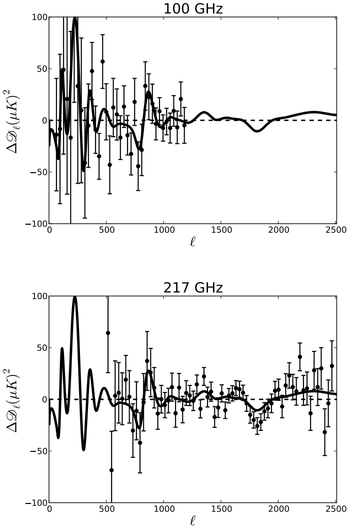


Results on Planck “Nominal mission" likeklihood

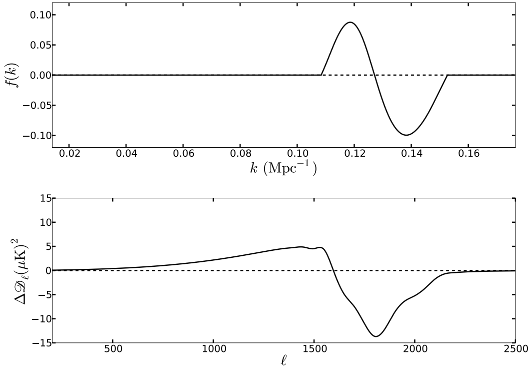


Maximum excursions locally 3.2σ and 3.9σ for $\lambda = 10^4$ and 10^3, respectively. After look-elsewhere-effect translates into $p = 1.74\%$ and $p = 0.21\%$, or 2.4σ and 3.1σ.

Where does this come from in the CMB multipole power spectrum?



Proof that signal is from around $\ell \approx 1800$



(Extract from parameters paper)

To the extremely high accuracy afforded by the Planck data, the power spectrum at high multipoles is compatible with the predictions of the base six parameter ΛCDM cosmology. This is the main result of this paper. Fig. 1 does, however, suggest that the power spectrum of the best-fit base ΛCDM cosmology has a higher amplitude than the observed power spectrum at multipoles $\ell \lesssim 30$. We will return to this point in Sect. 7.

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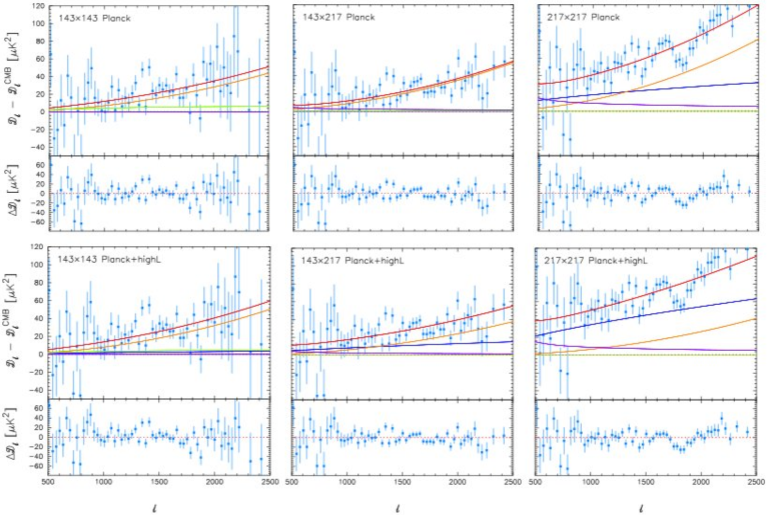

(Extract from parameters paper)

Table 6. Goodness-of-fit tests for the *Planck* spectra. The $\Delta\chi^2 = \chi^2 - N_\ell$ is the difference from the mean assuming the model is correct, and the last column expresses $\Delta\chi^2$ in units of the dispersion $\sqrt{2N_\ell}$.

Spectrum	ℓ_{min}	ℓ_{max}	χ^2	χ^2/N_ℓ	$\Delta\chi^2/\sqrt{2N_\ell}$
100×100	50	1200	1158	1.01	0.14
143×143	50	2000	1883	0.97	−1.09
217×217	500	2500	2079	1.04	1.23
143×217	500	2500	1930	0.96	−1.13
All	50	2500	2564	1.05	1.62

Conclusions:

- While low-ℓ power spectrum anomaly is at about 2σ, the high ℓ anomaly is at 3.1σ.
- Global χ^2 is not a good statistical method to test for residuals because expected signal is concentrated in a small number of degrees of freedom and any possible signal becomes drowned in the noise. Good for proving concordance but poor for detecting new physics.
- Results shown are for nominal mission. Preliminary full mission analysis gives lower statistical significance. We do not understand why and more investigation is needed.
- We must wait for a more detailed analysis using the full mission data.