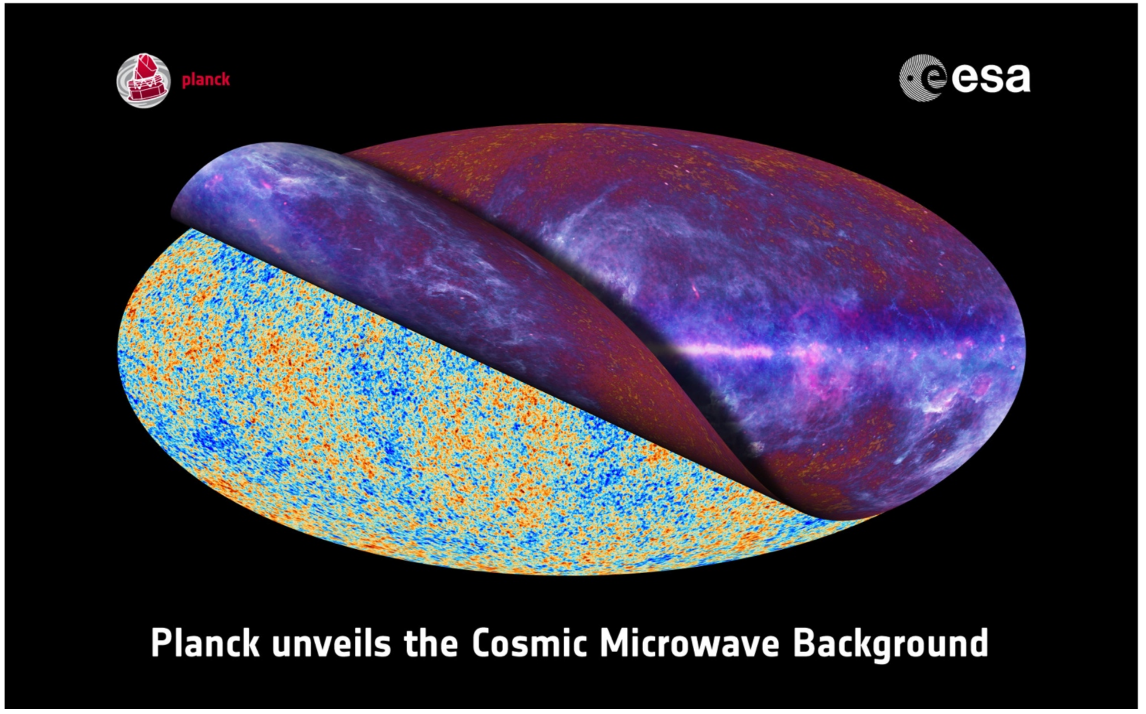
Semi-parametric approach to CMB cleaning

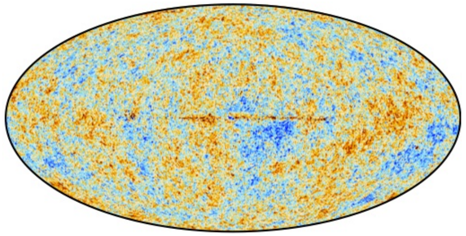
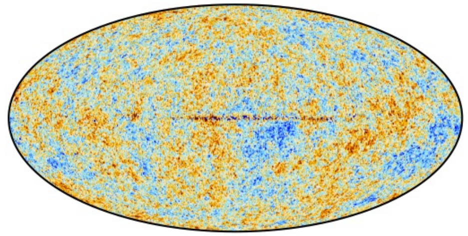
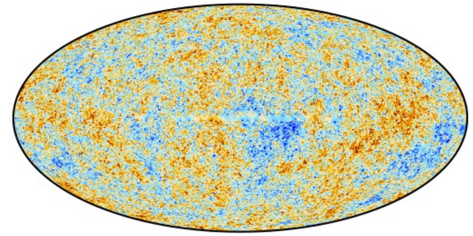
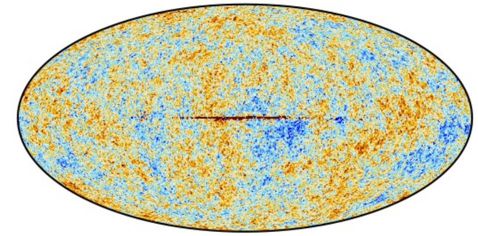


J.-F. Cardoso, CNRS (Télécom-ParisTech, IAP, APC)

on behalf on the Planck collaboration

47th ESLAB symposium, ESA/ESTEC, Noordwijk, The Netherlands, 2-5 April 2013.

Four CMB anisotropy maps delivered on March 21st to the Planck Legacy Archive

NILC	SEVEM	SMICA	C-R
$\ell_{\text{max}} = 3200$	$\ell_{\text{max}} = 3100$	$\ell_{\text{max}} = 4000$	Pixel-based
5 arc-min	5 arc-min	5 arc-min	$\sim$ 7 arc-min
$\ell_{\text{SNR}=1} = 1790$	$\ell_{\text{SNR}=1} = 1790$	$\ell_{\text{SNR}=1} = 1790$	$\ell_{\text{SNR}=1} = 1550$
non-parametric	non-parametric	semi-parametric	parametric

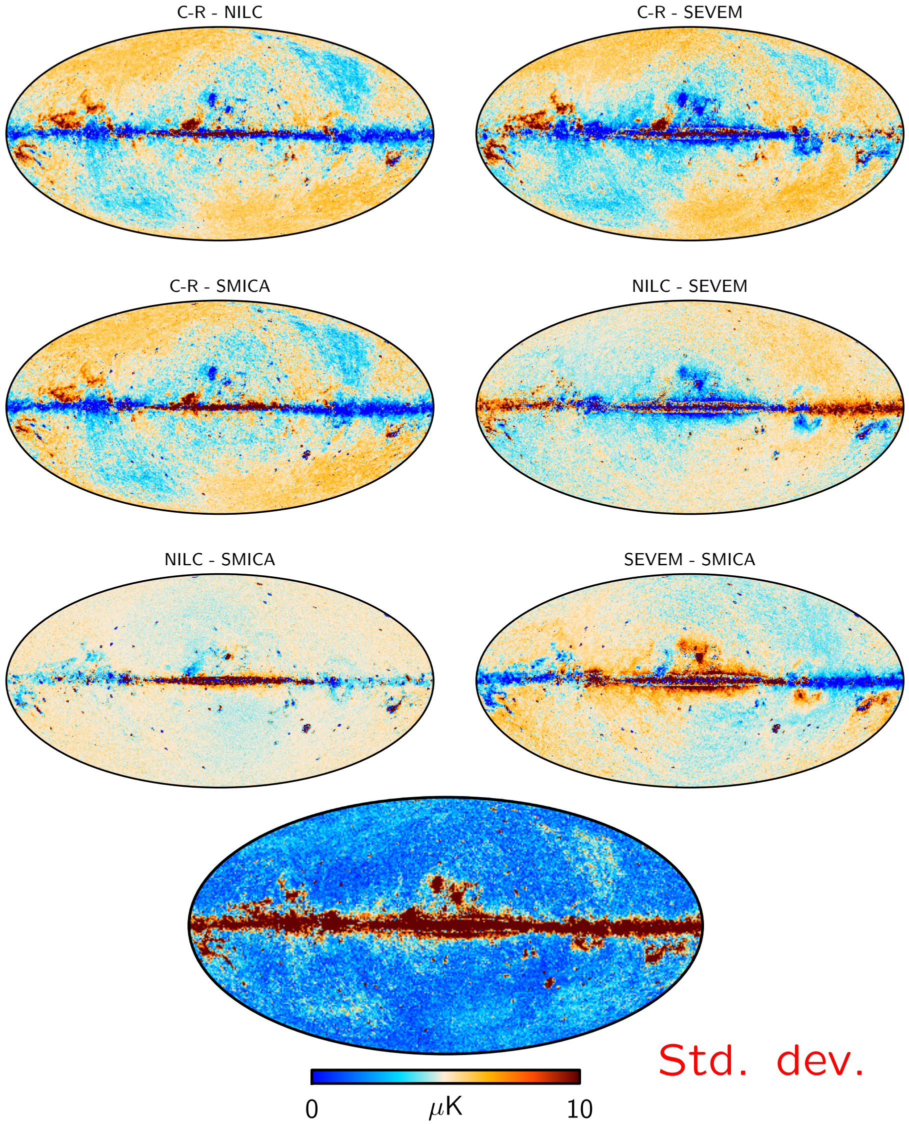
The SMICA product selected as the ‘Main product’ for CMB map.

This talk complements Mark Ashdown’s with focus on the SMICA pipeline:

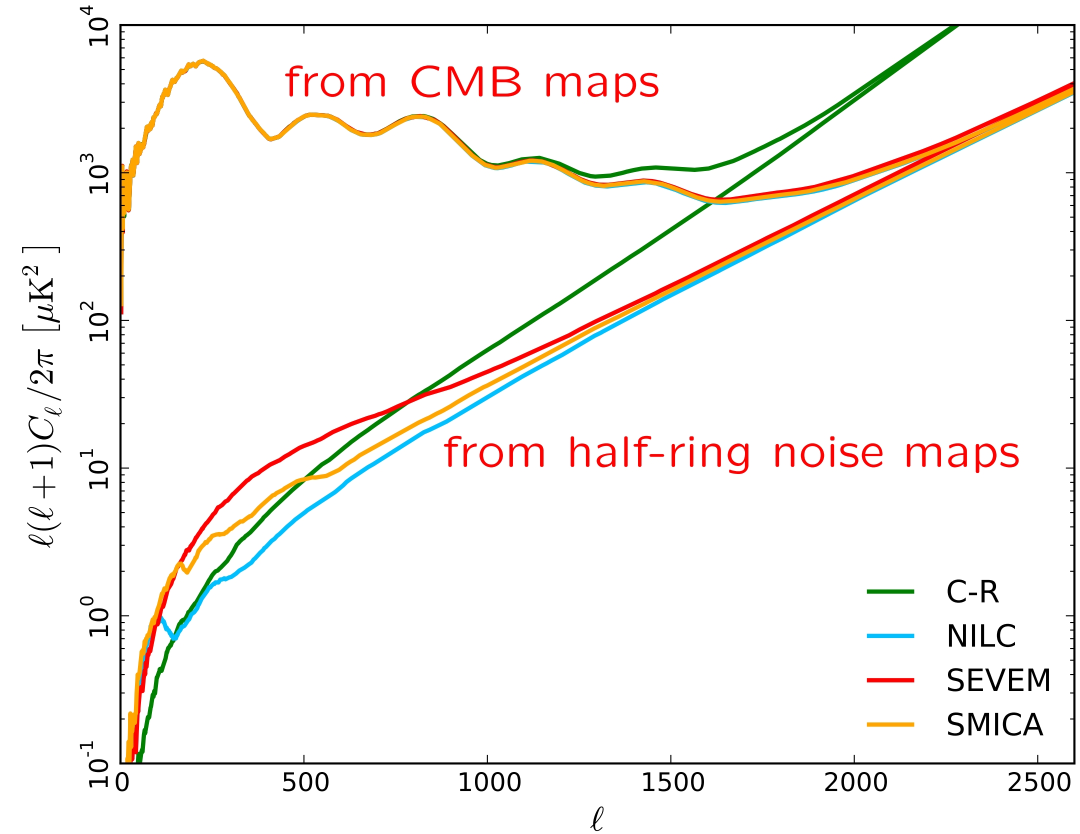
- More method comparison based on Planck data and simulations.
- More about SMICA characterization, operation and principles.
- Comments about the “semi-parametric” approach...

A simple comparison at large scale and fine scales.

Pairwise diff. ($N_{\text{side}} = 128, \Delta T \pm 30 \mu K$).



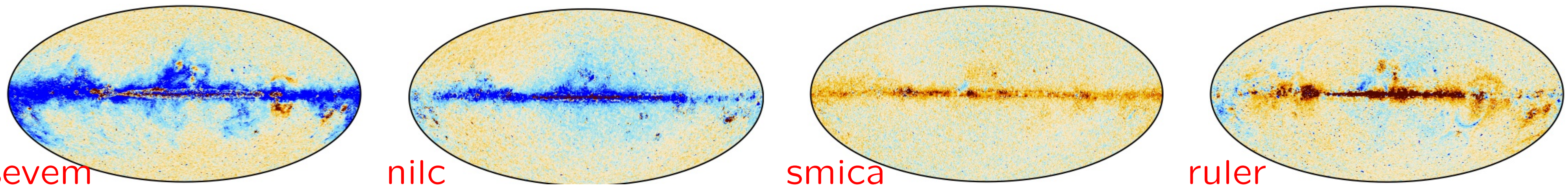
Std. dev.



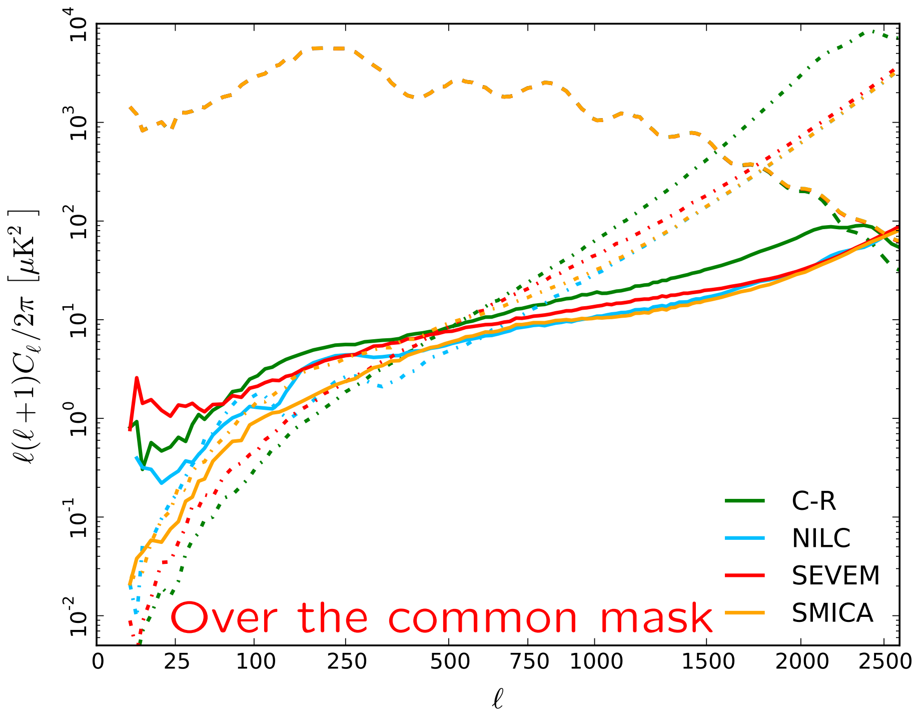
Angular spectra over the common mask.

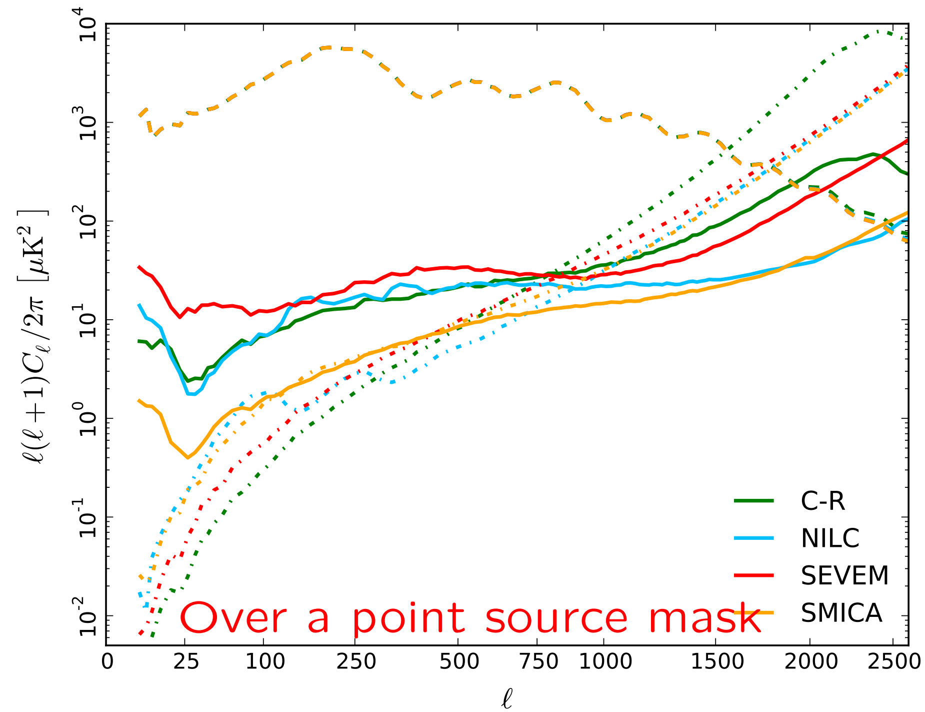
Comparison on the FFP6 simulations

- Large scale residuals ($N_{\mathrm{side}} = 128$. Color scale: $\pm 30\mu K$).



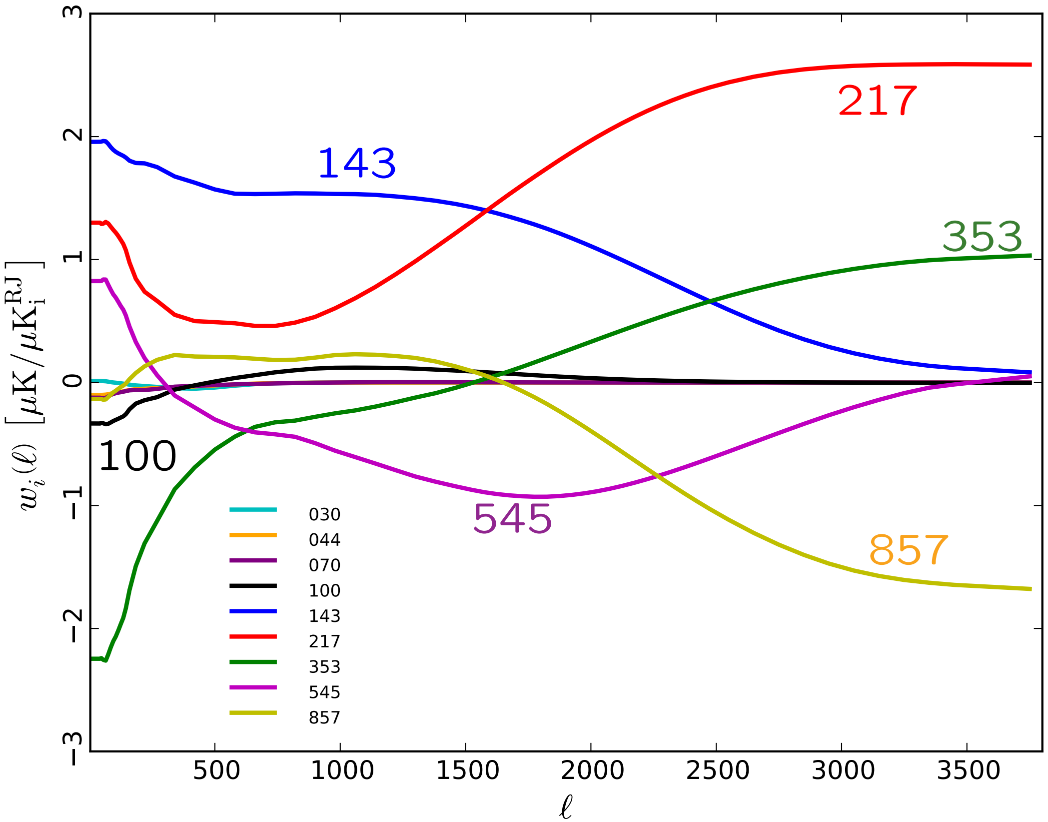
- Propagation of CMB, foregrounds, noise through each pipeline.



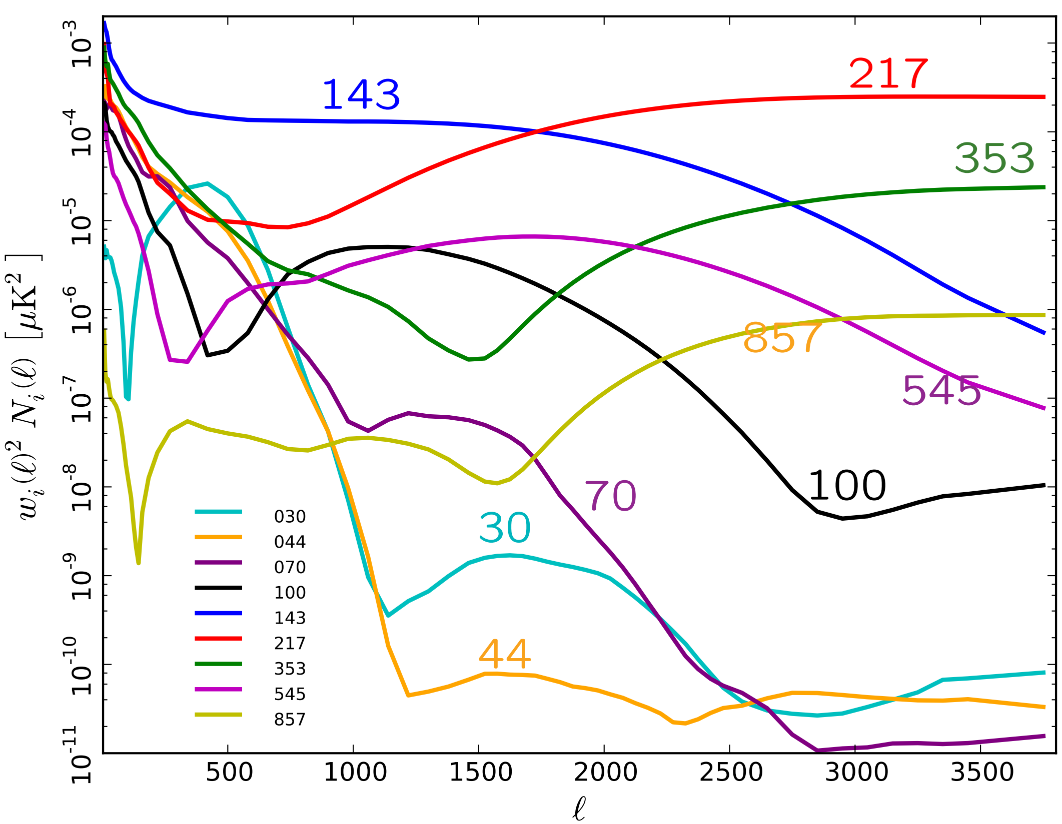


Dashed: CMB, Dotted: noise, Solid: foregrounds.

What SMICA does to signal. . . and to noise



SMICA channel weighting vs multipole

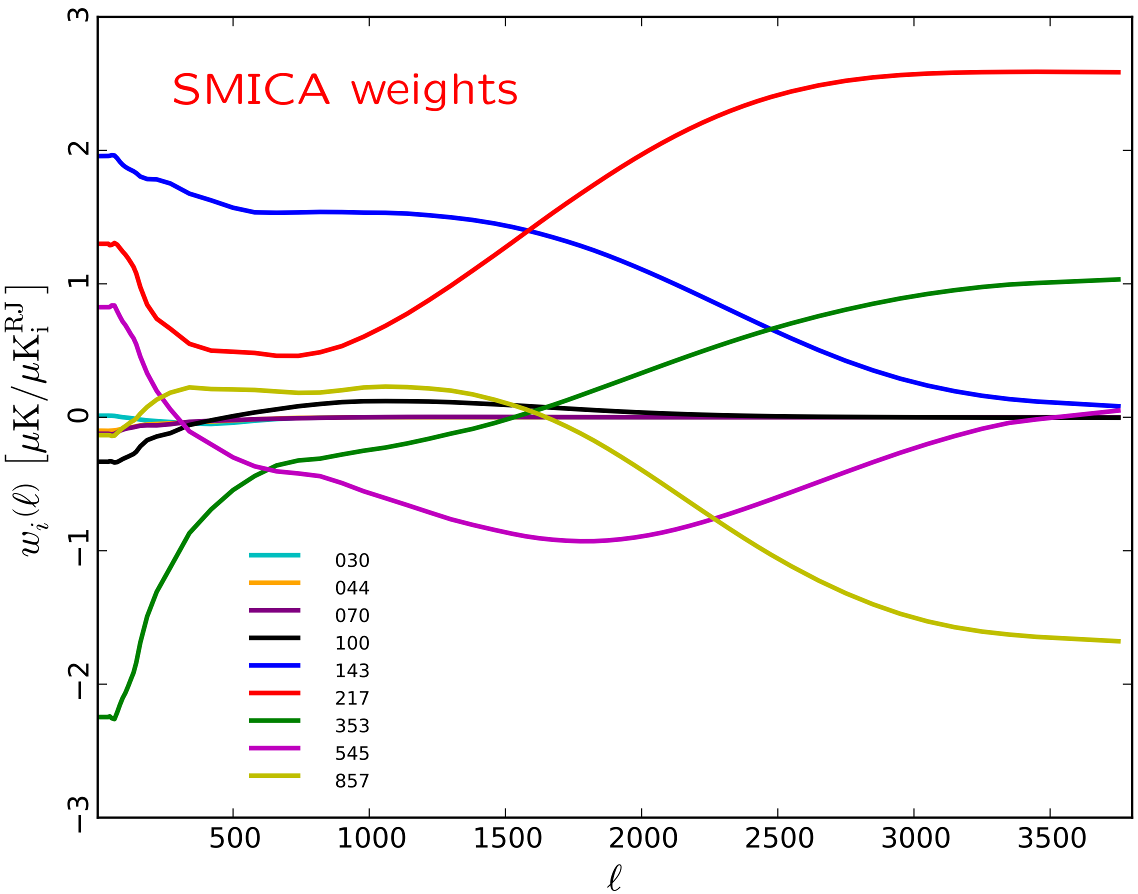


Noise contribution from each channel

The data (and common sense) are telling us to let the weights depend on angular frequency.

They do not strongly advise us to let them vary with position (See NILC performance).

SMICA filtering (where do those weights come from?)



Combine channels in harmonic space:

$$\widehat{s}_{\ell m} = \mathbf{w}_\ell^\dagger \mathbf{d}_{\ell m}$$

Assume coherent CMB:

$$\mathbf{d}_{\ell m} = \mathbf{a}\, s_{\ell m} + \text{contamination}_{\ell m},$$

Best weights for known $\mathbf{C}_\ell = \mathrm{Cov}(\mathbf{d}_{\ell m})$:

$$\mathbf{w}_\ell = \frac{\mathbf{C}_\ell^{-1}\mathbf{a}}{\mathbf{a}^\dagger \mathbf{C}_\ell^{-1}\mathbf{a}}$$

- But spectral matrix $\mathbf{C}_\ell$ is unknown...

$\longrightarrow$ At high ℓ, fear not and take

$$\widehat{\mathbf{C}}_\ell = \frac{1}{2\ell+1}\sum_m \mathbf{d}_{\ell m}\mathbf{d}_{\ell m}^\dagger$$

$\longrightarrow$ At low ℓ, model $\mathbf{C}_\ell(\theta)$ and fit

$$\mathbf{C}_\ell(\widehat{\theta}) = \max_\theta P(\widehat{\mathbf{C}}_\ell | \mathbf{C}_\ell(\theta))$$

SMICA semi-parametric model

- SMICA models the 9 Planck channels as noisy linear mixtures of CMB and 6 "foregrounds":

$$\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ \vdots \\ d_9 \end{bmatrix} = \begin{bmatrix} a_1 & F_{11} & \dots & F_{16} \\ a_2 & F_{21} & \dots & F_{26} \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ a_9 & F_{91} & \dots & F_{96} \end{bmatrix} \times \begin{bmatrix} s \\ f_1 \\ \vdots \\ f_6 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ \vdots \\ n_9 \end{bmatrix} \quad \text{or} \quad \mathbf{d}_{\ell m} = [\,\mathbf{a} \mid \mathbf{F}\,] \begin{bmatrix} s_{\ell m} \\ \mathbf{f}_{\ell m} \end{bmatrix} + \mathbf{n}_{\ell m}$$

- SMICA only uses the decorrelation between foregrounds and CMB.

The foregrounds must have 6 dimensions but are otherwise completely unconstrained: they may have any spectrum, any color, any correlation...

So the data model is very blind: all non-zero parameters are free!

$$\mathrm{Cov}(\mathbf{d}_{\ell m}) = [\,\mathbf{a} \mid \mathbf{F}\,] \begin{bmatrix} C_\ell^{\mathrm{cmb}} & 0 \\ 0 & \mathbf{P}_\ell \end{bmatrix} [\,\mathbf{a} \mid \mathbf{F}\,]^\dagger + \begin{bmatrix} \sigma_{1\ell}^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_{9\ell}^2 \end{bmatrix} = \mathbf{C}_\ell(\mathbf{a}, C_\ell^{\mathrm{cmb}}, \mathbf{F}, \mathbf{P}_\ell, \sigma_{i\ell}^2).$$

- Blind identifiability: can it be done? Maths say: yes!

If no foreground combination can mimick the CMB angular spectrum, then the semi-parametric elements $\mathbf{a}\, s_{\ell m}$ and $\mathbf{F}\ \mathbf{f}_{\ell m}$ are uniquely fitted.

Foregrounds, physical components and the mixing matrix

- Mixing matrix. The 9 Planck channels as noisy linear mixtures of components:

$$\mathbf{d}_{\ell m} = \mathbf{A}(\theta)\ \mathbf{s}_{\ell m}\ +\ \mathbf{n}_{\ell m}$$

- Some models for the mixing matrix $\mathbf{A} = \mathbf{A}(\theta)$:

Type	Mixing matrix	parameters θ	dim(θ)
physical, fixed	$\mathbf{A} = [\mathbf{a}_{\text{cmb}}\ \mathbf{a}_{\text{dust}}\ \mathbf{a}_{\text{CO}}\ \mathbf{a}_{\text{LF}}]$	$\theta = [\ ]$	0
physical, parametric	$\mathbf{A} = [\mathbf{a}_{\text{cmb}}\ \mathbf{a}_{\text{dust}}(T)\ \mathbf{a}_{\text{CO}}\ \mathbf{a}_{\text{LF}}(\beta)\ ]$	$\theta = (T, \beta)$	2
equivalent to ILC	$\mathbf{A} = [\mathbf{a}_{\text{cmb}}\ \mathbf{B}]$ (a square matrix)	$\theta = \mathbf{B}$	$N_{\text{chan}} \times (N_{\text{chan}} - 1)$
semi-parametric, SMICA	$\mathbf{A} = \mathbf{A}$ (any tall matrix)	$\theta = \mathbf{A}$	$N_{\text{chan}} \times N_{\text{comp}}$

- Sky-varying emission spectra: Sky-varying emission laws can be accounted for
  - locally by letting $\mathbf{A}$ depend on the pixel: $\mathbf{A}(\theta_{\text{pix}})$ (Commander) or
  - globally by adding columns to $\mathbf{A}$ (SMICA).

For instance, a sky-varying low-frequency emission $\mathbf{a}_{\text{LF}}(\theta_{\text{pix}})$ could be approximatively represented by two fixed columns over the whole sky: $[\ \mathbf{a}_{\text{LF}}(\langle\theta\rangle),\ d\mathbf{a}_{\text{LF}}/d\theta(\langle\theta\rangle)\ ]$

Summary/Conclusions

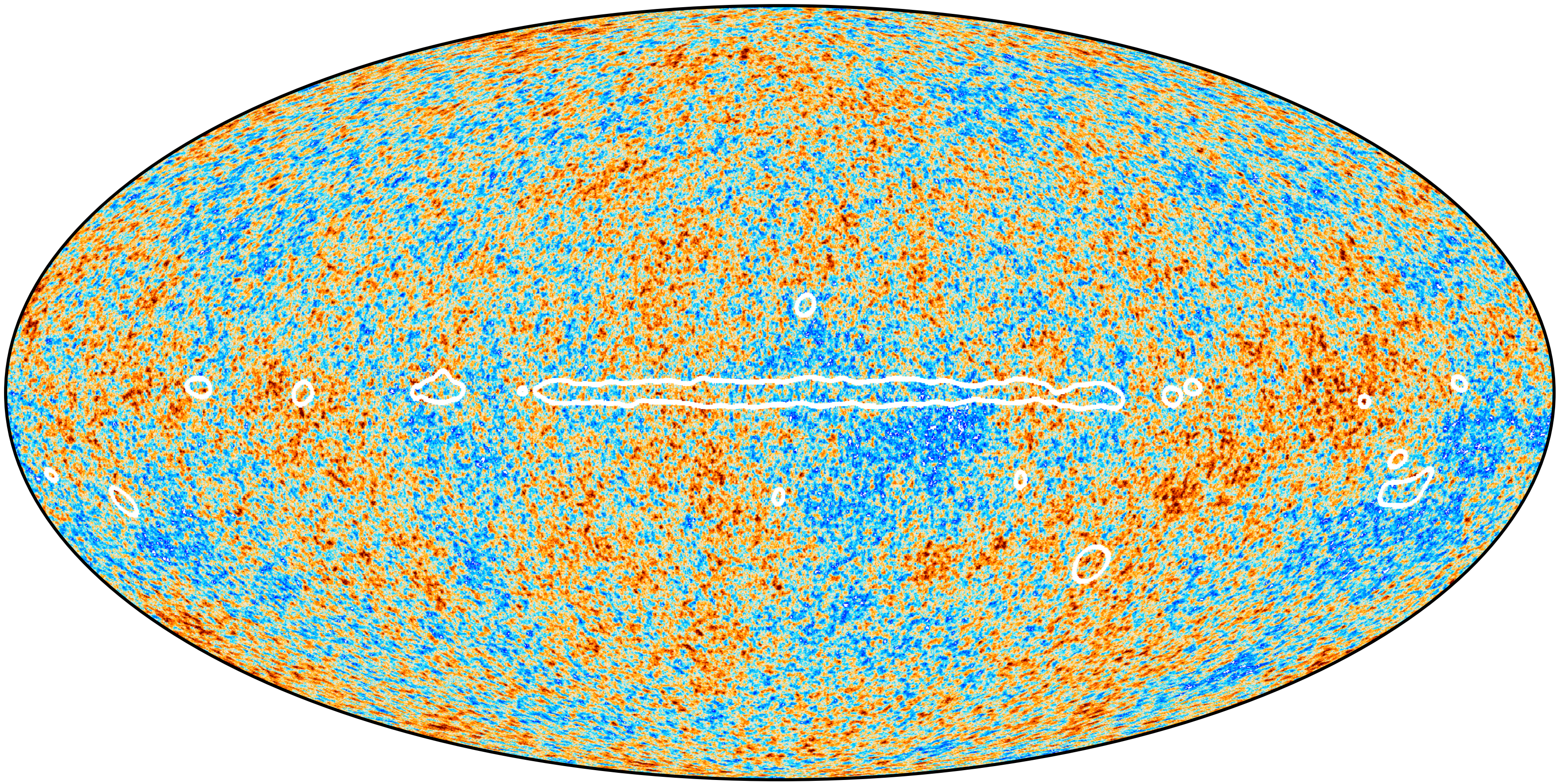
- SMICA equivalent to fitting CMB and foregrounds with arbitrary morphologies, spectra and inter-correlations. Totally blind processing (but for the beams).

- The only foreground assumption is that of coherence: they can be represented by, say, 6 templates.

- No prior from physics: let the data speak for themselves. The non-parametric model seems to live in a sweet spot between too many constraints (physical modeling) and not enough (ILC and similar non-parametric methods).

- Thanks to ESA and to the Planck collaboration for amazing data and to the foregrounds for living in a 6-dimension subspace (well, almost...).

Update: inpainted CMB maps delivered for SMICA and NILC (end of March)



Inpainting of 3% of the sky: 'large' bright regions (shown here) plus the masked point sources.

The inpainted SMICA map was used for PR. Good scientific value too.