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Excellence Cluster *Universe*

Based on
S.C., B. Garbrecht, Y. Zhu,
Non-gaussianities and curvature perturbations in hybrid inflation,
in preparation

New Insights in Hybrid Inflation

ESLAB Symposium – 2nd to 5th April 2013 – *The Universe seen by Planck*

Outline

I. Original, F-term, Dterm models (unified description)

II. Initial conditions of the field – Fine tuning problem

III. Usual regime: 1-field slow-roll + fast waterfall

IV. Mild waterfall regime (adiabatic)

V. Mild waterfall regime (adiabatic + entropic)

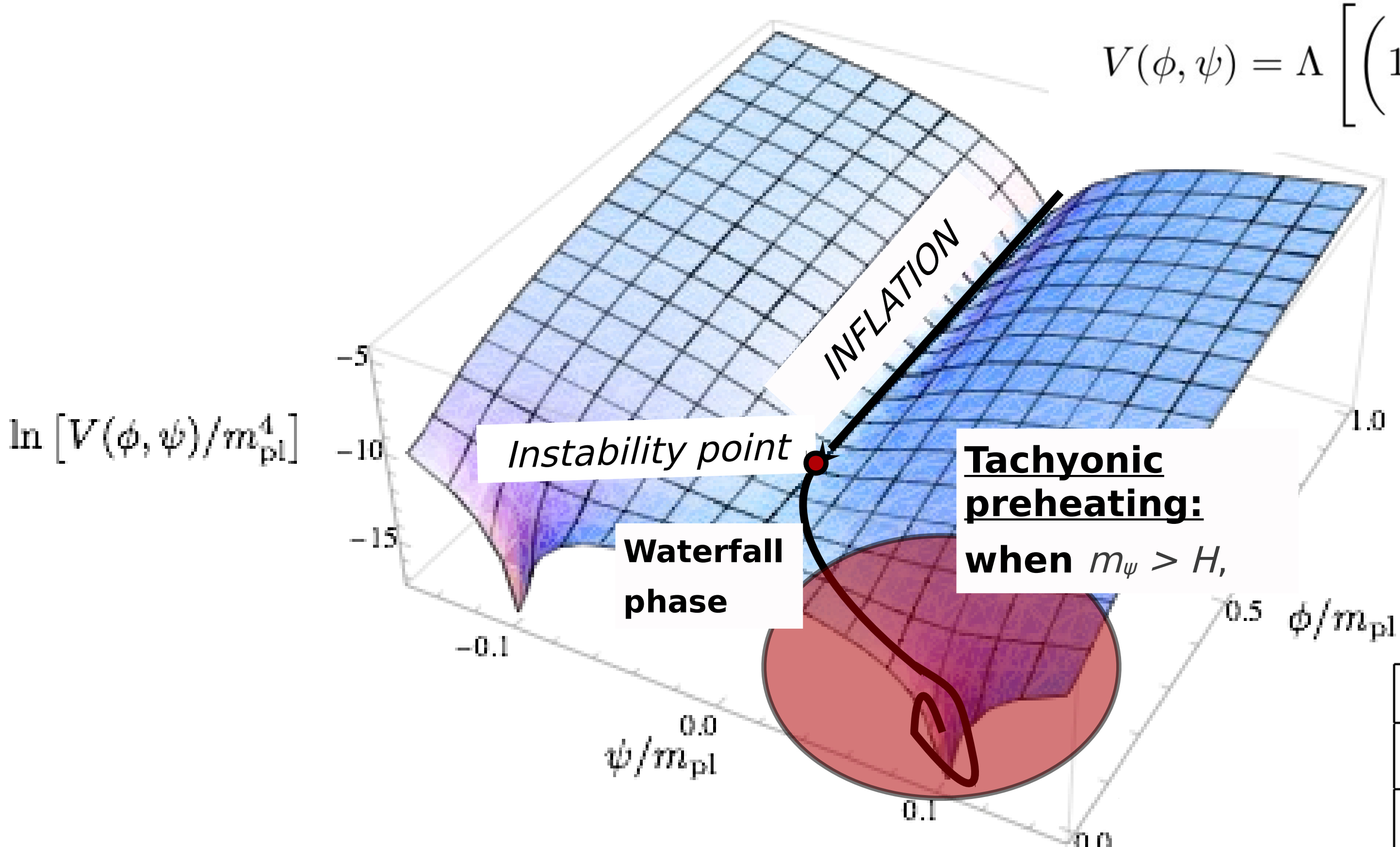
VI. New constraints from Planck...

VII. Conclusion

I. Original, F-term, D-term models

- **Classical Inflaton** ϕ **(slow-roll inflation in the valley)**
- **Higgs-type auxiliary field** ψ
- **Hybrid potential** *(A. Linde, astro-ph/9307002)*
- **F-term/D-term potential** *(E.Copeland et al.,astro-ph/9401011, P.Benitury,G.Dvali,hep-ph/9606342)*

$$V(\phi,\psi) = \Lambda\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + \left(\frac{\phi}{\mu}\right)^p + \frac{2\phi^2\psi^2}{M^2\phi_c^2}\right]$$



$$W_{F-term} = \kappa\hat{S}(\overline{\hat{H}}\hat{H} - M_{FI})$$
$$W_{D-term} = \kappa\hat{S}\overline{\hat{H}}\hat{H}$$
$$D = \frac{g}{2}(|H^2| - |\overline{H}^2| + m_{FI}^2)$$

Along the valley:

CW log corrections

SUGRA corrections (neglected)

	F-term	D-term
Λ	$\kappa^2 M_{\rm FI}^4$	$\frac{\kappa^4}{2g^2}\phi_{\rm c}^4 = \frac{g^2}{8}m_{\rm FI}^4$
$\phi_{\rm c}$	$\sqrt{2}M_{\rm FI}$	$\frac{g}{\sqrt{2}\kappa}m_{\rm FI}$
M	$2M_{\rm FI}$	$2\phi_{\rm c}$
$1/\mu$	$\frac{\mathcal{N}\kappa^2\log(2)}{4\pi^2\phi_{\rm c}}$	$\frac{g^2\kappa^2\log 2}{8\pi^2\phi_{\rm c}}$

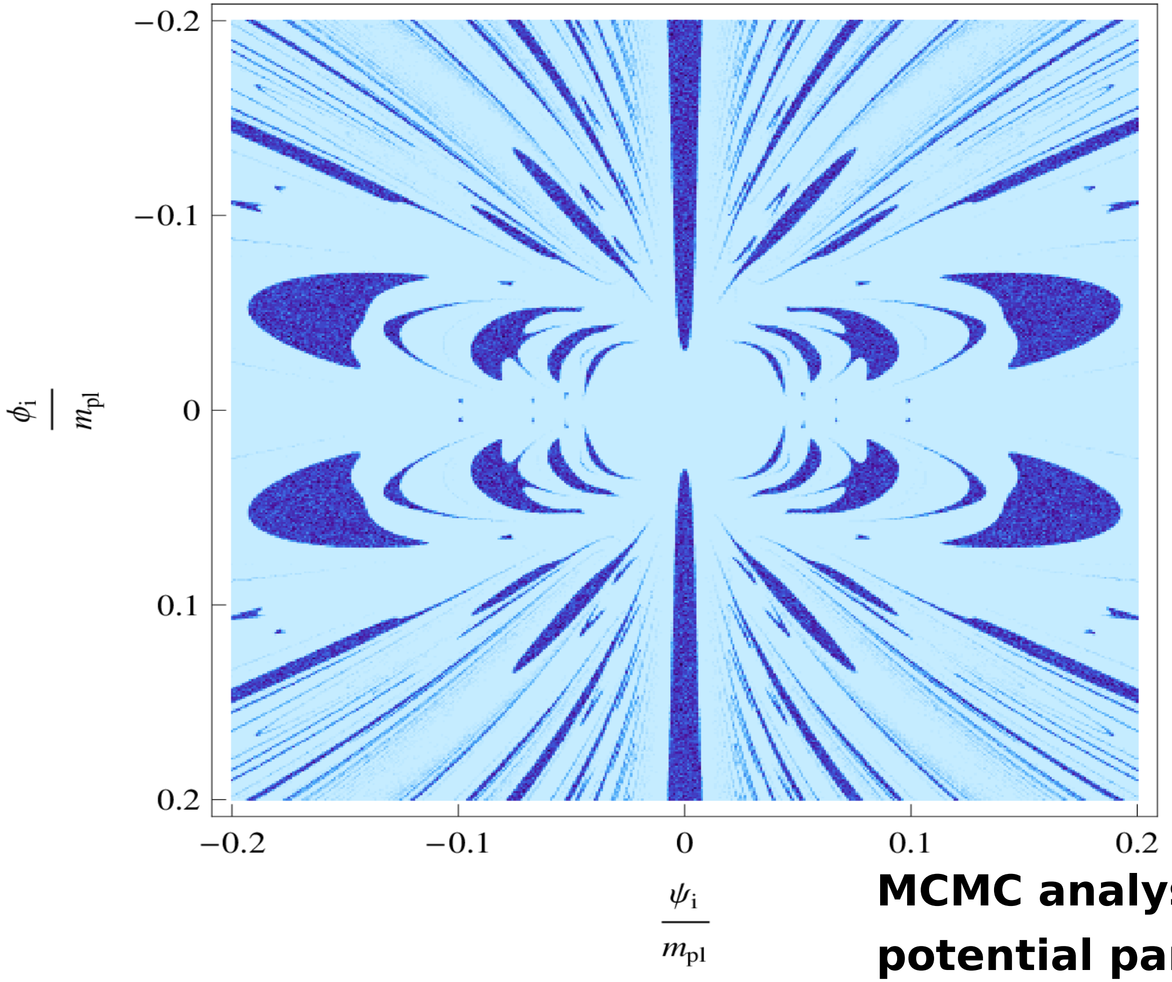
	Original model (5 parameters)	**F-term model** (2 parameters)	**D-term model** (3 parameters)
Natural Initial Field values			
Fast Waterfall k=aH along the valley			
Mild waterfall k=aH in phase 2 N>>60 (generic)			
Mild waterfall k=aH in phase 1 N≥60 (tuned)			

II. Initial conditions of the fields

Original Hybrid Model : Thin successful band along the valley - Fine-Tuning Problem

Mendes, Liddle, astro-ph/0006020, Tetradis, astro-ph/9707214

S.C., J. Rocher, arXiv:0809.4355, S.C, C. Ringeval, J. Rocher, arXiv:0909.0402



Depending on the potential parameters:

=> up to 20% successful IC

=> fractal boundaries

=> trajectories reach the valley (attractor)

Generic for other Hybrid Models

Original, F-term, shifted,

Smooth, Radion

MCMC analysis (initial fields, velocities, potential parameters) => Natural Constraints

Original model: $\mu > 0.3\, m_{pl}$ $\varphi_c < 0.004\, m_{pl}$ F-term: $M < 0.009\, m_{Pl}$

	Original model (5 parameters)	**F-term model** (2 parameters)	**D-term model** (3 parameters)
Natural Initial Field values	$\varphi_c < 0.004\, m_{pl}$ $\mu > 0.3\, m_{pl}$	$M < 0.009\, m_{Pl}$	
Fast Waterfall k=aH along the valley			
Mild waterfall k=aH in phase 2 N>>60 (generic)			
Mild waterfall k=aH in phase 1 N≥60 (tuned)			

III. 1-field slow-roll + fast waterfall

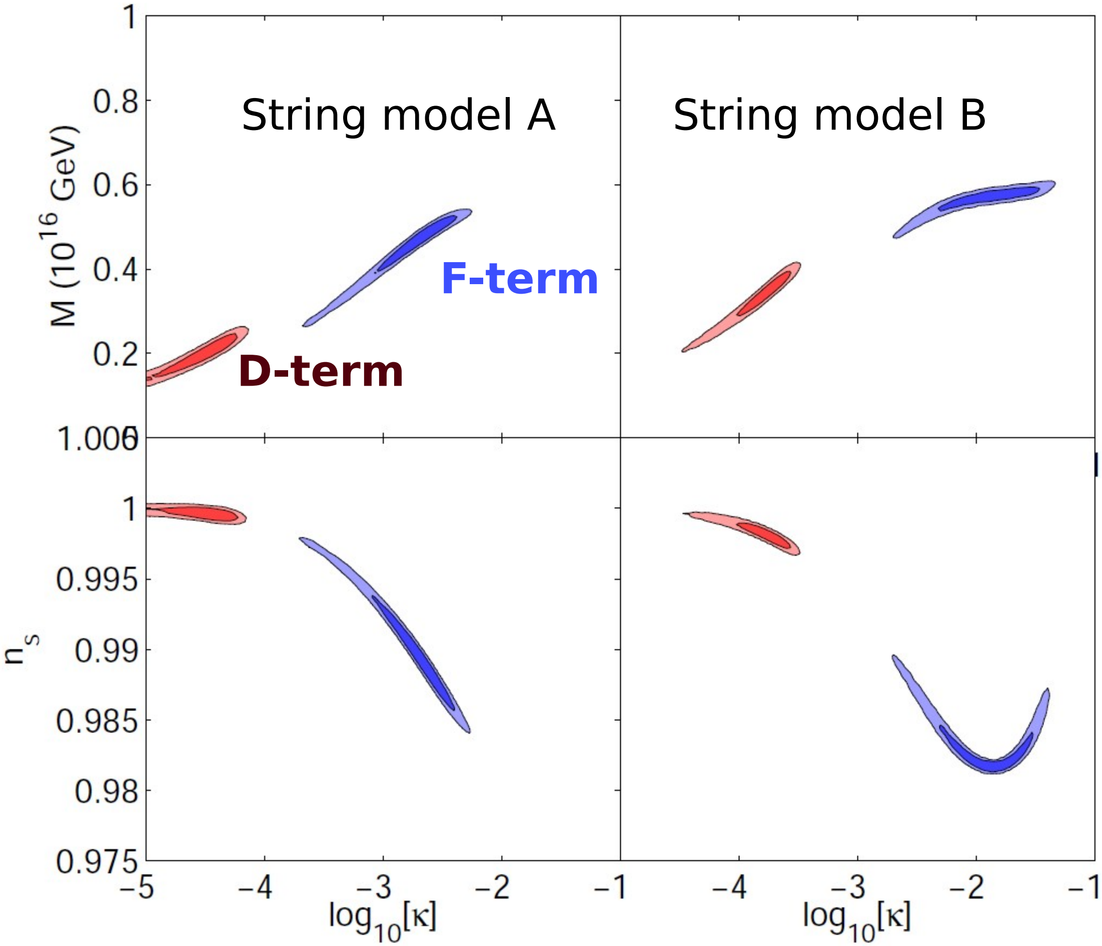
Usual regime: **=> Inflation along the valley**

=> Nearly instantaneous waterfall phase

=> Domain walls (ruled out) **or Cosmic Strings**

Observable Predictions: **Original model:** $n_s \geq 1$

F-term / D-term: *R.Battye, B.Garbrecht, A.Moss,arXiv:1001.0769*



F-term model, best fit:

$$\log_{10}\kappa = -1.94 \quad M/(10^{16} GeV) = 0.56$$

$$\Delta\chi^2 = 0.2$$

Allowed by WMAP+SDSS

+ BBN: excluded at 2.4σ

D-term model:

$$\log_{10}\kappa = -3.84 \quad M/(10^{16} GeV) = 0.33$$

$$\Delta\chi^2 = -2.8$$

Disfavored at 1.6σ

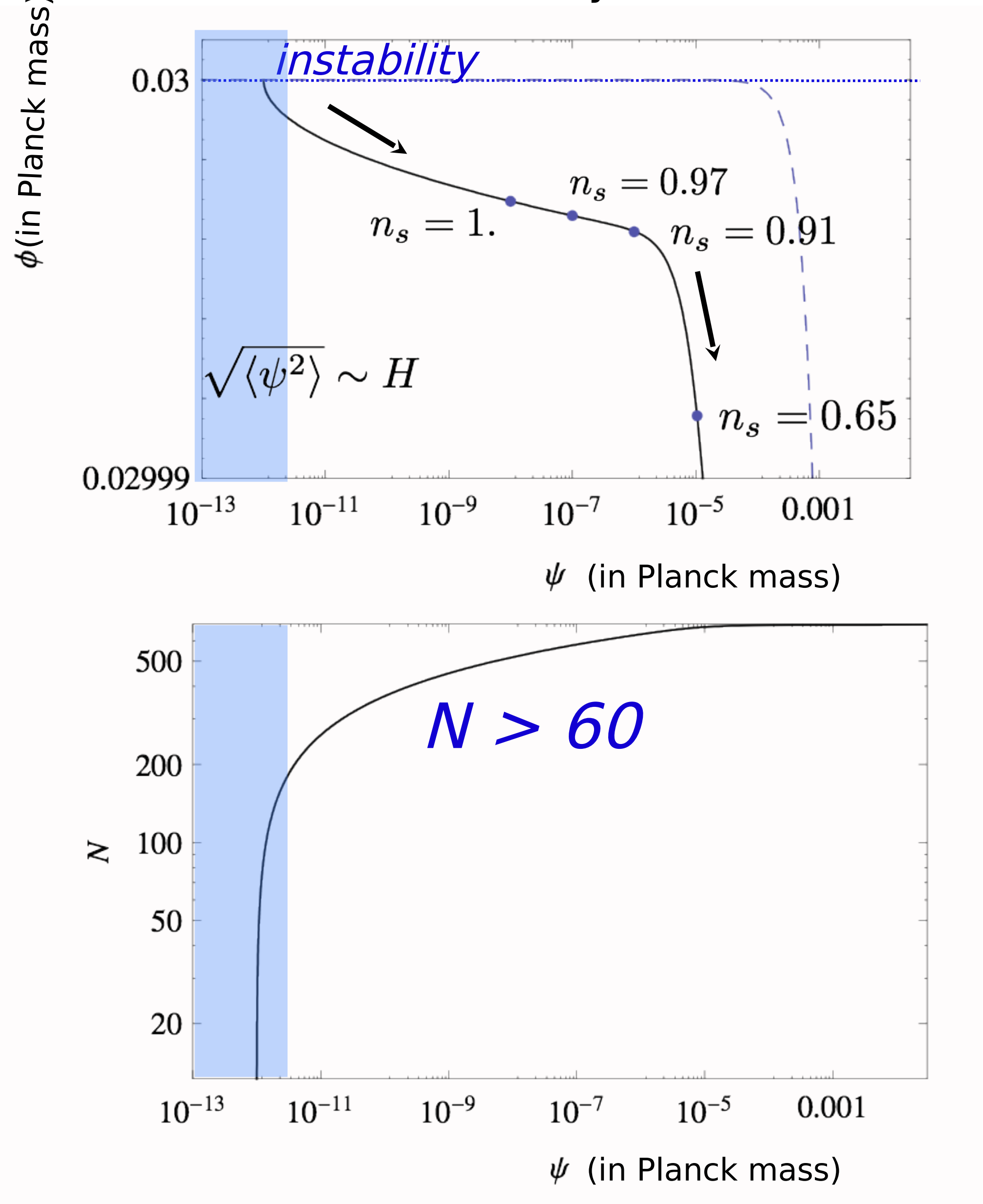
by WMAP+SDSS

+ BBN : excluded at 3.5σ

	Original model (5 parameters)	**F-term model** (2 parameters)	**D-term model** (3 parameters)
Natural Initial Field values	$\varphi_c < 0.004\, m_{pl}$ $\mu > 0.3\, m_{pl}$	$M < 0.009\, m_{Pl}$	
Fast Waterfall k=aH along the valley	$n_s > 1$ Domain walls Ruled out	$0.98 < n_s < 1$ Cosmic Strings tension with WMAP	$n_s \simeq 1$ Cosmic Strings Ruled out
Mild waterfall k=aH in phase 2 N>>60 (generic)			
Mild waterfall k=aH in phase 1 N≥60 (tuned)			

IV. Mild waterfall phase (adiabatic)

The exact 2-field classical dynamics of waterfall trajectories S.C., arXiv:1104.3494



$\phi_c = 0.03\ m_{pl}, M = 0.03\ m_{pl}, \mu = 636.4\ m_{pl}, \Lambda^4 = 10^{-24}\ m_{pl}^4$

Much more than 60 e-folds along the waterfall

Classical value of ψ emerges from The quantum diffusion regime

Topological defects stretched outside the Hubble radius

Observable modes leave the Hubble radius during the waterfall

Red power spectrum of adiabatic perturbations

IV. Mild waterfall phase (adiabatic)

NUMERICALLY: Homogeneous Multi-field dynamics

ANALYTICALLY: 2-fields, slow-roll, 2 phases

2 regimes:

$$3H\dot{\phi} = -\frac{2\Lambda^4\phi}{\mu^2}\left(1+\frac{2\mu^2\psi^2}{M^2\phi_{\mathrm{c}}^2}\right)$$

$$3H\dot{\psi} = -\frac{4\Lambda^4\psi}{M^2}\left(\frac{\phi^2-\phi_{\mathrm{c}}^2}{\phi_{\mathrm{c}}^2}+\frac{\psi^2}{M^2}\right)$$

Notation: $\phi \equiv \phi_{\mathrm{c}}\mathrm{e}^{\xi}$ $\quad \psi \equiv \psi_0\mathrm{e}^{\chi}$ **1st order expansion:** $\phi \simeq \phi_{\mathrm{c}}(1+\xi)$

IV. Mild waterfall phase (adiabatic)

NUMERICALLY: Homogeneous Multi-field dynamics

ANALYTICALLY: 2-fileds, slow-roll, 2 phases

2 regimes:

$$3H\dot{\phi} = -\frac{2\Lambda^4\phi}{\mu^2}\left(1 + \text{[redacted]}\right)$$

$$3H\dot{\psi} = -\frac{4\Lambda^4\psi}{M^2}\left(\frac{\phi^2 - \phi_c^2}{\phi_c^2} + \text{[redacted]}\right)$$

PHASE 1

Notation: $\phi \equiv \phi_c \mathrm{e}^{\xi}$ $\quad \psi \equiv \psi_0 \mathrm{e}^{\chi}$ **1st order expansion:** $\phi \simeq \phi_c (1 + \xi)$

IV. Mild waterfall phase (adiabatic)

NUMERICALLY: **Homogeneous Multi-field dynamics**

ANALYTICALLY: **2-fields, slow-roll, 2 phases**

2 regimes:

$$3H\dot{\phi} = -\frac{2\Lambda^4\phi}{\mu^2}\left(\text{[redacted]}\ \frac{2\mu^2\psi^2}{M^2\phi_c^2}\right)$$

$$3H\dot{\psi} = -\frac{4\Lambda^4\psi}{M^2}\left(\frac{\phi^2-\phi_c^2}{\phi_c^2} + \text{[redacted]}\right)$$

PHASE 2

Notation: $\phi \equiv \phi_c e^{\xi}$ $\psi \equiv \psi_0 e^{\chi}$ **1st order expansion:** $\phi \simeq \phi_c(1+\xi)$

IV. Mild waterfall phase (adiabatic)

NUMERICALLY: Homogeneous Multi-field dynamics

ANALYTICALLY: 2-fields, slow-roll, 2 phases

2 regimes:

$$3H\dot{\phi} = -\frac{2\Lambda^4\phi}{\mu^2}\left(\text{[redacted]}\frac{2\mu^2\psi^2}{M^2\phi_c^2}\right)$$

$$3H\dot{\psi} = -\frac{4\Lambda^4\psi}{M^2}\left(\frac{\phi^2-\phi_c^2}{\phi_c^2}+\text{[redacted]}\right)$$

PHASE 2

Notation: $\phi \equiv \phi_c e^{\xi}$ $\psi \equiv \psi_0 e^{\chi}$ **1st order expansion:** $\phi \simeq \phi_c(1+\xi)$

Mild waterfall phase ($N \gg 60$ **) - Original Model:** $M\mu \gg M_{pl}^2$

- F-term / D-term: $\kappa \ll M^2/M_{pl}^2$

<u>1st Case: *k = aH* in phase 2 ($N \gg 60$, generic)</u>

Power spectrum of adiabatic perturbations: $P_\zeta = \frac{\kappa^2 M_{Pl}^2 N_e^4}{18\pi^2 M^2}$ **for F-term**

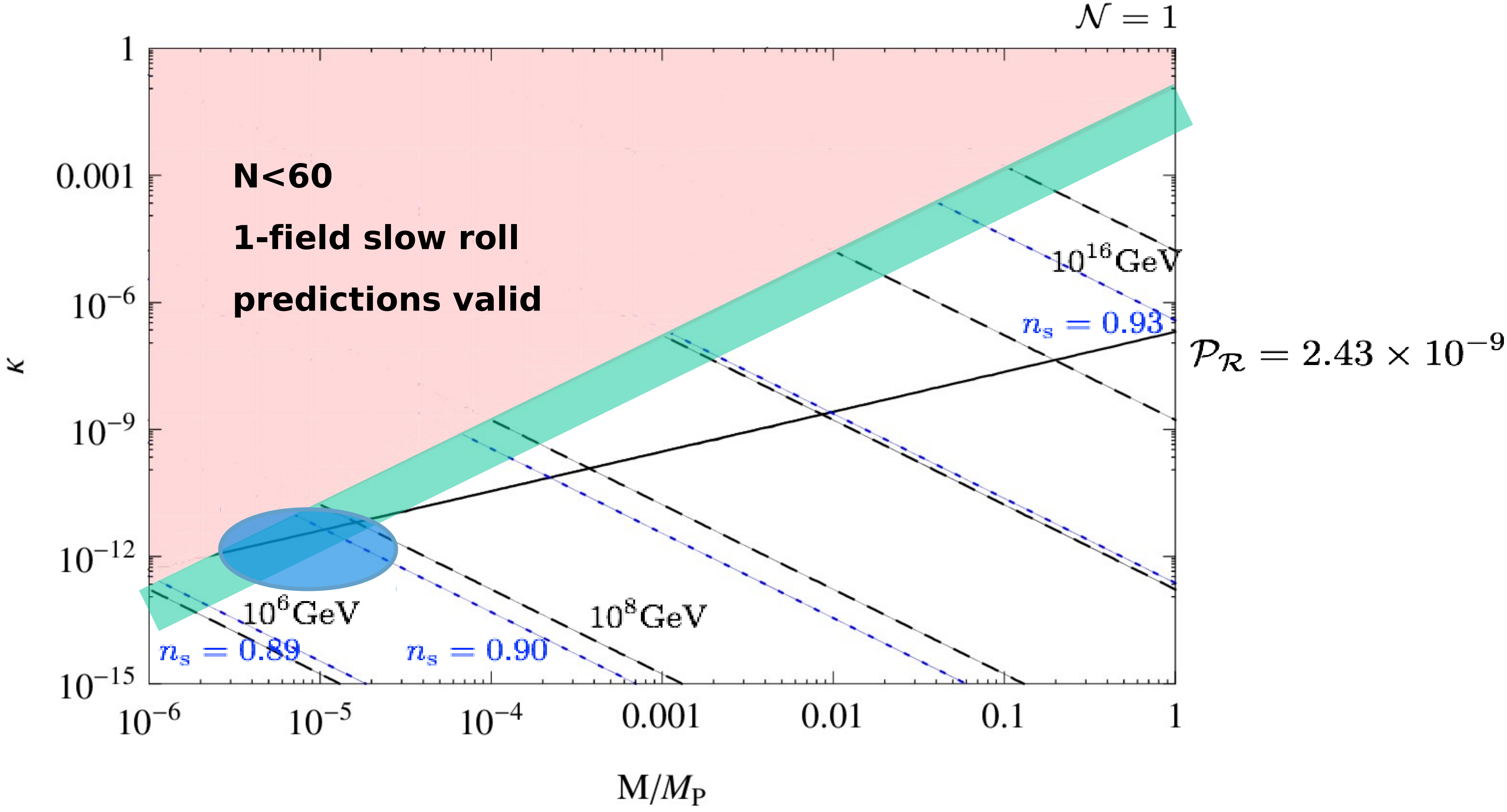
$P_\zeta = \frac{\kappa^4 M_{Pl}^2 N_e^4}{9\pi^2 g^2 m_{FI}^2}$ **for D-term**

•

Spectral index: $n_s = 1 - 4/N_e$

<u>2nd Case: *k = aH* in phase 1 ($N \geq 60$, tuned)</u> **Spectral index:** reaches unity

IV. Mild waterfall phase (adiabatic)



	Original model (5 parameters)	**F-term model** (2 parameters)	**D-term model** (3 parameters)
Natural Initial Field values	$\varphi_c < 0.004\, m_{pl}$ $\mu > 0.3\, m_{pl}$	$M < 0.009\, m_{Pl}$	
Fast Waterfall k=aH along the valley	$n_s > 1$ Domain walls Ruled out	$0.98 < n_s < 1$ Cosmic Strings tension with WMAP	$n_s \simeq 1$ Cosmic Strings Ruled out
Mild waterfall k=aH in phase 2 N>>60 (generic)	$\mu M \gg M_{\mathrm{p}}^2$ Effective 1-field: $n_{\mathrm{s}} = 1 - \frac{4}{N_{\mathrm{e}}}$ Nearly ruled out	$\kappa \ll \frac{M^2}{M_{\mathrm{p}}^2}$ Effective 1-field: $n_{\mathrm{s}} = 1 - \frac{4}{N_{\mathrm{e}}}$ Ruled out	$\kappa \ll \frac{m_{\mathrm{FI}}^2}{M_{\mathrm{p}}^2}$ Effective 1-field: $n_{\mathrm{s}} = 1 - \frac{4}{N_{\mathrm{e}}}$ Ruled out
Mild waterfall k=aH in phase 1 N≥60 (tuned)	n_s goes to unity possible agreement with CMB	n_s goes to unity possible agreement with CMB	n_s goes to unity possible agreement with CMB

V. Mild waterfall phase (adiabatic+entropic)

PRELIMINARY

- **2-field trajectories (turning)**
- **Potentially large local non-gaussianities**
- **Contribution of entropic perturbations to the PS of curvature perturbations**

Methods:

δN Formalism (analytically and numerically)

Linear Theory of Multi-field Perturbations (numerically)

k = aH **in phase 2**

Long waterfall phase

$$\kappa \ll \frac{M^2}{M_{\mathrm{p}}^2} \qquad \mu M \gg M_{\mathrm{p}}^2$$

Local non-gaussianities:

$$f_{NL} \approx -1/N_e \approx -0.03$$

Spectral index and Amplitude

As in the adiabatic case

k = aH **in phase 1**

Moderate waterfall

$$\kappa \approx M^2 / M_{Pl}^2 \qquad \mu M \approx M_{Pl}^2$$

Local non-gaussianities:

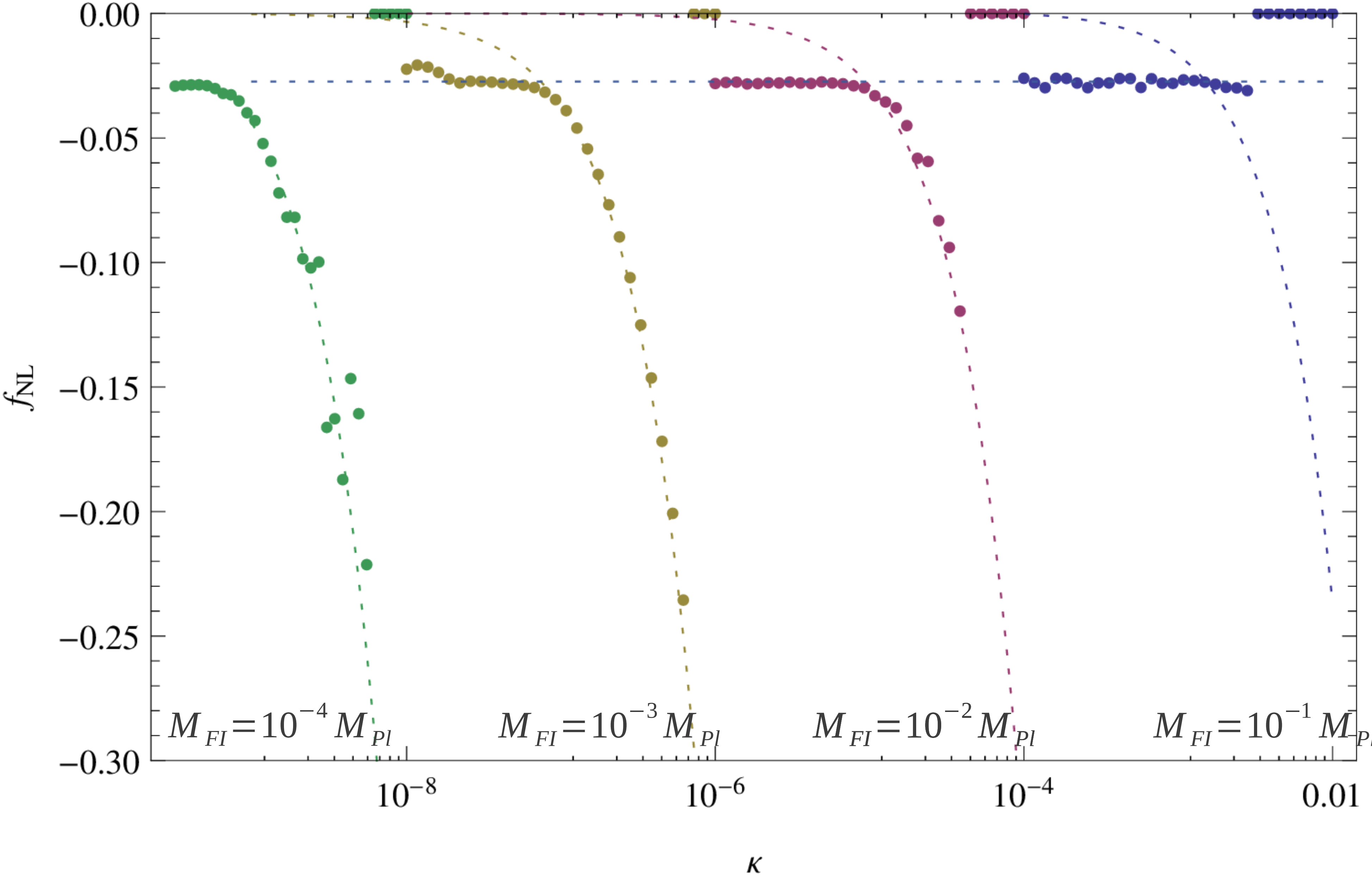
$$f_{NL} \approx \frac{-10\, p^{1/2} M_{Pl}^2 \varphi_c^{p/2-1} \chi_2}{3 M \mu^{p/2}} \qquad |f_{NL}| \leqslant 0.3$$

Amplitude of the power spectrum increases by

Several orders of magnitudes

V. Mild waterfall phase (adiabatic+entropic)

F-term, D-term *PRELIMINARY*



V. Mild waterfall phase (adiabatic+entropic)

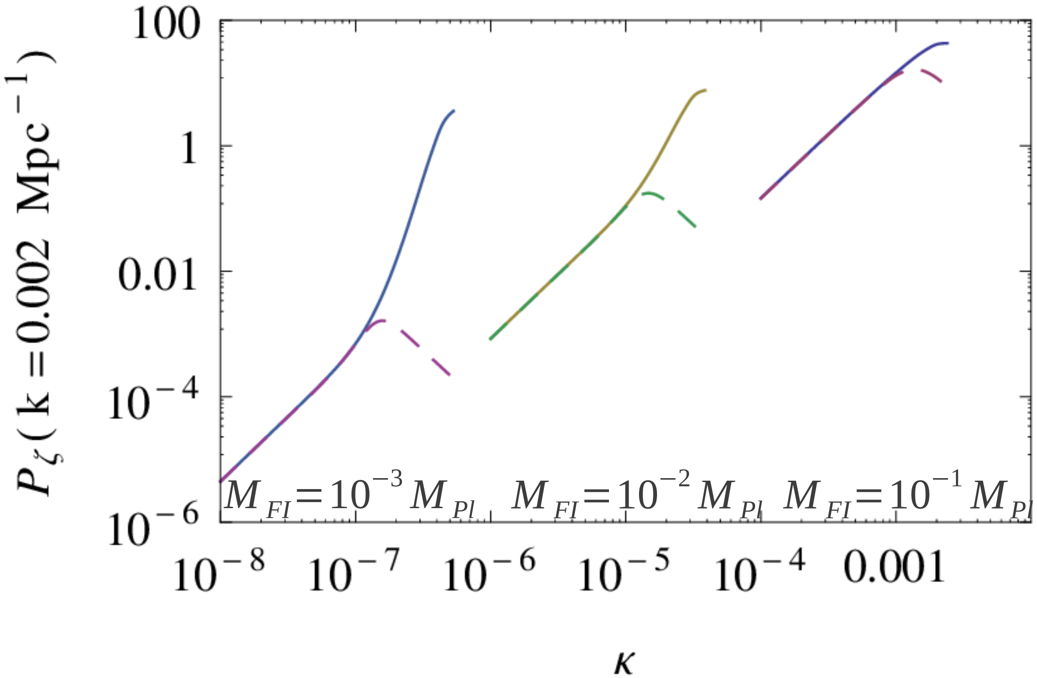
PRELIMINARY

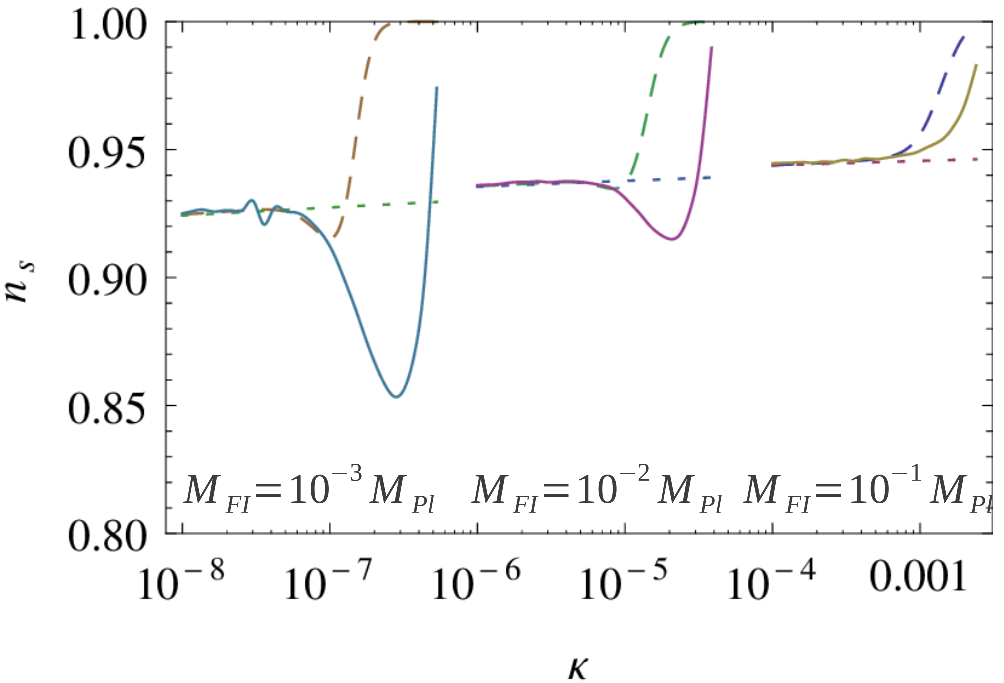
F-term model – Power spectrum

Adiabatic perturbations (dashed)

Curvature perturbations (adiabatic + entropic, plain)

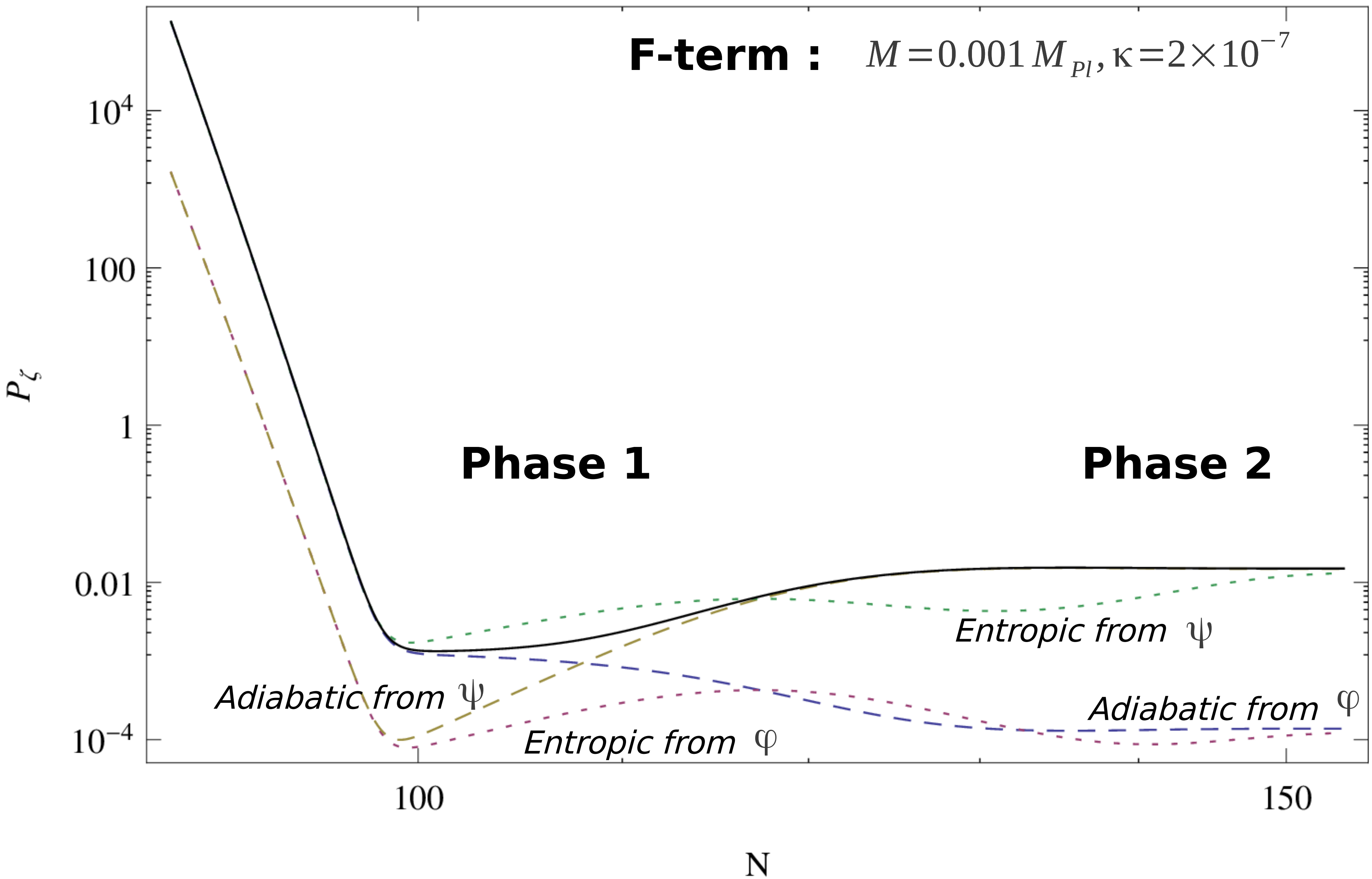
$n_s = 1 - 4/N_{exit}$ (dotted)





V. Mild waterfall phase (adiabatic+entropic)

PRELIMINARY



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Mild waterfall k=aH in phase 1 N≥60 (tuned)	Amplitude increases **due to entropic modes** |fNL| <0.3 **Ruled out**	Amplitude increases **due to entropic modes** **|fNL| <0.3** **Ruled out**	Amplitude increases **due to entropic modes** **|fNL| <0.3** **Ruled out**

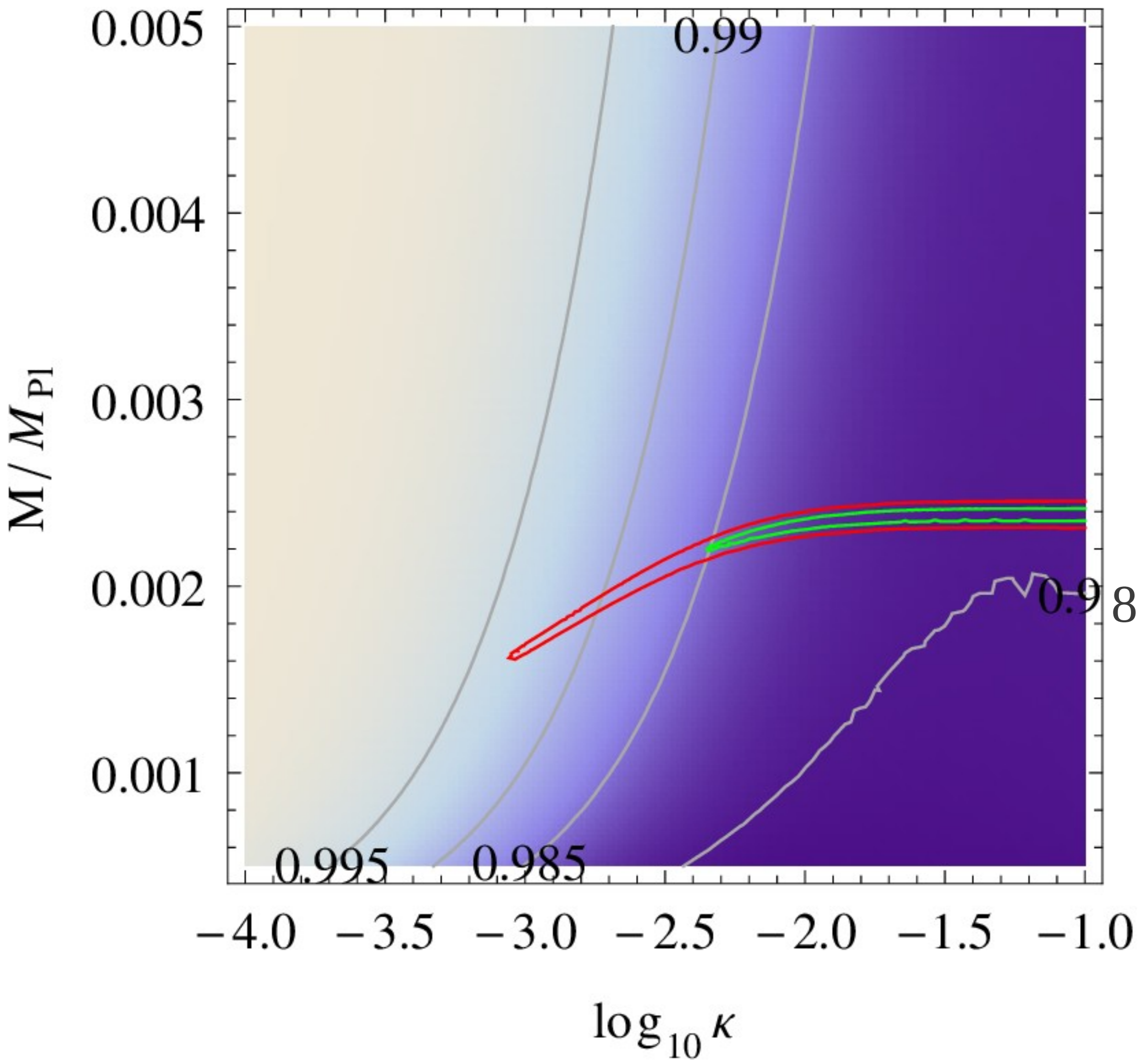
VI. New constraints from Planck

PRELIMINARY

F-term model, regime of quasi-instantaneous waterfall + cosmic strings

Planck results : Cosmic strings parameters not degenerated with n_s **and** A_s

Discrepancy with BBN bounds



- **Gaussian uncorrelated likelihoods**
- **AH string model**
- **3σ (green) and 5σ (red) contours**
- **Best fit :** $\log_{10}\kappa = -1.92$

  $M/(10^{16} GeV) = 0.54$
- **Disfavored at** 2.15σ

VII. Conclusion

	Original model (5 parameters)	**F-term model** (2 parameters)	**D-term model** (3 parameters)
Natural Initial Field values	$\varphi_c < 0.004\, m_{pl}$ $\mu > 0.3\, m_{pl}$	$M < 0.009\, m_{Pl}$	
Fast Waterfall k=aH along the valley	$n_s > 1$ Domain walls Ruled out	$0.98 < n_s < 1$ Cosmic Strings Disfavored by Planck	$0.98 < n_s < 1$ Cosmic Strings Disfavored by Planck
Mild waterfall k=aH in phase 2 N>>60 (generic)	$\mu M \gg M_{\mathrm{p}}^2$ Effective 1-field: $n_{\mathrm{s}} = 1 - \frac{4}{N_{\mathrm{e}}}$ Nearly ruled out	$\kappa \ll \frac{M^2}{M_{\mathrm{p}}^2}$ Effective 1-field: $n_{\mathrm{s}} = 1 - \frac{4}{N_{\mathrm{e}}}$ Ruled out	$\kappa \ll \frac{m_{\mathrm{FI}}^2}{M_{\mathrm{p}}^2}$ Effective 1-field: $n_{\mathrm{s}} = 1 - \frac{4}{N_{\mathrm{e}}}$ Ruled out
Mild waterfall k=aH in phase 1 N≥60 (tuned)	Amplitude increases **due to entropic modes** |fNL| <0.3 **Ruled out**	Amplitude increases **due to entropic modes** **|fNL| <0.3** **Ruled out**	Amplitude increases **due to entropic modes** **|fNL| <0.3** **Ruled out**

VII. Conclusion

Still to be done: - **Full MCMC analysis of F/D-term with Planck data**

- **Regime where 10<N<60 during the waterfall ?**

VII. Conclusion

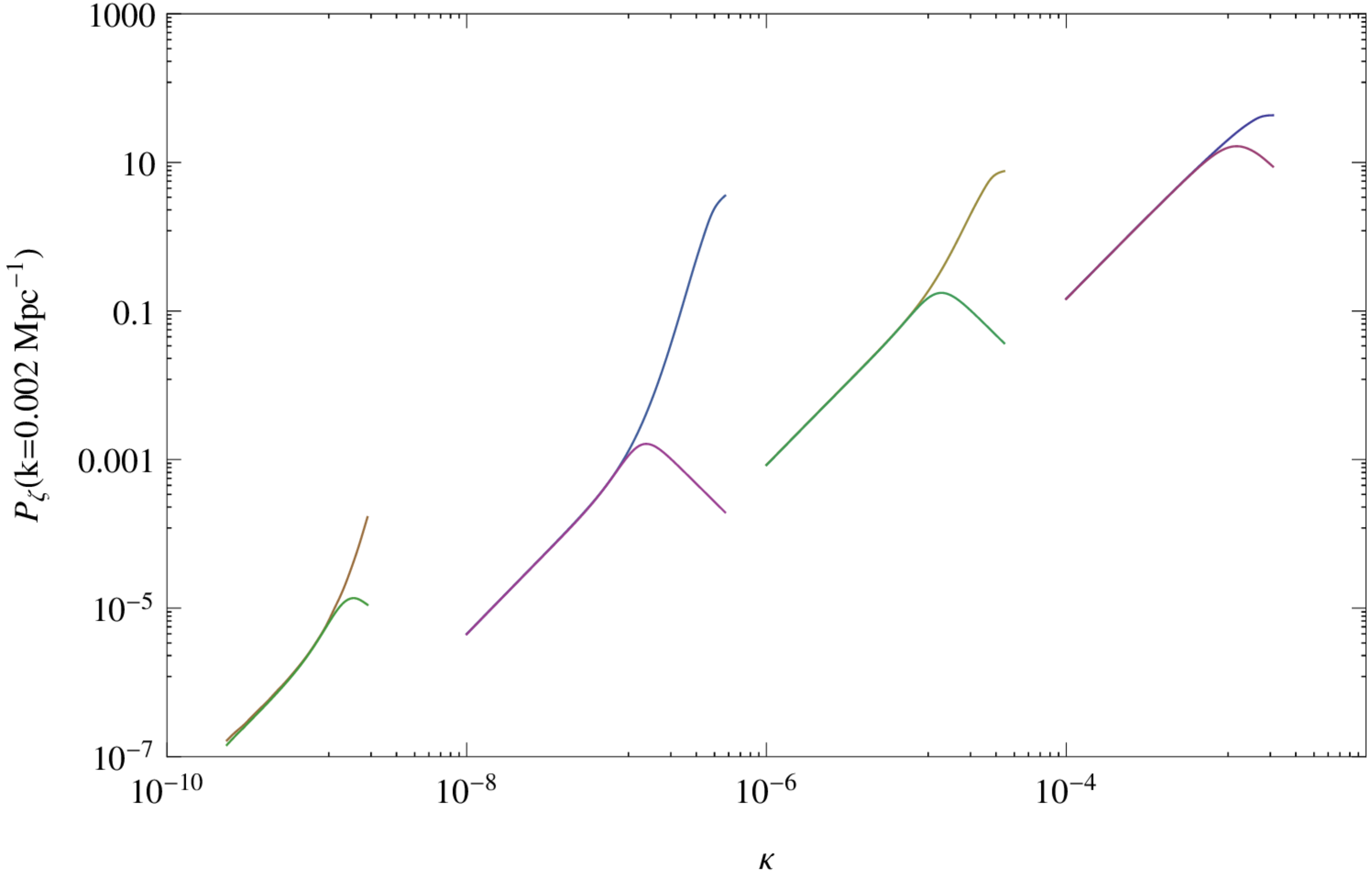
Still to be done: **- Full MCMC analysis of F/D-term with Planck data**

- Regime where 10<N<60 during the waterfall ?

Thank you for your attention...

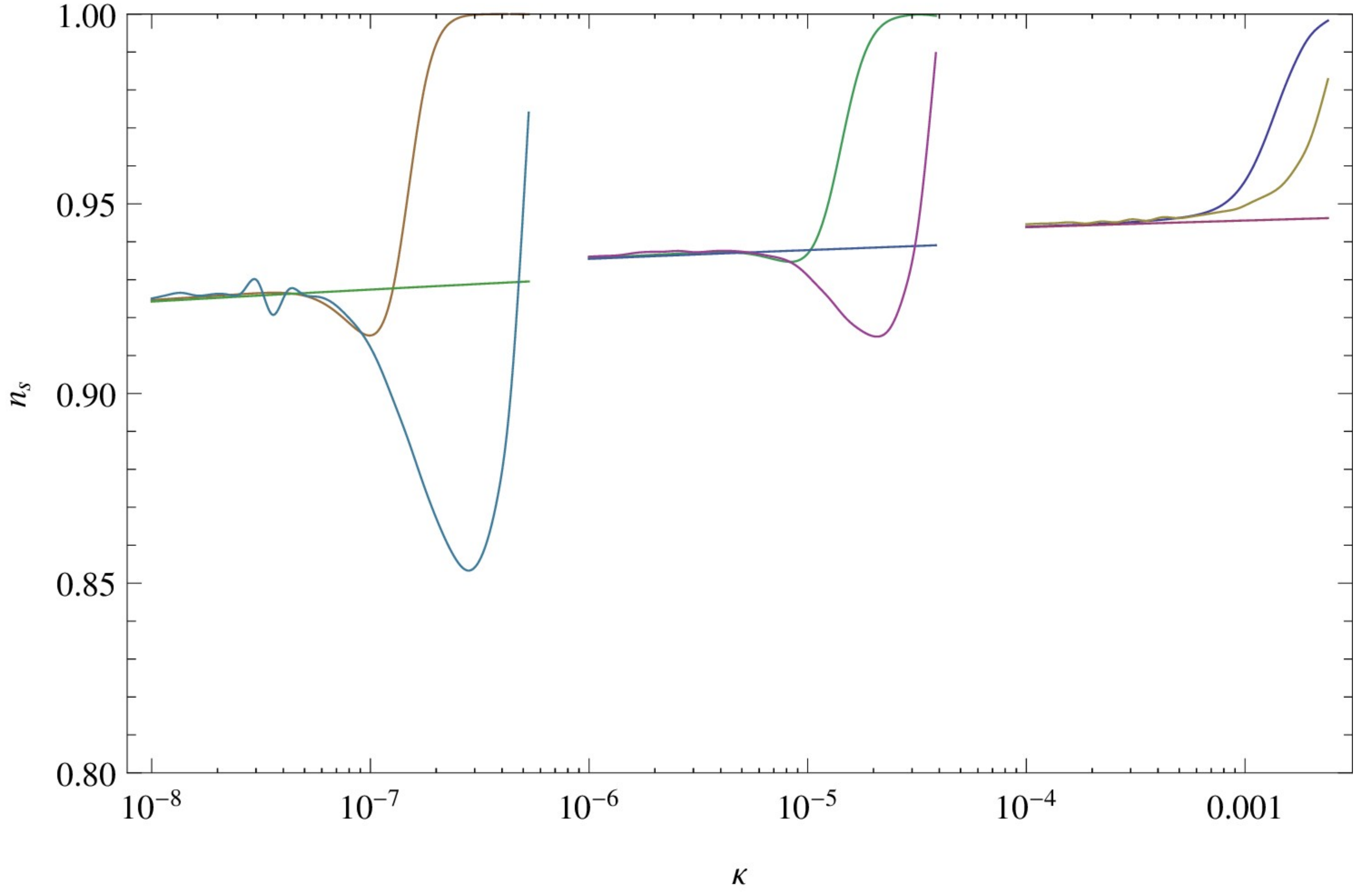
V. Mild waterfall phase (adiabatic+entropic)

PRELIMINARY



V. Mild waterfall phase (adiabatic+entropic)

PRELIMINARY



1. Basics on Inflation

- Horizon Problem
- Flatness Problem
- Topological defects

Number of e-folds:

$$N_{\rm end} \equiv \ln \frac{a_{\rm end}}{a_{\rm i}} > 60$$

- **Inflation :** **Period of accelerated expansion of the universe, that is** $\ddot{a} > 0$ **, where a is the scale factor**
- **Simplest realisation :** **Fill the universe with an homogeneous scalar field** ϕ **, slowly rolling along its potential (ex:** $V(\phi) = m^2\phi^2$ **)**
- **Dynamics :** **Einstein equations in homogeneous FLWR universe + Klein-Gordon equation**

$$H^2 = \frac{8\pi}{3m_{\rm p}^2}\left[\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)\right]$$

$$\frac{\ddot{a}}{a} = \frac{8\pi}{3m_{\rm p}^2}\left[\cancel{-\dot{\phi}^2} + V(\phi)\right]$$

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

Slow-roll approximation

- **Cosmological Perturbations :**

$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

- **Power spectrum of scalar pert of the metric, in SR approximation:**

$$\mathcal{P}_\zeta(k) = C\left(\frac{k}{k_*}\right)^{n_{\rm s}-1}$$

$$C \sim \frac{H_*^2}{\pi\epsilon_{1*}}$$

$$n_s - 1 = -2\epsilon_{1*} - \epsilon_{2*}$$

$$\epsilon_1 = \frac{m_{\rm p}^2}{16\pi}\left(\frac{\frac{\mathrm{d}V}{\mathrm{d}\phi}}{V}\right)^2 \ll 1$$

$$\epsilon_2 = \frac{m_{\rm p}^2}{4\pi}\left[\left(\frac{V'}{V}\right)^2 - \frac{V"}{V}\right] \ll 1$$

Other realisation : **Fill the universe with TWO scalar fields**

F.L. equations:

$$H^2 = \frac{8\pi}{3m_{\mathrm{p}}^2}\left[\frac{1}{2}\left(\dot{\phi}^2 + \dot{\psi}^2\right) + V(\phi,\psi)\right]$$

$$\frac{\ddot{a}}{a} = \frac{8\pi}{3m_{\mathrm{p}}^2}\left[-\dot{\phi}^2 - \dot{\psi}^2 + V(\phi,\psi)\right]$$

K.G. equations:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0 \qquad \ddot{\psi} + 3H\dot{\psi} + \frac{\mathrm{d}V}{\mathrm{d}\psi} = 0$$

2. δN formalism vs. linear multi-field perturbations

2.1. δN formalism:

- Based on the separable Universe approximation
- i : initial flat hypersurface, f : final hypersurface of uniform density

Curvature Perturbation: $\zeta = \delta N_{\mathrm{i}}^{\mathrm{f}}$

- Curvature perturbation when a pivot scale k_p leaves the Hubble radius

$$\zeta \simeq \sum_{i=1}^{n} N_{,i}\delta\phi_i + \frac{1}{2}\sum_{i,j=1}^{n} N_{,ij}\delta\phi_i\delta\phi_j \qquad N_{,i} \equiv \frac{\partial N^{\mathrm{f}}}{\partial \phi_i^{\mathrm{i}}}, N_{,ij} \equiv \frac{\partial^2 N^{\mathrm{f}}}{\partial \phi_i^{\mathrm{i}} \partial \phi_j^{\mathrm{i}}}$$

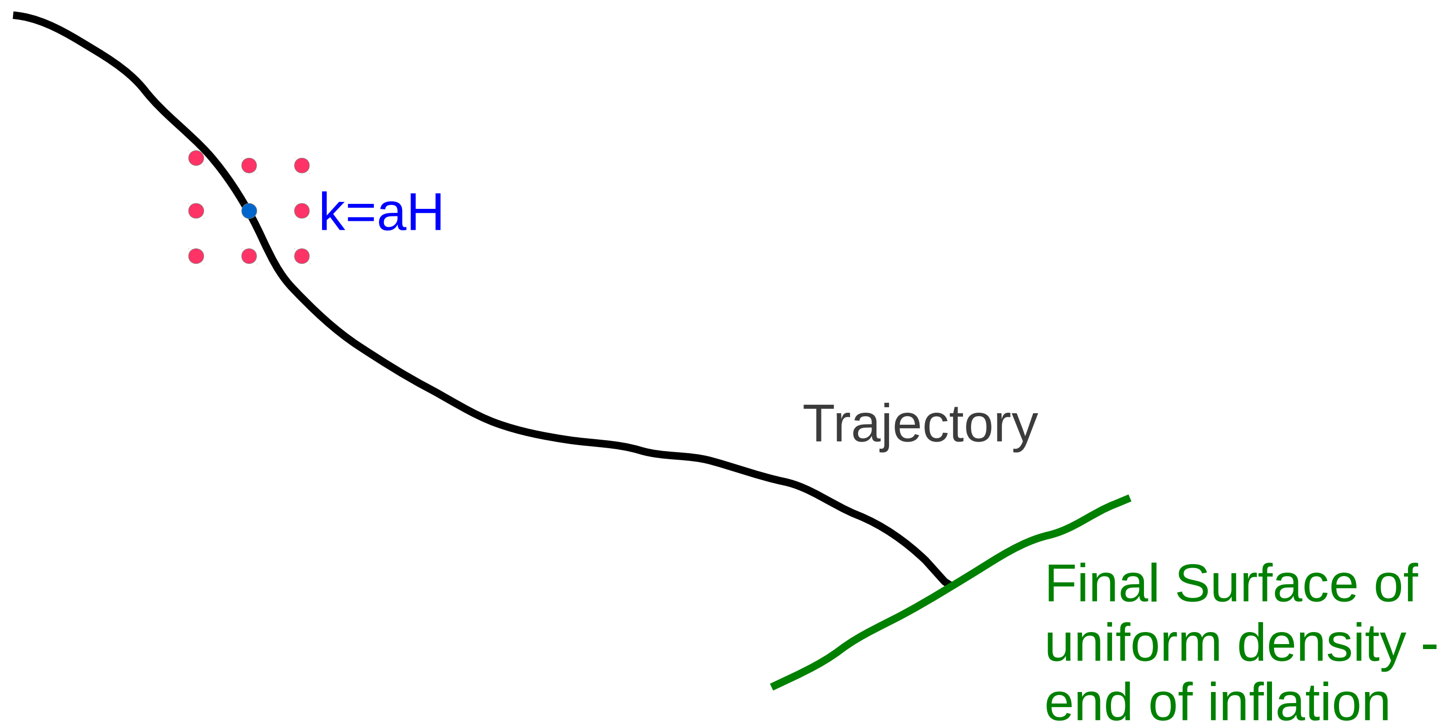
- Level of (local) non-gaussianities: $f_{\mathrm{NL}} = -\frac{5}{6}\frac{\sum_{i,j} N_{,i}N_{,j}N_{,ij}}{\left(\sum_i N_{,i}^2\right)^2}$
- Power spectrum Amplitude: $A_\zeta^2 = \frac{H_*^2}{4\pi}\sum_i N_{,i}^2$
- Spectral index: $n_{\mathrm{s}} - 1 = -2\epsilon_{1*} + \frac{2\sum_{ij}\dot{\phi}_{i*}N_{,j}N_{,ij}}{H_*\sum_i N_{,i}^2}$

ONLY needs the background dynamics

2. δN formalism vs. linear multi-field perturbations

2.1. δN formalism:

- Numerical Implementation:
  1. 1st Integration of the background dynamics, to find N_{end}
  2. 2nd Integration, up to k_p=aH
  3. Take a grid of 9 initial conditions around this point
  4. Integration until the final surface of uniform density – find N
  5. Calculate $N_{,i}$ and $N_{,ij}$
  6. Compute f_{NI}, A_s, n_s



2. δN formalism vs. linear multi-field perturbations

2.2. Linear Theory of multi-field perturbations:

- Metric (longitudinal gauge): $\mathrm{d}s^2 = a^2\left[-\left(1+2\Phi\right)\mathrm{d}\eta^2 + \left(1-2\Psi\right)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\right]$
- First order Einstein equations and perturbed Klein Gordon equation
- Quantification of field perturbations → Initial conditions (k>>aH)
- **Perturbations orthogonal to the trajectory can source the curvature perturbations**
- Snapshot of the equations...

2. δN formalism vs. linear multi-field perturbations

2.2. Linear Theory of multi-field perturbations:

$$-3\mathcal{H}(\Phi' + \mathcal{H}\Phi) + \nabla^2\Phi = \frac{4\pi}{m_{\mathrm{p}}^2}\sum_{i=1}^{n}\left(\phi_i'\delta\phi_i' - \phi_i'^2\Phi + a^2\frac{\partial V}{\partial\phi_i}\delta\phi_i\right)$$

$$\Phi' + \mathcal{H}\Phi = \frac{4\pi}{m_{\mathrm{p}}^2}\sum_{i=1}^{n}\phi_i'\delta\phi_i \ ,$$

$$\Phi'' + 3\mathcal{H}\Phi' + \Phi\left(2\mathcal{H}' + \mathcal{H}^2\right) = \frac{4\pi}{m_{\mathrm{p}}^2}\sum_{i=1}^{n}\left(\phi_i'\delta\phi_i - \phi_i'^2\Phi - a^2\frac{\partial V}{\partial\phi_i}\delta\phi_i\right)$$

$$\delta\phi_i'' + 2\mathcal{H}\delta\phi_i' - \nabla^2\delta\phi_i + \sum_{j=1}^{n}a^2\delta\phi_j\frac{\partial^2 V}{\partial\phi_i\partial\phi_j} = 2(\phi_i'' + 2\mathcal{H}\phi_i')\Phi + 4\phi_i'\Phi'$$

$$\zeta = \Phi + \frac{\mathcal{H}}{\sigma'^2}\sum_{i=1}^{n}\phi_i'\delta\phi_i$$

$$\zeta' = \frac{2\mathcal{H}}{\sigma'^2}\nabla^2\Phi - \frac{2\mathcal{H}}{\sigma'^2}\left[a^2\sum_{i=1}^{n}\phi_i'\frac{\partial V}{\partial\phi_i} - \frac{a^2}{\sigma'^2}\left(\sum_{i=1}^{n}\phi_i'\frac{\partial V}{\partial\phi_i}\right)\left(\sum_{i=1}^{n}\phi_i'\delta\phi_i\right)\right]$$

$$= \frac{2\mathcal{H}}{\sigma'^2}\nabla^2\Phi - \frac{2\mathcal{H}}{\sigma'^2}\perp_{ij}a^2\frac{\partial V}{\partial\phi_i}\delta\phi_j \ ,$$

$$v_i = \sqrt{2}ak^{3/2}\delta\phi_i$$

$$\lim_{k/aH\to+\infty} v_{k,i}(\eta) = \frac{\sqrt{8\pi}}{m_{\mathrm{p}}}k\mathrm{e}^{-ik(\eta-\eta_{\mathrm{i}})}$$

2. δN formalism vs. linear multi-field perturbations

2.2. Linear Theory of multi-field perturbations:

- Numerical Implementation:

  1. 1st integration of the background – to find N_{end}
  2. Loop over the wavelength modes
  3. Integration of the background, up to k = C aH, with C>>1
  4. Loop over the fields
  5. Fix the initial conditions for the perturbations for the field *i*
  5. Integration of the Background + Perturbations until the end of inflation
  6. Compute the exact power spectrum of curvature perturbations

$$\mathcal{P}_{ab} = \frac{k^3}{2\pi^2} \sum_{m=1}^{n_\sigma} \left[\nu_m^a(k)\right]^* \left[\nu_m^b(k)\right]$$

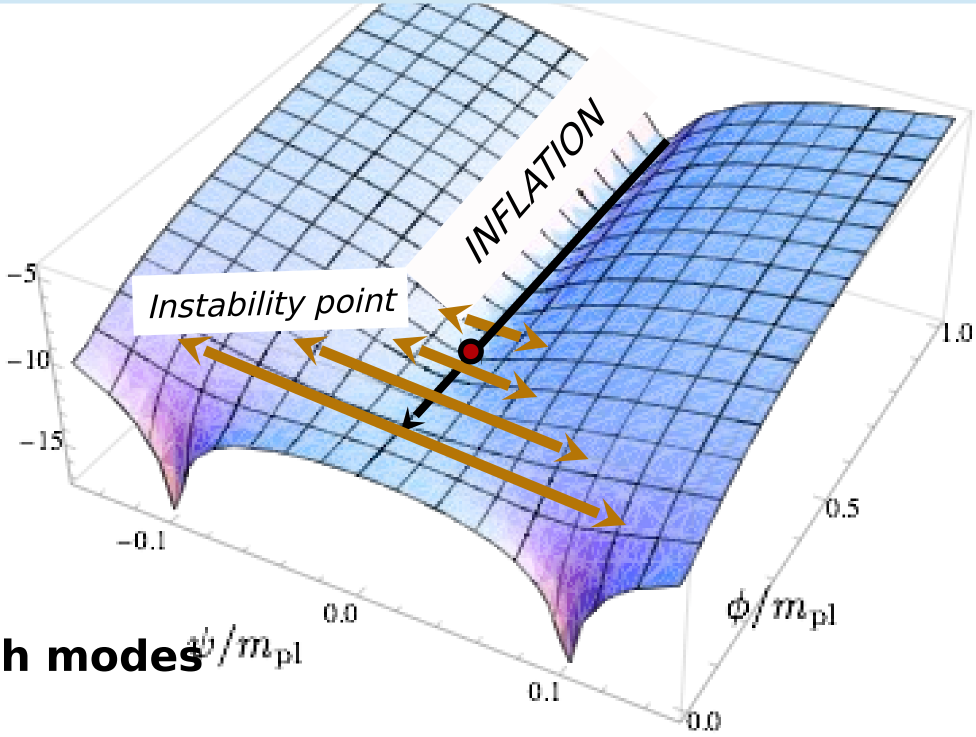
2. δN formalism vs. linear multi-field perturbations

δN formalism	Multi-field perturbations
Background dynamics only	Background + Perturbations
Several assumptions	Exact power spectrum
Straightforward calculation of the level of non-gaussianities	Possibility to follow the mode evolution and to separate the contributions from adiabatic/entropic perturbations
Numerical implementation: • Easy • Fast	Numerical implementation: • Not so easy • Rather slow

4. The waterfall phase

WATERFALL PHASE

- **Initially,** $\bar{\psi} = 0$ **and** $\psi(x,t) = \delta\psi(x)$
- **Inflation driven by** ϕ
- $H \simeq \text{cst}$ **and** $N \simeq Ht$



- **After Fourier mode expansion,**

$$\delta\ddot{\psi}_k + 3H\delta\dot{\psi}_k + \left[\frac{k^2}{a^2} + m_\psi(\phi)^2\right] = 0$$

- **Exponential growth of long wavelength modes**

- **In typically less than 1 e-fold:**

→ **Linear theory no longer valid (need of lattice simulations),**

→ **Long wavelength fluctuations interpreted as classical waves,**

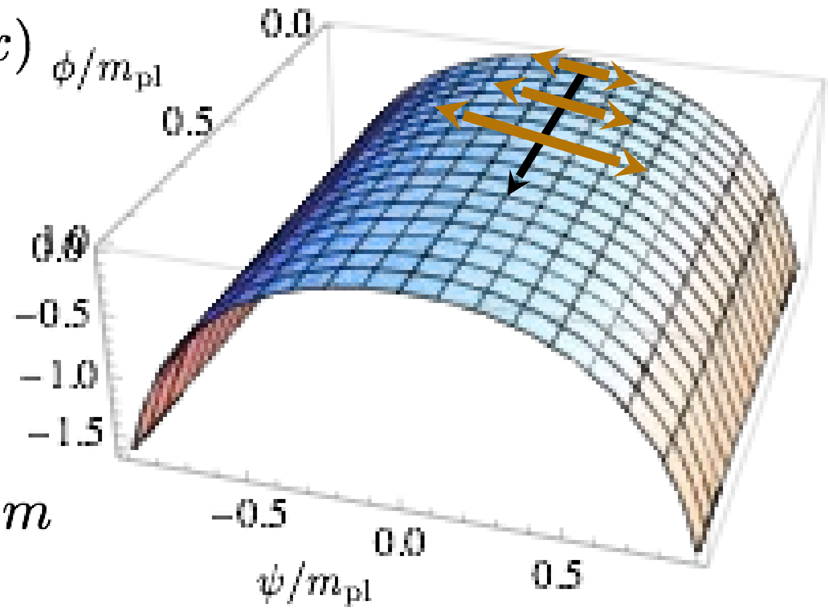
→ **Rapid energy transfer of the homogeneous scalar field into the energy of inhomogeneous oscillations**

TACHYONIC PREHEATING

4. The waterfall phase

WATERFALL PHASE - Case of Study

- **Initially,** $\bar{\psi} = 0$ **and** $\psi(x,t) = \delta\psi(x)$
- **Inflation driven by** ϕ
- $H \simeq \mathrm{cst}$ **and** $N \simeq Ht$



- **After Fourier mode expansion,**

$$\delta\ddot{\psi}_k + 3H\delta\dot{\psi}_k + \left[\frac{k^2}{a^2} - m^2\right] = 0$$

- **For long wavelength modes,** $k/a \ll m$

$$\delta\psi_k \propto \mathrm{e}^{-\frac{3}{2}Ht(1-\sqrt{1+4m^2/9H^2})}$$

- **In the regime,** $9H^2 \ll 4m^2$ **exponential growth** $\delta\psi_k \propto \mathrm{e}^{mt}$

Nearly instantaneous waterfall phase

- **In the regime** $4m^2 \ll 9H^2$ **, one has** $\delta\psi_k \propto \mathrm{e}^{Htm^2/3H^2}$

Long phase of inflation before the mode explosion $N \simeq Ht > 3H^2/m^2 \gg 1$

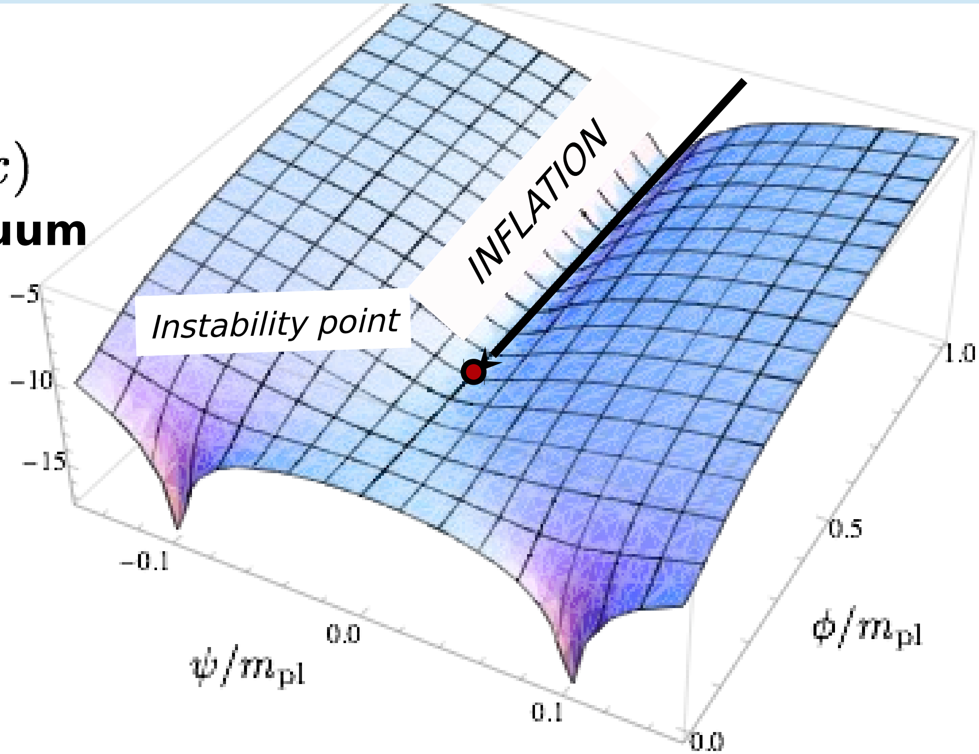
4. The waterfall phase

WATERFALL PHASE - Hybrid inflation

- **Initially,** $\bar{\psi} = 0$ **and** $\psi(x,t) = \delta\psi(x)$
- **Inflation driven by** ϕ **in the false vacuum**
- $H \simeq \text{cst}$ **and** $N \simeq Ht$

- **After Fourier mode expansion,**

$$\delta\ddot{\psi}_k + 3H\delta\dot{\psi}_k + \left[\frac{k^2}{a^2} + m_\psi(\phi)^2\right] = 0$$



In the usual regime,
the transition between $m_\psi^2 = 0$
and $-m_\psi^2 > H^2$
is nearly instantaneous (N < 1).

Does it exist a regime
for which N >60 between $m_\psi^2 = 0$
and $-m_\psi^2 > H^2$ **?**

YES...

Numerically for the original model: S.C., arXiv:1104.3494
Analitically: H. Kodama et al., arXiv:1102.5612
For the F-term and D-term Models: S.C, B. Garbrecht, arXiv:1204.3540

5. Hybrid models – Instantaneous waterfall regime

ORIGINAL MODEL - PHASE OF INFLATION

- Along the valley $\psi = 0$ **, effective potential** $V(\phi) = \Lambda^4 \left(1 + \frac{\phi^2}{\mu^2}\right)$

- In the false vacuum regime $\phi \ll \mu$

- Slow-roll parameters in the slow-roll approximation:

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} \simeq \frac{m_\mathrm{p}^2}{16\pi}\left(\frac{V'}{V}\right)^2 = \frac{m_\mathrm{p}^2}{4\pi\mu^2}\frac{\left(\frac{\phi}{\mu}\right)^2}{\left[1+\left(\frac{\phi}{\mu}\right)^2\right]^2} \ll 1$$

$$\epsilon_2 \equiv \frac{\mathrm{d}\ln\epsilon_1}{\mathrm{d}N} \simeq \frac{m_\mathrm{p}^2}{4\pi}\left[\left(\frac{V'}{V}\right)^2 - \frac{V''}{V}\right] = \frac{m_\mathrm{p}^2}{2\pi\mu^2}\frac{\left(\frac{\phi}{\mu}\right)^2 - 1}{\left[1+\left(\frac{\phi}{\mu}\right)^2\right]^2} < 0$$

- Scalar spectral index: $n_\mathrm{s} = 1 - 2\epsilon_{1*} - \epsilon_{2*} > 1$

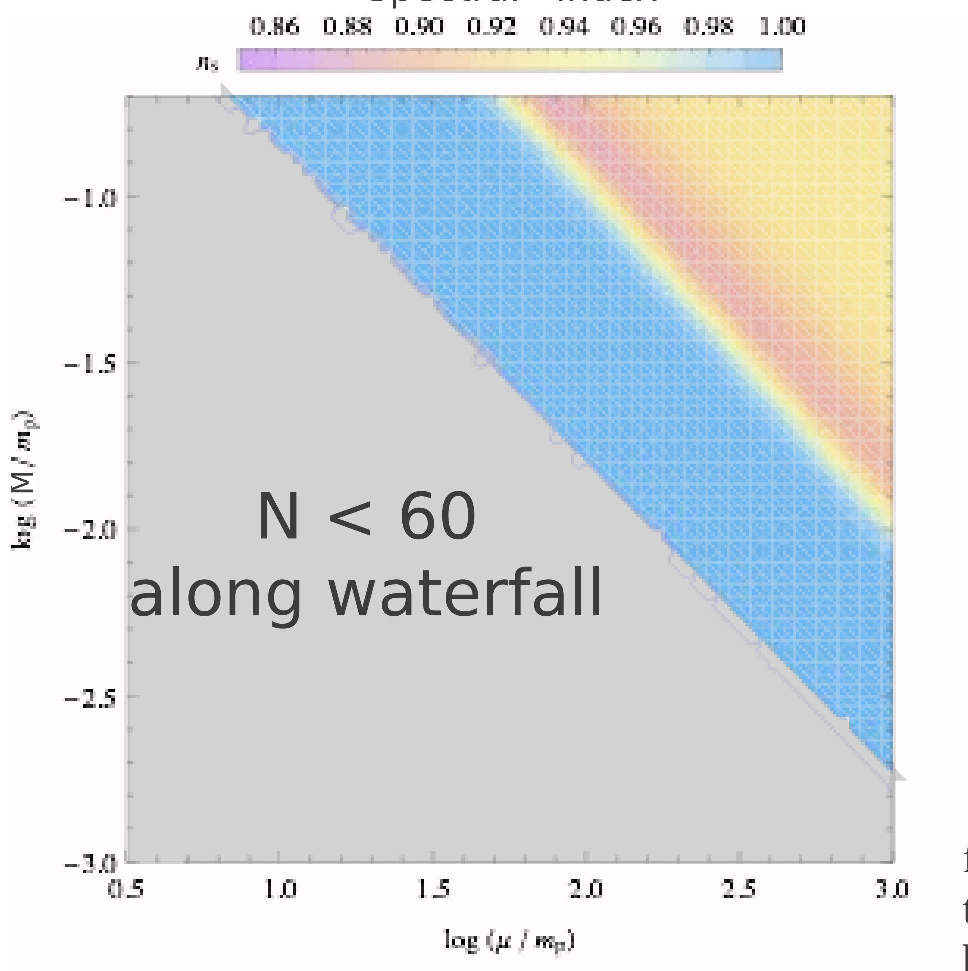
BLUE power spectrum strongly disfavored by CMB observations

Formation of domain walls at the end of inflation

6. Origininal Hybrid model – Mild waterfall regime

S.C., arXiv:1104.3494

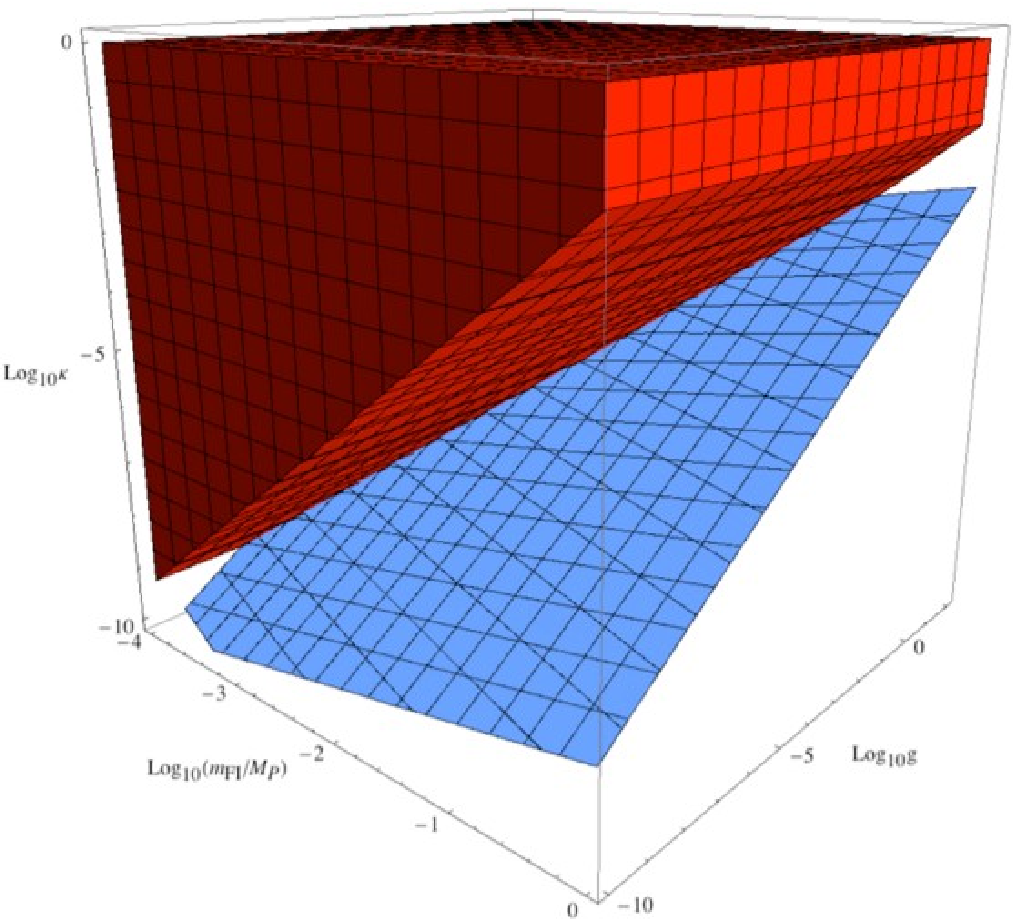
- **Power spectrum (spectral index) of adiabatic perturbations**
- **Possible agreement with CMB constraints**
- **but entropic modes contribution not yet included**

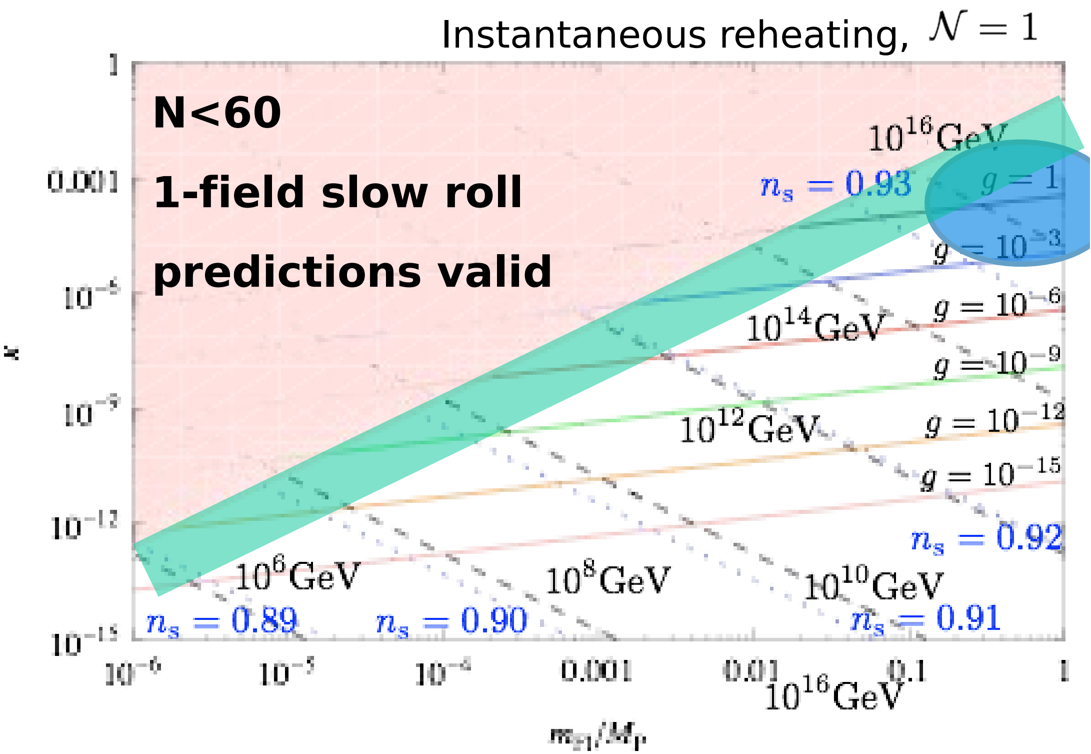


for a pivot scale leaving
the Hubble radius 60 e-folds
before the end of inflation

7. F-term and D-term: Mild waterfall regime

Parameter Space Analysis for D-term:





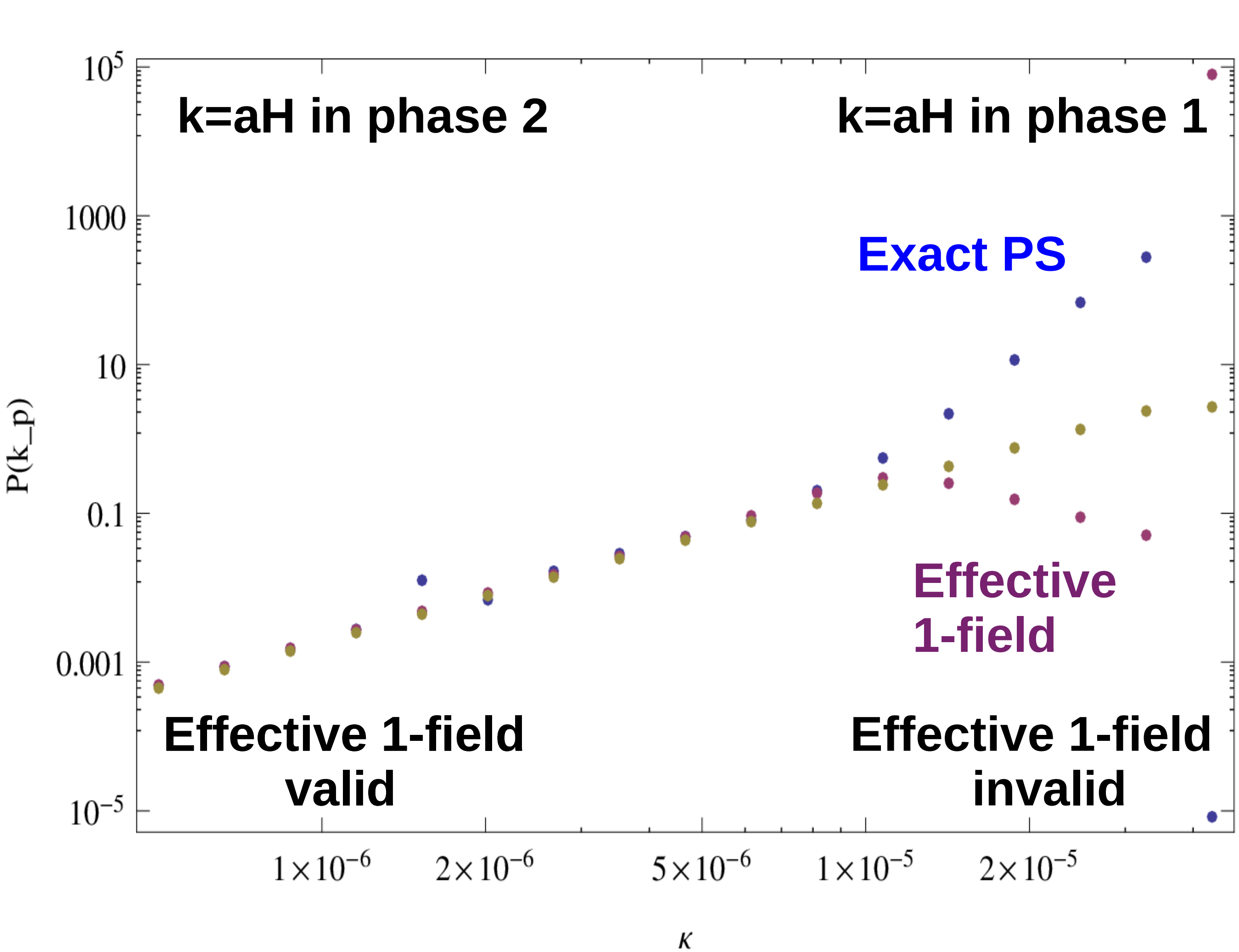
k=aH in phase 2:   strong tension with CMB observations provided $g \approx 1, m_{\rm FI} \approx M_{\rm p}, \kappa \approx 10$

k=aH in phase 1: when $N \gtrsim 60$ **the spectral index goes from** $n_{\rm s} = 1 - \frac{4}{N_{\rm e}}$ **to unity**

Possible agreement with CMB

9. Power spectrum of curvature perturbations

Numerical results (M=0.05M$_{pl}$ but qualitatively valid for other values)**:**



Effective
1-field
k=aH in phase 2
(extrapolation)

And the spectral index decreases when k=aH in phase 1 instead of increasing up to unity